THE ELEMENTS OF

APPLIED ELECTRICITY

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BY

BHIM CHANDRA CHATTERJEE,
B. A., B. L., B. Sc., A. M. I. E. E.
HEAD OF THE DEPARTMENT OF ELECTRICAL ENGINEERING,
BENARES HINDU UNIVERSITY.

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श्रीकृष्णार्पणमस्तु ।

PREFACE.

This book contains in as compact a form as is consistent with clear exposition, a resumé of those important conclusions in Electrical Engineering that are generally admitted by the Heads of the Profession throughout the world to be correct. There is therefore no claim to originality as regards the matter. As regards the exposition, the theory as a whole is exhibited in what appears to the author to be a logical sequence, and is clenched at every suitable point by a number of worked examples. A large number of examples is offered which the student may, and should work for himself.

A long experience of teaching and practice has convinced the author that it is not possible to learn the art and science of Engineering from books only. They are to be regarded as a useful and indeed indispensable auxiliary to the spoken word of an energetic teacher, and to much practice in the workshop and drawing office.

The omission of matter best apprehended from lectures or from practice in the drawing office and workshop is deliberate. The reader is hereby warned that he cannot become an Engineer merely by reading this or any other book.

In conclusion the following acknowledgements must be made :—

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It gives me great pleasure to express my thanks to my colleagues, Professor L.D. Coueslant, Head of

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CONTENTS.

CHAPTER I.

THE MAGNETIC EFFECTS OF A CURRENT.

CHAPTER II.

ELECTROMAGNETS.

Elementary Propositions in Electromagnetics.

Applications of the Law of Magnetic Circuit.

CHAPTER III.

Hysteresis and Eddy Current.

Hysteresis Loss.

CHAPTER IV.

Direct Current Generators.

STARTING, STOPPING AND RUNNING GENERATORS IN SERIES AND PARALLEL.

CHAPTER V.

DIRECT CURRENT MOTORS.

EFFICIENCY.

EFFICIENCIES OF DIFFERENT TYPES OF MOTORS.

CHAPTER VI.

ALTERNATING CURRENT.

CHAPTER VII.

INDUCTANCE.

CHAPTER VIII.

CAPACITANCE.

ERRATA.

			Page.	Line.
Considered	for	con i dered	... 8	18
...x 2.54 x...	,,	...x .54 x	... 10	1
so	,,	also	... 113	11
omit ; so	—	—	do	do
Friction	for	Fraction	... 140	19
watts	,,	volts	... 144	20
(Fig.4.27)	,,	(Fig.4.26)	... 153	do
shut	,,	shunt	... 156	16
reduced	,,	reduce	... 167	10
Revolution	,,	Revoluion	... 172	27
$\frac{2\ p\phi xn}{...}$	,,	$\frac{2p\phi}{...}$	... 173	2
20/0.8 × 8	,,	20/0.08	... 177	19
P	,,	p	... 181	8
S	,,	N	... 183	10
220	,,	22	... 186	5
25	,,	·25	... 190	17
5500	,,	44	... 191	1
$\frac{...(.078+.09)}{220\times 25}$		$\frac{...(.078\times .09)}{220\times 25}$	... 192	16
$3n^2$	,,	$2n^2$	... 195	7
$\frac{...v}{3}$	,,	$\frac{...}{6}$	... 196	4

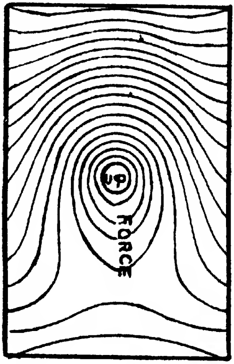
$\frac{\cdots\cdots\cdots\cdots}{510.576 \times 150}$	,,	$\frac{\cdots\cdots}{540.576 \times 150}$	do	15
$=97\%$	,,	$=92\%$		
$E - R_a I_a$	,,	$EI_a - R_a$	199	1
$\frac{\log 1.93 - \log .18}{\cdots}$	,,	$\frac{\log I7 - \log .18}{\cdots}$	216	2
$r_2 - r_3$	,,	$r_2 r - 3$	do	13
reactance	for	rectance	246	13
$1/2\pi f$	,,	$1/2\pi C$	do	14
$E = I\ r$	,,	$Z = Ir$	252	1
.... $\|{-2}\|$	,,	.. .. $\|^2$	265	22
$I = I^m/\sqrt{2}$	,,	$I = I^m/\sqrt{3}$	267	12
574	,,	57.4	291	18
Fig.6 45	,,	Fig. 6.3	314	—
,, 6.39	,,	,,6.25	317	—
when the pressures	,,	whenr the pessures	319	12
difference	,,	deffeerence	do	25
$\frac{\cdots\cdots \times 1 \times 4}{10^9 \times 0.01}$	,,	$\frac{\cdots \times 1 \times 5}{10^9 \times 0.01}$	349	13
·0201 volts	,,	0.005023 volts	do	14
$\frac{\cdots \times 1000 \times 4}{10^9 \times 0.01}$	,,	$= \frac{\cdots \times 100}{10^9 \times 0.01} = 20.096$ volts	do	20
3.19	,,	2.19	356	20
$\log_e$ (......)	,,	log (...)	379	3
$\frac{X^2 i}{R^2}$	,,	$\frac{X^2}{R^2 i}$	379	18

			P.	L.
dl for d		...	397	9
f d l ergs	„	f d f ergs ...	397	11
$t=-\frac{L}{R}[\log_e$	„	$t=-\left[\frac{L}{R}\log_e\right.$ ...	402	5
$=44(1[2.718^{-5})$	for	$=44(1-2.718^{-3})$ ...	403	9
$=i\ (1-e^{\frac{-Rt}{L}})$	„	$=i\ (1-e\frac{-Rt}{L})$ ...	406	15
$-.3182\,t$		$.3182t$	406	22
$V_L=L\frac{di}{dt}=.....\times 100\times L$	for	$V_L=\frac{di}{dt}$	409	18
$C=Q/V$	„	$C=Q/VK$ ...	429	10
$Q/RK;\therefore C=RK,$	„	$R/Q;\therefore C=R/K,$ ...	do	11
if $K=1,$	„	$K=1$ ...	do	do
$E=Q/C+Ri$	„	$=Q/C-Ri$ ...	446	15
$=\int_t^0 dt/RC$	„	$=\int_q^0 dt/RC$ ...	447	11

CHAPTER I.

THE MAGNETIC EFFECTS OF A CURRENT.

1. Force on a Conductor (carrying current) in a Magnetic Field.—When a wire, carrying a current, is placed in a magnetic field, perpendicular to the lines of force, it causes the field to be distorted, and this tends to force the wire in such a direction that the lines shall take up again their normal position (**Principle of Electric Motor**). (Fig. 1·01).



Magnetic lines due to conductor carrying current placed in a magnetic field.

Fig. 1·01

Thus a conductor, carrying a current, when placed in a magnetic field, is repelled from the field, by a certain mechanical force acting at right-angles both to the conductor itself and to the lines of force in the field.

If the field is uniform, the magnitude of this repelling force is found as follows :—

Let H = magnetizing force, or intensity of the field,

L = length of conductor across the field in cm,
I = amperes of current flowing in the conductor,
F = repelling force.

Thus, F (in dynes) $= \frac{H\ L\ I}{10}$. (P. 282, Vol. I).

If the conductor is inclined at an angle α,

$$F = \frac{H\ L\ I \sin \alpha}{10} \qquad \text{.........(1)}$$

Example 1—A copper wire carrying a current of 5 amps. is placed in a magnetic field of 40,000 lines per sq. cm. What is the force in pounds on each centimetre of the wire (a) if it lies perpendicular to the direction of the magnetic field, (b) if it lies parallel to the field, (c) if it makes an angle, α, with the direction of the field?

Solution :—

$$F = \frac{I\ L\ B \sin \alpha}{10},$$

$$\therefore F, \text{ per cm.} = \frac{I\ B \sin \alpha}{10}$$

(a) $\sin \alpha = \sin 90° = 1$
$I = 5$ amp.
$B = 40{,}000$

$$\therefore F, \text{ per cm.} = \frac{5 \times 40{,}000 \times 1}{10} \text{ dynes}$$

$$= 20{,}000 \text{ dynes.}$$

$$= \frac{20{,}000}{981} = 20.4 \text{ grams.}$$

$$=\frac{20.4}{453.6}=.0449 \text{ lb.}$$

(b) $\sin \alpha = \sin 0 = 0$,

$\therefore$ F, per cm. $=0$.

(c) For any angle α,

$$\text{F, per cm.} = \frac{\text{I B} \sin \alpha}{10} \text{ dynes.}$$

$$= \frac{5 \times 40{,}000}{10} \sin \alpha \text{ dynes.}$$

$$= 20{,}000 \sin \alpha \text{ dynes.}$$

$$= .0449 \sin \alpha \text{ lb.}$$

Mutual Action of Currents.

2. Force between two infinitely Long Parallel Conductors.—In a long straight conductor a current i_1 produces a magnetic field $H=2i_1/l$ at a distance l from it, and a second straight conductor, carrying a current i_2, parallel to the first, will experience a force Hi_2 or $2i_1 i_2/l$ per unit length (Fig. 1·02). By Ampere's hand rule it is seen that when the currents are in the same direction the force urges i_2 towards i_1; and when in opposite directions the force drives i_2 away from i_1. Thus in two infinitely long parallel conductors, currents in the same direction attract each other, and those in opposite directions repel each other.

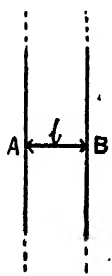


Fig. 1·02

The same conclusion is arrived at by drawing the magnetic field due to the two parallel currents. When the currents are in the same direction the lines or tubes of force surrounding the wires, would, by their tendency to shorten, urge the wires together. When the currents are in opposite directions, there are no tubes of force surrounding both the wires, and since they are more crowded in the space between the wires than in that outside, the lateral pressures of the tubes will urge the wires apart. Thus we deduce the following laws :—

(1) Two PARALLEL currents attract or repel one another according as they flow in the same or in opposite directions.

(2) Two NON-PARALLEL currents attract one another if both approach or both recede from the point of meeting of their directions, while they repel one another if one approaches and the other recedes from that point.

(See Figs. 1·03, 1·04, 1·05.)

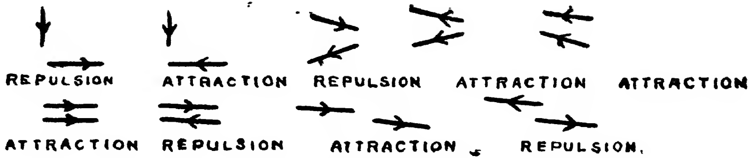


Fig. 1.03

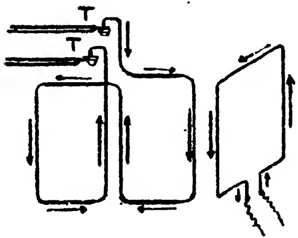


Fig. 1.04

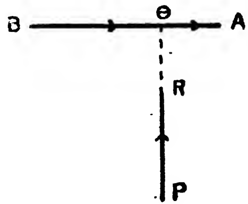


Fig. 1.05

Attraction and repulsion of two parallel and non-parallel currents.

3. Repulsion between Conductor and Return Conductor.—

Let I = current in amperes flowing in a circuit of two parallel conductors, one being a return for the other;

l = distance, in centimetres, between the conductors;

L = inductance;

d = diameter of each conductor in centimetres,

g = acceleration of gravity in C.G.S. units.

Then, the two conductors repel each other by a mechanical force exerted by the magnetic field of the circuit on the current in the conductors.

Now, $$F=\frac{I^2}{2g}\,\frac{dL}{dl}\,10^7 \text{ grms.} \qquad (1)$$

And, the inductance of two parallel conductors, is :—

$$L=\left[4\log_e\left(\frac{2l}{d}\right)+\mu\right]10^{-9} \text{ henrys.} \quad (2)$$

Hence, differentiated $\frac{dL}{dl}=\frac{4\times10^{-9}}{l}$.

$$\therefore \quad F=\frac{I^2}{50gl} \text{ grams.}$$

$$F=\frac{20{\cdot}4\ I^2\ 10^{-6}}{l} \text{ grams.}$$

It will be seen that if the conductors are close together, and the current is very large, as the momentary short-circuit current of a large alternator, the forces may become appreciable.

Example 2.—A 6600-volt 10,000-kw. quarter-phase alternator feeds through single conductor cables having a distance of 20 cm. (8 in.) from each other. A short-circuit occurs in the cables, and the momentary short-circuit current is 12 times full-load current. What is the repulsion between the cables ?

Solution:—

Full-load current is, per phase, 758 amps. Hence, short-circuit current, $I = 12 \times 758 = 9096$ amp., and, $l = 20$ cm. Hence, $F = 84$ grams per cm.
Or multiplied by 30.5/454,

$$F = 5.6 \text{ lb per foot of cable,}$$

that is, pulsating between 0 and 11.2 lb. per foot of cable, and hence, sufficient to lift the cable from its supports and throw it aside.

Problems such as the opening of disconnecting switches under short-circuit, etc. can also be investigated in a similar way.

Example 3.—Two parallel conductors carry currents of 200 amperes and 150 amperes respectively in the same direction. Find the force between them per centimetre length if their centres are 4 inches apart.

Solution:—

The field produced by the current of 200 amperes at the centre of the other wire is, in C. G. S. units,

$$B=\frac{2\,I}{l}$$

$$=\frac{2\,I}{l\times 10}$$ in practical units, since the current is given in amperes (See p. 270, Vol. I).

$$=\frac{2\times 200}{10\times 4\times 2{\cdot}54}=3{\cdot}93.$$

The force due to this field on one centimeter length of the second wire is

$$\frac{BLI}{10}\text{ dynes}=\frac{3{\cdot}93\times 150\times 1}{10}\ \ldots\ldots\ldots\ldots\text{(Art. 1)}.$$

$$=58{\cdot}95\text{ dynes.}$$

The force is one of attraction.

Example 4.—A storage battery having a discharge rating of 8000 amps, is connected to the switchboard by copper bus-bars which have a cross-section of 1 in. by 8 inches, and which are spaced 6 ins. centre to centre, the 8 ins. face being in parallel vertical planes. Assuming that the current may be con idered to be concentrated at the centre of cross-section, what must be the distance between supporting brackets in order that the bus-bars may not deflect more than 1/4 in. ?

Solution :—

Let Fig. 1·06 represent cross-sections of the bus-bars. Assume that the bus-bars are sufficiently long, so that, for all practical purposes, they may be considered as infinite in length.

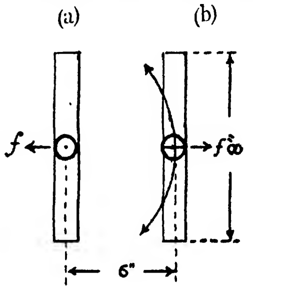


Fig. 1·06

The current flows in the direction indicated. The flux-density at the centre of bus-bar "b" due to the current in "a" is given by the equation,

$$B=\frac{2\,I}{10\,l} \text{ (Art. 2.)}$$

$$=\frac{2\times 8000}{6\times 2.54\times 10}=104.982 \text{ gausses,}$$

and the lines of force have the direction shown by the dotted arrow. Using the left-hand rule, the direction of the force on "b" will be to the right; similarly, the force on "a" will act towards the left. The direction of these two forces, therefore, agrees with the known fact that 'parallel conductors carrying currents in opposite directions mutually repel each other'.

The force per inch of length on bus-bar "b" is then

$$f=\frac{Bl\,I}{10}$$

$$= \frac{104.982 \times .54 \times 8000}{10}$$

$= 213319.36$ dynes per inch.

$= .479$ lb. per inch.

Each bus-bar is then equivalent to a uniformly loaded beam, which, for simplicity, may be considered to be like a beam fixed at the ends, the ends being the points of attachment to the supporting brackets. The deflection of such a beam is given by the formula

$$\delta = \frac{w\ l^4}{384\ E\ I}$$

where w is the load per unit length,

l is the length between supports,

E is the modulus of elasticity of the material of the beam,

I is the moment of inertia of the cross-section.

The latter is equal to

$$I = \frac{1}{12}\ b\ h^3$$

where b is the width of the beam, and h is the depth.

Substituting $\delta = \frac{1}{4}$ inch,

$w = .479$ lb. per inch,

$E = 15 \times 10^6$ (the value for hard drawn copper)

and $I = 1/12\ bh^3 = 1/12 \times 8 = 2/3$,

$$1/4 = \frac{.479\ l^4}{384 \times 15 \times 10^6 \times 2/3}$$

or $l^4 = \dfrac{384 \times 10^7}{.479 \times 4} = \dfrac{96 \times 10^7}{.479} = 2004.175365$

$\therefore \quad l = 211.5$ inch

$= 17$ ft. 7.5 inches.

4. Strength of a Magnetic Shell.—If a thin sheet of steel or iron is magnetised so that opposite polarities are distributed uniformly over the flat faces, the distribution is then said to be 'Lamellar' as compared with the 'Solenoidal' distribution in a long thin wire. Such a magnetised sheet is termed a 'MAGNETIC SHELL'.

The magnetic moment per unit area of the shell is called the '*Strength*' of the shell. Thus, if ϕ be the strength of the shell, a its area, σ = pole-strength per unit area, l = thickness of the shell, then,

$$\phi = \sigma \times l$$

and the Moment of the shell $= a \times \sigma \times l$.

5 Equivalent Magnetic Shell.—Consider any closed circuit ABCD travers-ed by a current of strength I (Fig. 1.07). The circuit ABCD may be divided into a number of elements, each of area a, by a net-work of conductors. If now in each element we imagine a current of strength I to flow in the same direction, the side of each element not

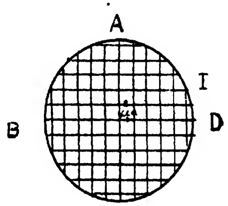


Fig. 1·07

situated at the boundary, will have equal and opposite currents flowing in it, which will neutralise each other. Therefore, the total current in the elements is zero, except at the boundary, where the resultant of the currents in the elements is the current in ABCD. Each of these elements may be replaced by a small MAGNETIC SHELL, the boundary of which coincides with that of the element, and the magnetic moment of which is such that its effect on a distant point is the same as that due to the current in the element which it replaces.

Further, let a represent (Fig. 1·08) a small circular mesh conveying a current I, and P a point on the axis of the circuit distance d cms. from its centre. Since the circuit is very small, the distance of all parts of the circuit from P is approximately equal to d.

Hence the intensity of the field at P,

$$=\frac{2\pi r^2 I}{d^3} \text{.....(Vol. I,P. 271)}$$

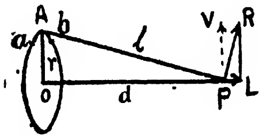


Fig. 1.08.

Replacing the circuit by a magnetised lamina of which the magnetic moment is M, the intensity, due to the magnet, at $P=2M/d^3$.

Hence the lamina is equivalent to the circuit when $\pi r^2 I = M = ml$ (m = pole-strength of the shell).

$$i.e. \text{ when } I = \frac{m}{\pi r^2} l. \quad = \sigma l = \phi.$$

The same holds good for all other elements into which the circuit may be resolved.

Hence, if a current of strength I (C. G. S. units) flow in a circle, we can replace it by a magnetic shell of the same contour and of strength $\phi = I$. This is AMPERE'S THEOREM.

5 A. Force between two Coils (one small and the other large) **at right angles to each other.**—From Amper's Law we know that forces would exist between two circuits carrying currents.

If two currents flow in two circular coils AB and CD mutually at right angles to each other, we have seen (vol. I, P. 272), that the field at the centre of AB is $2\pi i_1/r$, where r is the radius and i_1 the current. The megnetic moment of the small coil CD is αi_2 where α is its area and i_2 the current in it. Hence a couple $2\pi\alpha i_1 i_2/r$ will act on CD tending to twist its plane into that of AB when the two are at right angles, or a couple $2\pi \alpha i_1 i_2 \sin\theta/r$ when the planes of the two coils are inclined to each other at an angle θ.

6. Force between two Equal Co-axial coils:—The force on unit length of coil AB due to the current in the coil CD is $2 i_1 i_2/x$ and the total force on AB is the product of this and the circumference.

$\therefore$ $F = (2 i_1 i_2/x) \times 2\pi r = 4\pi r i_1 i_2/x$, where x is the distance between the coils and r the radius and i_1 and i_2 the current in the coils.

Note :—This can also be deduced from the following Art. 7 by putting $r_1 = r$.

7. Force between two very Nearly Equal Co-axial coils.—Consider two circular co-axial coils of very nearly the same radius r, situated a small distance x (Fig. 1.09) apart. The force on each unit of length of either coil is $2\,i_1\,i_2/ab$ (Art. 2) in the direction ab. The component of this, normal to the axis, taken all round the coils, will, by symmetry, vanish. The component parallel to the axis is—

$(2\,i_1\,i_2/ab)(bc/ab) = 2\,i_1\,i_2\,x/(ab)^2$ for unit length, and for the whole circle, since total length is $2\pi\,r$, r_1 being very nearly equal to r

$$F = \frac{2\,i_1\,i_2,\,x.\,2\,\pi\,r}{(r_1-r)^2+x^2} = \frac{4\,\pi\,i_1\,i_2\,r.x}{(r_1-r)^2+x^2} \quad \ldots \quad (1)$$

The force is zero when $x=0$, i.e. when the coils are in the same plane, and is a maximum when $x/(k^2+x^2)$ is a maixmum, put k^2 in place of $(r_1-r)^2$.

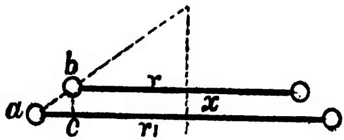


Fig. 1.09.

$$\text{Then } \frac{d}{dx}\left(\frac{x}{k^2+x^2}\right) = \frac{(k^2+x^2)-2x^2}{(k^2+x^2)^2} = \frac{k^2-x^2}{(k^2+x^2)^2}$$

If this is equal to zero, $k^2=x^2$. Determine

$$\frac{d^2}{dx^2}\left(\frac{x}{k^2+x^2}\right)$$

and substitute k^2 for x^2, the result is negative, and therefore $k^2=x^2$, or $(r_1-r)^2=x^2$ corresponds to a maximum. The force between the coils is therefore a maximum when $x=r_1-r$. and its value is—

$$2\,\pi\,i_1\,i_2\,r_1/(r_1-r) \quad \ldots \quad \ldots \quad (2).$$

8. Force between Two Coils, one Large and the other Small.—Let A be a small coil of radius r (Fig. 1.10), and B a larger one of radius r_1. Let H be the field at the left side of A due to B. Let the equivalent magnetic shell for A have a pole-strength m per unit area and a thickness dx; then

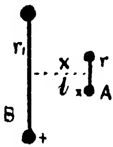


Fig 1. 10.

Force on the left side of $A = m\pi r^2 H$................(1)

$\therefore$ Field at the right side of $A = H - \dfrac{dH}{dx}\,dx.$

$\therefore$ Force on the right side of $A = m\pi r^2 \left(H - \dfrac{dH}{dx}\,dx\right)$...(2)

$\therefore$ Resulting force on $A = (1) - (2) = m\pi r^2 \dfrac{dH}{dx}\,dx.$

But, mdx is the moment per unit area, or the strength of the magnetic shell, and is therefore equal to the current I in A. Again, the field H at A due to B is given by

$$H = \frac{2\pi r_1^2 I_1}{(r_1^2 + x^2)^{\frac{3}{2}}}, \text{ (Vol. I. P. 271.)}$$

$$\therefore \frac{dH}{dx} = -\frac{6\pi r_1^2 I_1 x}{(r_1^2 + x^2)^{\frac{5}{2}}}.$$

Hence, force on $A = -\dfrac{6\pi^2 \cdot r^2 r_1^2 I I_1 x}{(r_1^2 + x^2)^{\frac{5}{2}}}$

$$= -\frac{6\,\pi^2\, r^2\, r_1^2\, I\, I_1\, l}{(r_1^2 + l^2)^{\frac{5}{2}}} \quad \ldots (3).$$

This is zero if $1 = 0$, and numerically greatest if $l = r_1/2$.

Again, if l be great compared with r_1, so that r_1^2 in the denominator may be neglected, equation (3) becomes

$$\text{Force on } A = \frac{6\,\pi^2\, r^2\, r_1^2\, I\, I_1}{l^4} \quad \ldots\ldots\ldots\ldots\ldots\ldots (4).$$

The force between two small "end on" magnets of moments M, M_1, is $6MM_1/l^4$ (Vol. I, P. 57.) In the case of two equivalent current circuits $M = \pi\, r^2\, I$, and $M_1 = \pi\, r_1^2\, I_1$; and the expression for the force becomes $\frac{6\,\pi^2\, r^2\, r_1^2\, I\, I_1}{l^4}$.

which is identical with expression (4) above.

Hence, we find that the force is always proportional to the product of the current strengths. Measuring instruments depending on the mutual action between currents are known as ELECTRO-DYNAMO-METERS.

Example 5.—Two circular coils of wire are placed with their planes parallel to each other at a distance 5 cms. apart. The larger coil has a radius of 10 cms. and 30 turns of wire, the smaller a radius of 2 cms. and 20 turns of wire. Calculate approximately in grammes weight the mechanical force between the coils when a current of one ampere is passed through both, proving any formula used. Show how the principles thus illustrated are applied practically in the ampere balance. (Lond. Univ., B. Sc. Hon., 1906).

Solution :—

If n and n_1 be the respective number of turns in the two coils, the formula (3) becomes—

$$F = \frac{6\pi^2 r^2 r_1^2 n n_1 I I_1 l}{(r_1^2 + l^2)^{\frac{5}{2}}} \text{ dynes.}$$

Here, $r_1 = 10$ cms., $r = 2$ cms., $n_1 = 30$, $n = 20$, $l = 5$ cms., $I = I_1 = 1/10$ C. G. S. unit of current.

$$\therefore \quad F = \frac{6 \times (3.14)^2 \times 2^2 \times 10^2 \times 20 \times 30 \times 5}{10 \times 10 \times (10^2 + 5^2)^{\frac{5}{2}} \times 981} \text{ grms.,}$$

(since 1 amp. $= 10^{-1}$ C. G. S. unit of current, and 1 grm. $= 981$ dynes).

$$= \frac{6 \times (3.14)^2 \times 2^2 \times 10^2 \times 20 \times 30 \times 5}{10 \times 10 \times 174600 \times 981} \text{ grms,}$$

$$= \frac{(3.14)^2 \times 2^2 \times 10}{97 \times 981} = \frac{394.384}{95157} \text{ grms,}$$

$= .0042$ grm. wt. (approx).

9. Field midway between Two Similar Co-axial Circular Coils the distance apart being equal to the radius :—

From P. 271, Vol. I, the field at any point on the axis of a circular coil carrying a current is

$$\frac{2\pi n r^2 I}{(d^2 + r^2)^{\frac{3}{2}}}.$$

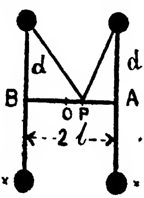


Fig. 1.11.

Let B (Fig. 1.11) represent the coil under consideration here, and B A its axis. Therefore, the strength of the field at any point P on B A is

$$H = \frac{2 \pi n d^2 I}{(x^2 + d^2)^{\frac{3}{2}}},$$

x being the distance B P, and d the radius of the coil.

As already shown, H is maximum at the centre B of the coil, i. e., when $x=0$, and decreases gradually as we pass away from the centre, becoming zero at infinity. It is important to note that the rate of change of H with respect to x, i. e. $\frac{dH}{dx}$ is not constant as we pass away from the centre along the axis. There is, however, a point on the axis at which this RATE OF CHANGE IS CONSTANT. We proceed to find that point.

Assuming that it is constant, we have

$$\frac{dH}{dx} = c,$$

i. e.
$$\frac{d}{dx}\left[\frac{2 \pi n d^2 I}{(x^2 + d^2)^{\frac{3}{2}}}\right] = c.$$

Now, omitting $2 \pi n d^2 I$ which is constant, and putting $\frac{1}{(x^2+d^2)^{\frac{3}{2}}} = (x^2+d^2)^{-\frac{3}{2}}$, we have

$$\frac{d'}{dx}(x^2+d^2)^{-\frac{3}{2}} = c' = -3x(x^2—d^2)^{-\frac{5}{2}}$$

$$\text{or, } \frac{d^2}{dx^2}(x^2+d^2)^{-\frac{3}{2}} = 0 = -3\left\{(x^2+d^2)^{-\frac{5}{2}} - 5x^2(x^2+d^2)^{-\frac{7}{2}}\right\}$$

Dividing throughout by $-3\ (x^2+d^2)^{-\frac{5}{2}}$, we have

$$5x^2\ (x^2+d^2)^{-1} = 1.$$

$$\therefore \qquad 5x^2 = x^2 + d^2, \ 4x^2 = d^2,$$

$$\text{or,} \qquad x = \frac{d}{2}.$$

Thus, the rate of change of the field is constant at the point on the axis whose distance from the centre of the circle is A^2. Now, placing another similar coil with its axis coincident with that of B, and at a distant apart $2\,l$ equal to d, and substituting $d/2$ for x in the expression for H, we have—

$$\frac{4\,\pi\,n\,d^2\,I}{\left(\frac{5d^2}{4}\right)^{\frac{3}{2}}} = \frac{32\,\pi\,n\,I}{5\sqrt{5}\,.d}.$$

which gives the strength of the field at the centre O due to current I in the two coils.

Such an arrangement of coils is made use of in the HELMHOLTZ TANGENT GALVANOMETER, the rate of change of field being most uniform about the middle of the common axis. Any diminution of the field due to one coil as we pass away from the middle is compensated for by an equal increase of the field due to the other coil, for the rate of change of the field is here constant and occurs in opposite directions for the two coils.

EXERCISES I.

(1) A point lies on the axis of a circular loop of wire in which flows a current of i amperes. The radius of the loop is r centimetres, and the point is x

centimetres distant from the plane of the loop. Show that the field intensity at the point is

$$H = \frac{0.2\pi r^2 i}{(r^2 + x^2)^{\frac{3}{2}}}.$$

(2) The current in two long parallel wires are equal but flow in opposite directions. Show that the field intensity at any point on a line joining the centres of the wires is

$$H = \frac{0.2\,i}{x} + \frac{0.2\,i}{D-x},$$

when D = the distance in centimetres between the axes of the wires, x = the distance in centimetres of the point from the axis of one wire.

(3) A conductor 25 cm. long is moved through a uniform magnetic field of 10,000 lines per sq. cm. with a velocity of 35 metres per second. Find the e.m.f. between the ends of the conductor.

If the ends of the conductor are joined through an external circuit of such resistance that a current of 50 amp. flows through the conductor, find the retarding force.

Find the power required to keep the conductor moving.

CHAPTER II.

ELECTROMAGNETS.

10. Ohm's Law and the Magnetic Circuit.—As the magnetic circuit is analogous to the electric circuit, so the law governing the magnetic flux is analogous to Ohm's law governing electric currents. Expressing the amount of the flux by ϕ, the magnetomotive force by M, the reluctance by R, we have

$$\phi = \frac{M}{R}. \qquad \text{(Vol. I, p.287)}$$

11. The Law of Magnetic Circuit.—In a magnetic circuit the total flux through the core will be

$$\phi = A\mu\, H,$$

$$= \frac{4\,\pi\, NI.\ \mu\, A}{10\, l},$$

$$\text{or}\quad \phi = \frac{\dfrac{4\,\pi\, NI}{10}}{\dfrac{l}{\mu\, A}},$$

where A is the area of the section of the core, l is the mean length of the circuit, and μ its permeability The numerator is the M. M. F. of the solenoid of N number of turns and carrying I amperes of current, and the denominator is the reluctance of the magnetic circuit.

12. Permeability.—There are (1) Absolute Permeability and (2) Relative Permeability.

The ABSOLUTE PERMEABILITY of air and of all other non-magnetic materials, like conductivity, depends upon the units selected and with the ampere-turns, the flux line and inches taken as units, the absolute permeability of air is 3.192 perms per inch cube.

This value of absolute permeability of air is derived from the practical unit of reluctance which is termed "Rel." A magnetic circuit has a reluctance of 1 rel when a M. M. F. of 1 amp.-turn produces a flux of 1 line in it. And, just as the ohm is the resistance of a column of mercury 41.85 in. long and 0.00049 in. in diameter at the temperature of melting ice, so the rel is the reluctance of a prism of air, or any other non-magnetic material 3.19 in. long, and 1 sq. in. in cross-section. Reluctivity, which is specific reluctance, is, therefore, 1/3.19 or .313 rels per in. cube.

Hence, the unit of absolute permeability, (which is the reciprocal of reluctivity) is 1/.313 or 3.19 perms per in. cube, for all non-magnetic materials.

When the permeability of air is given as unity it is the RELATIVE PERMEABILITY. If the permeability of a material is said to be n, it means that its permeability is n times as great as the permeability of air and it indicates the relative permeability of the substance. Its absolute permeability will be n times the absolute permeability of air, i.e., n × 3.19 perms per cubic inch.

Elementary Propositions in Electromagnetics.

13 (1) **Magnetomotive Force**—The magnetomotive force, or magnetizing power of an electromagnet varies directly as the number of turns of wire and the amperes of current flowing through them, that is, one ampere flowing through twenty coils or turns will produce the same magnetomotive force as twenty amperes flowing through one coil or turn.

If N = number of turns in the coil,

I = amperes of current flowing,

$1.257 = \frac{4\pi}{10}$ (to reduce to C. G. S. units),

Magnetomotive force $= 1{\cdot}257 \times N\,I =$ M.M.F.

14 (2) **Intensity of Magnetic Force.**—Intensity of magnetic force in an electromagnet varies in different parts of the magnet, being strongest in the middle of the coil, and weaker towards the ends. In a long electromagnet, say a length 100 times the diameter, the intensity of magnetic force will be found nearly uniform along the axis, falling off rapidly close to the ends.

In a long magnet, such as described above, and in an annular ring wound evenly over its full length, the value of the magnetic force, except near the ends is determined by the following expression:—

$H = 1.257\frac{NI}{l}$, in which l is in centimetres.

If the length is given in inches, then

$$H = .495 \frac{NI}{l_1}, \text{ in which } l_1 \text{ is in inches.}$$

If intensity of the magnetic force is to be expressed in lines per sq. inch,

$$H_1 = 3.193 \times \frac{NI}{l_1}.$$

15. (3) **Value of H, the Magnetic Force, at the Centre of a Single Turn of Conductor**.—In a single ring or turn of wire of radius r, carrying I amperes of current,

$$H = \frac{2\pi}{10} \times \frac{I}{r} = .6284 \times \frac{I}{r}.$$

16. (4) **Force on a Conductor** carrying current in a magnetic field, $F = \frac{H\ L\ I \sin \alpha}{10}$ (See Art. 2).

17. (5) **Work done by Conductor (carrying current) in moving across a Magnetic Field**.—If the conductor be moved across the field of force, the work done will be determined as follows :

Let b = breadth of the field in and across which the conductor is moved,

w = work done in ergs,

$$\text{Then, } w = Fb = \frac{b\ H\ L\ I}{10} = \frac{\phi\ I}{10},$$

since bL = area of field,

$\phi = b\ L \times H$ = number of lines of force cut.

18. (6) **Rotation of a Conductor (carrying current) around a Magnet Pole**.—If a conductor (carrying current) is so arranged that it can rotate about the pole of a magnet, the torque or the force producing the rotation is found as follows:

The total number of lines of force radiating from the pole is 4π times the pole strength m.

$$\therefore w = \frac{4\pi m I}{10} = 1.257 \text{ m I}.$$

Dividing by the angle 2π, the torque,

$$T = \frac{2 m I}{10} = .2 \text{ m I}.$$

19. (7) **Every Electric Circuit** tends to place itself so as to embrace the maximum flux. In other words, an electric circuit tends so to alter its configuration as to make the magnetic flux through it a maximum, (MAXWELL'S RULE).

20. (8) **Two Electric circuits** (or conductors carrying currents) are urged by mutual forces to change their configurations so that their mutual magnetic flux may be a maximum. (S. P. Thompson).

21. The Law of Traction.—The formula for the pull or lifting power of an electromagnet when the poles are in actual contact with the armature or keeper is as follows:

$$\text{Pull (in dynes)} = \frac{B^2 A}{8\pi} \quad \text{(See Chap, VII.)}$$

$$\text{Pull (in grammes)} = \frac{B^2 A}{8 \pi \times 981}.$$

$$\text{Pull (in pounds)} = \frac{B^2 A}{11,183,000}.$$

$$\text{In inch measure : Pull (in pounds)} = \frac{B_1^2 A_1}{73,134,000}.$$

22. Different types of Electromagnets —

(1) Bar Electromagnet.

(2) Horse-shoe Electromagnet.

(3) Club-foot Electromagnet, having only one coil in one limb of a horse-shoe electromagnet, the other core being left uncovered.

(4) Iron-clad Electromagnet having an iron shell or casing external to the coils and attached to the core at one end. The armature is generally a circular disc or lid of iron.

(5) Coil-and-plunger Electromagnet in which a detached iron core is attracted into a hollow coil solenoid of copper wire which carries the current.

(6) Stopped-coil-and-plunger Electromagnet has a short fixed core and a movable core to be sucked into the coil, and finally attracted in close proximity with the fixed core. The action partakes of the nature of weak long range pull of the coil and plunger, and the powerful short range pull of the common form of electromagnet.

(7) Electromagnet with consequent poles.

(8) Circular or ring Electromagnet.

(9) Miscellaneous forms.

(10) Polarized electromagnet—being a combination of a permanent magnet and an electromagnet.

23. Use of Electromagnets.—

(1) For temporary adhesion, or lifting power, e. g., traction of an armature in contact, magnetic lathe chuck.

(2) Attraction of an armature at a distance, e. g. electric trembling bell.

Applications of the law of Magnetic Circuit.

24. Series Circuits.—A circuit consisting of a number of reluctances in series will have a total reluctance given by

$$\frac{l_1}{\mu_1 A_1} + \frac{l_2}{\mu_2 A_2} + \cdots + \frac{l_n}{\mu_n A_n},$$

and the resultant flux through the circuit will be—

$$\phi = \frac{\frac{4\pi}{10} NI}{\frac{l_1}{\mu_1 A_1} + \frac{l_2}{\mu_2 A_2} + \cdots + \frac{l_n}{\mu^n A_n}},$$

$$= \frac{\frac{4\pi}{10} NI.}{\Sigma \frac{l}{\mu A}} \qquad \text{(VOL., I P. 288)}$$

where Magnetic reluctance $= \Sigma \frac{l}{\mu A}$.

25. The Magnetic Circuit.—THE CALCULATION OF AMPERE-TURNS required to produce a given Magnetic Flux is best represented by examples.

Eaxmple 1.—Find the ampere-turns necessary to produce a flux of 50,000 C. G. S. lines across the space between the parallel plane faces of the pole-pieces A, D. The pole-pieces are of cast iron, the area of each pole-face being 10 sq. cms.,

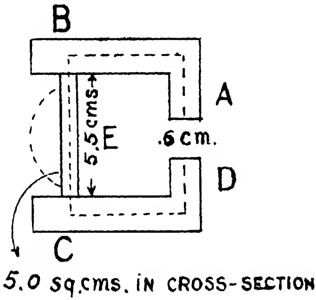


Fig. 2·01

and the sum of the lengths of the mean magnetic circuit in the portions A B and C D, (the dotted line of induction) amounts to 12 cms. The length of the air-gap between the pole-pieces is 6 cm., and the exciting coil is wound on the cylindrical core E, made of wrought iron, 5.5 cms. long and 5·0 sq. cms. in cross-section.

Solution :—

All the lines of force produced by the exciting coil at the middle cross-section of the core E do not pass across the plane faces of the pole-pièces, a large number leaking out into the surrounding air-space. The leakage coefficient of the magnetic circuit, i. e. the ratio of the maximum flux through E to the useful flux across A D must be known before the problem can be solved.

Consider each of these portions separately, and find the corresponding values of H. In the air-gap

$H=B=\frac{50,000}{10}=5000$. The flux through the core E and the pole-pieces is, on account of leakage, $1.4 \times 50,000=70,000$. Hence, B in the polepieces $=70,000/10=7000$. Referring to the B—H curve for cast iron, we find the corresponding value of H to be 48. Next, in the wrought-iron core we have $B=70,000/5=14000$. A reference to the curve for this material gives $H=20$. The total M. M. F. round the circuit is thus:

Air-gap............................5000 × .6

Cast-iron pole-pieces................48 × 12

Wrought-iron core................20 × 5.5

Total......................= 3686, approximately.

The corresponding ampere-turns are .8 × 3686 = 2949, approximately.

Note that by far the largest portion of the M.M.F. is employed in maintaining the flux across the air-gap. Such is frequently the case in practice.

After the ampere-turns necessary to force the flux through each of the parts has been figured, these component ampere-turn values are totalled. The resulting total will then be the number of ampere-turns required to force the flux through the entire circuit. The following example illustrates the process.

Example 2.—Calculate the number of ampere-turns required to develop a flux of 480,000 lines in the magnetic circuit of Fig. 2.02.

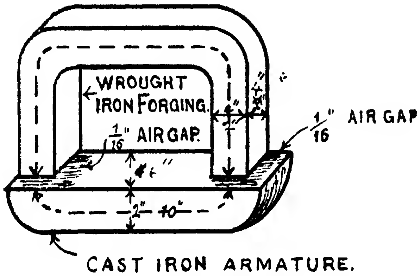


Fig. 2.02

Solution :—

Take the component parts of the circuit one at a time.

First, take the wrought iron yoke which has a sectional area of $4 \times 2 = 8$ sq. in. and length 20 inches. Hence, if the total flux is to be 480,000 lines, the flux density or lines per square inch will be $480{,}000 \div 8 = 60{,}000$. Now, in wrought iron, with a flux density of 60,000 lines (60 kilolines) per sq. in., the m. m. f. gradient is found from table to be 40. That is, 40 amp.-turns are required per inch length of the magnetic circuit to produce a flux density of 60,000 lines. Therefore, the amp.-turns required to magnetize the 20-in, yoke of the specified flux density would be: $20 \times 40 = 800$.

Now, the .air gaps are. ach 1/16 in. long. Assume they are 4.32×2.16 sq. in. each. The lines of force

spread out at an air gap, and there occupy a cross-sectional area greater than that of the iron yoke. Therefore, the area of the magnetic circuit at the air-gap is: $4.32 \times 2.16 = 9.33$ sq. in. The flux density in the air gap is: $480{,}000 \div 9.33 = 51400$ lines per sq. in., and to produce this flux density the ampere-turns per inch length would be 16000. For the 1/16- in. air gap there would be required: $16{,}000 \div 16 = 1000$ amp.-turns. Hence, for the two 1/16-in. air-gaps: $2 \times 1000 = 2000$ amp.-turns are necessary.

Again, the cast-iron armature has a sectional area of $6 \times 2 = 12$ sq. in. Therefore, the flux density in it is: $480{,}000 \div 12 = 40{,}000$ lines per sq. in., and to produce this flux density in cast iron requires 80 amp.-turns per in. length of the magnetic circuit. Hence, to magnetize this 10 in. long armature there will be required: $80 \times 10 = 800$ amp.-turns.

Adding the ampere-turns required for the different components, we have—

Wrought-iron yoke	...	800 amp.-turns.
Two air-gaps	...	2000 ,,
Cast-iron armature	...	800 ,,
Total	...	3600 amp.-turns.

Thus, 3600 amp.-turns are necessary to drive a flux of 480,000 lines through the magnetic circuit of Fig. 2.02.

26. The Difficulties encountered in calculating Magnetic Circuit should be considered.

Magnetic circuits cannot be computed with the same accuracy as electric circuits for two reasons.

First—Magnetic leakage. Magnetic flux unlike electric current cannot be confined to certain paths. We can compute, with accuracy, the m.m.f. that a given helix will develop, but the effective flux that this force will push through a magnetic circuit cannot be computed exactly because of magnetic leakage.

Second—The reluctance of iron and its permeability varies with its saturation.

27. Hints on the Design of Electromagnets.—

(1) PERMISSIBLE FLUX DENSITIES IN A MAGNETIC CIRCUIT should not be exceeded, and the iron in it should be worked somewhat below its saturation point. For average work with grades of iron ordinarily obtainable, the flux density should not exceed about 110,000 per sq. in. for annealed sheet iron; 90,000 per sq. in. for unannealed cast steel and wrought-iron forgings; and 50,000 lines per sq. in. for grey cast iron.

(2) DISSIPATION OF HEAT BY A MAGNET.—Generally 0.8 watt of I^2R loss or heat developed by the coil is taken for every square inch of coil surface exposed to the air. However, 0.5 watt of I^2R loss per sq. in. is more safe. It is usually assumed that the heat dissipated through surfaces of the magnet structure, in addition to that dissipated directly from the surface of the coil, amounts to from 50 to 75 per

cent. of that radiated directly from the coil surface. As a general proposition, no coil that is to carry relatively heavy currents should be thicker than 2 inches.

(3) CALCULATION OF SIZE OF WIRE FOR A MAGNET COIL that will provide a required number of ampereturns when connected across a given voltage.—The following formula is used for the purpose:

$$R = \frac{K \times N}{S} \times \frac{l}{12} = \frac{K \times N \times l}{12 \times S} \dots \quad \text{(ohms)}$$

wherein R = resistance, in ohms, of all the turns of any magnet winding; K = a constant, numerically equal to the resistance, in ohms, of a circular mil-foot of the conductor of the winding; N = number of turns in the winding; l = length in inches of an average turn of the winding, or the length of a mean turn; S = cross-sectional area of the conductor in circular mils.

If the magnet coil is to operate on some certain fixed voltage, as magnet coils usually do, the current through the coil will be, by Ohm's Law, I = E ÷ R. Now, substituting the expression for R obtained above in this Ohm's law formula, we have—

$$I = \frac{E}{R} = \frac{E}{\frac{K \times N \times l}{12 \times S}} = \frac{E \times S \times 12}{K \times N \times l} \quad \text{(amp.)}$$

wherein I = current, in amperes, through the coil, and E = e. m. f. impressed on the coil in volts. Now,

multiplying both sides of this last equation by N, we have—

$$IN=\frac{E \times S \times 12 \times N}{K \times l \times N} \qquad \text{(amp.-turns)},$$

$$=\frac{E \times S \times 12}{K \times l} \qquad \text{(amp.-turns)},$$

and hence—

$$S=\frac{I \times N \times K \times l}{12 \times E} \qquad \text{(cir. mils)}.$$

For soft-drawn copper wire operating at about 130° F., K becomes 12 ohms. Thus, where a winding will operate at about 130° F., which is a fair average operating temperature,

$$S=\frac{I \times N \times l \times 12}{E \times 12.},$$

$$=\frac{I \times N \times l}{E} \qquad \text{(cir. mils)}.$$

(4) THE MAXIMUM PERMISSIBLE THICKNESS OF MAGNET COILS of solid wound cotton-covered wire—without ventilating ducts—intended to carry continuously relatively heavy currents, is 2 inches. In thicker coils the heat developed in the inner turns travels slowly to the surface from which it may be radiated. The inner turns of such coils thus become excessively hot. Where the wire comprising the winding is insulated with a non-combustible, or heat-resisting material, windings may be thicker than 2 inches.

Example illustrating the design of a constant-voltage magnet coil to produce a certain number of ampere-turns is given below.

Example 3.—Design a winding to produce 40,000 amp.-turns for the winding space. Assume the voltage available to be 220.

Solution:—

(1) DETERMINE WIRE SIZE.—Assume that the coil will be 2.25 in. thick, the maximum permissible thickness. The magnet core is 3.25 ins. in diameter. Hence, the diameter of mean turn is 5.5 ins. The circumference of mean turn equals: $5.5 \times 3.14 = 17.27$ ins. Substituting these values in the formula, we have—

$$S = \frac{I \times N \times l}{E}$$

$$= \frac{40{,}000 \times 17.27}{220} = 3{,}140 \text{ (cir.mils.)}$$

Hence, to produce 40,000 amp-turns under the conditions outlined, a 3,140-cir. mils winding should be used. A 3,140-cir. mils conductor corresponds to No. 17 S.W.G. Remember that a 3,140-cir. mil conductor will then produce 40,000 amp.-turns, regardless of how few or how many turns of this conductor are wound into the coil.

(2) ASCERTAIN JUST HOW MANY AMPERE-TURNS THE WIRE SIZE AS ABOVE WILL PRODUCE.—Since with a given voltage, wire size, and mean length of turn,

the amount of this No. 17 wire wound on the coil will not affect the number of ampere-turns developed, we will find the number of ampere-turns developed by 1 lb. of wire. Then the ampere-turns developed by a greater or lesser amount of the wire will be the same number.

The length of a mean turn is 17.27 in., $=17.27 \div 12 = 1.44$ ft. 1 lb. of No. 17 wire contains 105.34 ft. Then, 1 lb. of No. 17 would provide: $105.34 \div 1.49 = 73.15$ turns. A coil containing 1 lb. of No. 17 has a resistance of .342 ohm. Then a 1-lb. coil would, on 220 volts, pass $220 \div 0.342 = 643.27$ amp. Therefore, the ampere-turns of No. 17 wire for the conditions of this example are: 643.27 amp. $\times$ 73.15 turns $=47055$ amp.-turns. Whether 1, 10, 100 or 1000 turns of No.17 wire were wound on the core (with a mean diameter of turn of 5.5 in. and an applied voltage of 220) the ampere-turns would remain 47055.

(3) DETERMINE HEAT RADIATING SURFACE OF COIL.—The outside diameter of the coil will be 7.75 in. Therefore, its circumference will be: $7.75 \times 3.14 = 24.335$ ins. Now, if the coil has 8 separate cloisons, and the total length of the coil be 18 in., the exposed area of the coil will be: $(24.335 \times 18) + (38.86 \times 16) = (483 + 622) = 1060$ sq. in. Assume that the pole-piece and frame provide a radiating surface 60 per cent. as great as that of the winding. Then the total equivalent radiating surface is $1060 \times 1.60 = 1696$ sq. ins.

(4) DETERMINE WATTS POWER LOSS PERMISSIBLE IN THE COIL.—Assume that each square inch of

coil surface will radiate the heat produced by .8 watt. Then the coil effectively radiates the heat due to $.8 \times 1696 = 1356.8$ watts.

(5) DETERMINE PERMISSIBLE CURRENT IN COIL.—With a pressure of 220 volts, the current that will develop 1356.8 watts is: $I = P \div E = 1356.8$ watts $\div 220$ volts $= 6.16$ amperes. Therefore, the permissible current in the coil is 6.16 amps.

(6) DETERMINE AMOUNT OF WIRE REQUIRED.—Through a coil of 1 lb. of No. 17 wire (as calculated in (2)), 643.27 amp. will flow. As determined in (5), the permissible current through the coil of this example is 6.16 amp. To pass a current of 6.16 amp., a coil of No. 17 wire weighing: $643.27 \div 6.16 = 104.4$ lbs. would be required. We will then use 104.4 lbs. of No. 17 if it will fit in the winding space available. Since there are 105.34 ft. in 1 lb. of No. 17 bare copper wire, the length of wire in the 104.4 lbs. coil required in this problem would be: $104.4 \times 105.34 = 10997.5$ ft.

(7) CHECK WIRE SIZE TO ASCERTAIN IF IT CAN BE WOUND IN SPACE AVAILABLE.—We must find room for 10997.5 ft. of No. 17 wire which must be insulated. The thickness of the coil is 2.25 in., and its length is 18 ins. The cross-section of the coil is then: $2.25 \times 18 = 40.5$ sq. in. Assume that double cotton covered magnet wire will be used. 1 sq. in. will contain 222 such wires or turns. Hence, 40.5 sq. in. will contain: $40.5 \times 222 = 8991$ turns. The mean turn has, as deter-

mined in (2), a length cf 1.44 ft. Then, the total length of wire that can be wound in the coil is: $1.44 \times 8991 = 12947$ ft. It is, then, evident that there is ample room for the 10997.5 ft. that is necessary, as calculated in (6).

MAGNET COILS OPERATING ON CONSTANT CURRENT, such as coils of constant-current or series generators and series street lighting magnets, always have practically the same current flowing through them. Therefore, with such coils the wire size merely determines the I^2R loss or heating in the coil. Where such a coil is to be designed, divide the ampere-turns required by the amperes flowing in the constant-current circuit; the result will be the number of turns required. Use a size wire that will carry this current without excessive heating.

28. In the Practical Design of Magnetic Circuits:—(1) Assume the total flux, ϕ. (2) Lay out a circuit with sufficient cross—sectional area of the magnetic path which will carry this flux effectively and without over-saturation. (3) Compute the ampere-turns necessary for its development in the magnetic circuit. The general proportions of the circuit are tentatively assumed, the dimensions being based on previous experience and trial calculations. (4) Examine similar magnetic circuits already in successful operation. (5) It is usually more practical to consider the component parts of the circuit, one part at a time; that is,

to compute the ampere-turns necessary to develop the required flux in each part.

The first tentative plan is developed to a conclusion and, if it does not work out as desired, it must then be altered and recalculated accordingly. As permeability varies with the saturation, so it is almost impossible to effectively design magnetic circuits without consulting data.

Example 4. A bobbin has a winding length of 2.25 ins., and the inner and outer diameters of the available winding space are 3.25 ins. and 7.5 ins. respectively, and if 27.5 volts is to produce 5000 amp.-turns of excitation, of what diameter must the copper wire be ?

(N.B.—The resistance of a bar of copper 1 ft. long and 1 sq. inch in cross-section is 9 microhms.)

Solution :—

The size of wire can be determined quite apart from the winding length. For,

$$\text{resistance per turn} = \frac{\text{volts}}{\text{ampere-turns}} = \frac{27.5}{5000}$$

$$= .0055 \text{ ohms.}$$

$$\text{and, length of a turn} = \pi \times \frac{3.25 + 7.5}{2} = 16.9 \text{ inches}$$

$$\text{Now, } R = \frac{\rho l}{A}.$$

$$\therefore A = \frac{\rho l}{R} = \frac{9 \times 16.9}{12 \times 10^6 \times .0055} = .002345 \text{ sq. in.}$$

$\therefore$ diameter $= \sqrt{\frac{4}{\pi}A} = \sqrt{.002345 \times 1.274} = .0553$in.

$$\left(\text{Cf :—Circular mils} = \frac{I\ N\ L\ K}{E \times 12} \text{ (Art. 27)} \right)$$

The nearest standard sizes are No. 18 S. W. G. (.048″ diam), and No. 17 S. W. G. (.056″ diam.), so the winding might be made up partly of each size, or entirely of the larger wire so as to be on the high side with the ampere-turns.

The following further calculations are made to illustrate the relations of "Winding and Spool" and "Weight and Resistance."

Taking No. 17 S. W. G. for the winding and adding 12 mils for double cotton covering, overall diameter $= .068''$.

$$\text{Depth of winding} = \frac{7.5 - 3.25}{2} = 2.125''.$$

$$\therefore \text{winding space} = 2.125 \times 2.25 = 4.78 \text{ sq. inches.}$$

$$\therefore \text{number of turns} = \frac{4.78}{(.068)^2} = 1039.$$

$$\therefore \text{total length of winding} = 1039 \times \frac{16.9}{36},$$

$$= 485 \text{ yards.}$$

$$\therefore \text{weight of copper} = \frac{485}{1000} \times 28.48 \text{ s.} = 13.8 \text{ lbs.}$$

$$\text{Resistance of winding} = \frac{9 \times 485 \times 3}{10^6 \times .00246} = 5.32 \text{ ohms,}$$

.00246 sq. inch being the area of No. 17 S. W. G,

$\therefore$ current in winding$=\dfrac{27.5}{5.32}=5.17$ amperes.

$\therefore$ ampere-turns$=5.17\times 1039=5372$.

The excess over the 5000 required, is due to the use of No. 17 S. W. G. instead of .055″ diam. wire.

Current density$=\dfrac{5.17}{.00246}=2100$ amperes per sq. in.

Yard per volt$=\dfrac{485}{27.5}=18$.

Example 5. Calculate the length of winding required per volt for a current-density of 800 amperes per sq. inch, if the working temperature of the wire is 50° C (=112° F.).

Solution :—

Specific resistance of copper at 60°F.$=0.668\times 10^{-6}$ ohms per in cube.

$\therefore$ At 112° F, specific resistance =

$0.668\times 10^{-6}\ \left\{1+(112-60)\times .00238\right\}$,

$=0.668\times 10^{-6}\times 1.124=.751\times 10^{-6}$ ohms per in. cube.

$\therefore$ length per volt.$=\dfrac{10^6}{.751\times 800}$ inches,

$=\dfrac{10^4}{.751\times 8\times 36}=46.2$ yards.

Example 6. Calculate the size and total length of wire to give 40,000 ampere-turns when supplied at 220 volts, with current-density and temperature as in Example 5, the mean length of turn being 17 inches.

Solution:—

From Example 5, total length

$$= 46.2 \times 220 = 10164 \text{ yds.}$$

$$\text{Resistance per turn} = \frac{220}{40{,}000} = .0055 \text{ ohm.}$$

$$\therefore \text{resistance per 1000 yds.} = \frac{.0055 \times 1000 \times 36}{17},$$

$$= 11.65 \text{ ohms at } 112^0 \text{ F.}$$

$$\therefore \text{resistance per 1000 yds at } 6^0\text{F.} = \frac{11.65}{1.124},$$

$$= 10.37 \text{ ohms.}$$

No. 17 S. W. G. is very near this, but is about 6.3 per cent. too big in cross-section.

CHECK CALCULATIONS.—Resistance of No. 17 S. W.G. $= 9.74$ ohms per 1000 yds. at 60^0F. $= (9.74—1.124)$ $= 10.95$ ohms per 1009 yds. at 112^0F.

$$\therefore \text{total resistance} = 10.95 \times \tfrac{10164}{1000} = 111.3 \text{ ohms.}$$

$$\therefore \text{current} = \frac{220}{111.3} = 1.98 \text{ amperes.}$$

$$\therefore \text{current-density} = \frac{1.98}{.002463} = 805 \text{ amperes per sq.in.}$$

$$\text{Number of turns} = \frac{10164 \times 36}{17} = 21548.$$

$\therefore$ ampere-turns = $1.98 \times 21548 = 42665$, slightly above the value owing to the thicker wire used.

Again calculate for No. 18 S. W. G. This will be found to be more suitable in the present case.

Example 7. Calculate the magnetic field at the centre of a long solenoid of wire, consisting of 300 turns wound as a helix 45 cms. long and 5 cms. in diam., when traversed by a direct current of 9 amperes. How many lines of force are embraced by the centre of this solenoid ?

Solution :—

By the formula, $H = \frac{1.257\ CN}{l}$, we have—

$$H = \frac{1.257 \times 9 \times 300}{45} = 60 \text{ lines per sq. cm.}$$

The flux F, or total flow of lines in air will be—

$$F = H \times A,$$

where A is the cross-sectional area in sq. cms.

But $A = 3.1416 \times 2.5^2$,

$= 19.625$ sq. cms.

$\therefore\ F = 60 \times 19.625$,

$= 1178$ lines at the centre of solenoid.

In the above example, if iron were present instead of air, the number of lines would be very greatly increased, as iron is a better medium for magnetic induction than air. The permeability of air is unity, while that of iron reaches much higher values, according to the kind of iron used, and on the magnetising force employed.

If H for a current-carrying coil be as above, and we insert an iron core whose permeability, $\mu = 120$ for that value of H, then the lines of induction per sq. cm. (i. e., B) would be equal to the permeability into the magnetic force, or—

$$B = H\mu,$$

i. e. $$B = 60 \times 120 = 7200 \text{ lines.}$$

The total flux F in the iron would then be:

$$F = B \times A,$$

$$= 7200 \times 19.625 = 141300 \text{ lines, or about}$$

80 times that through air alone.

Example 8. Find the number of ampere-turns required to magnetise up to 16000 lines per sq. cm. a soft-iron ring made of round iron 2 inches thick and 20 inches in mean diameter, the specific magnetic conductance (permeability) of the iron being 800.

Solution :—

The length of the ring, $l = 3.1416 \times 20$ inches, or $3.1416 \times 20 \times 2.5 = 157.08$ cms.

$$B = 16,000, \text{ and } \mu = 800.$$

$$\text{Now, } H = \frac{B}{\mu} = \frac{16,000}{800} = 20.$$

By formula $H = \dfrac{1.257\ CN}{l}$ where CN is the ampere-turns required, we have

$$20 = \frac{1.257\ CN}{157.08}.$$

$$\therefore CN = \frac{157.08 \times 20}{1.257} = 2500 \text{ ampere-turns}$$

29. The Constant-Current Electromagnet.—Such magnets are, in most cases, direct-current electromagnets, and the series operating magnets of constant-current arc lamps in alternating current circuits.

MECHANICAL WORK OF THE CONSTANT-CURRENT ELECTROMAGNET :—

Let I = current (constant in strength), in amperes, flowing in the magnet winding ;

l = length or stroke of the electromagnet in cms. from its initial position 1, to its final position 2 ;

n = number of turns in the magnet winding ;

ϕ = total magnetic flux (which varies from a minimum value ϕ_1 in the initial position, to a maximum value ϕ_2 in the final position of the electromagnet) ;

L = inductance of the magnet winding in henrys, which varies with ϕ, from L_1 to L_2 ;

Then, we have

$$L = \frac{n\,\phi}{I}\,10^{-8} \text{ henrys ; (Ch. VII)}$$

and the e. m. f. induced in the magnet winding,

$$E = n\frac{d\phi}{dt}10^{-8} = I\frac{d\,L}{dt}\text{ volts.}$$

Evidently this e. m. f. is induced by the consumption of power

$$P = I.\ E. = I^2\,\frac{dL}{dt}\text{ watts,}$$

and therefore, the energy,

$$W = \int^{1} Pdt = I^2 (L_2 - L_1) \text{ joules.} \qquad (1)$$

Now the energy stored in the magnetic field is:

$$W_1 = \frac{I^2 L_1}{2} \text{ joules (in position 1). (Chap. VII.)}$$

$$W_2 = \frac{I^2 L_2}{2} \quad \text{,, (in position 2).}$$

Therefore,

$$W' = W_2 - W_1 = \frac{I^2}{2}(L_2 - L_1) \text{ joules, ...} \quad (2)$$

represents the increase of stored magnetic energy during the motion of the armature. Here we assume that the inductance, in any fixed position of the armature, does not vary with the current, in other words, that the magnetic saturation is absent.

Now, the total consumption of energy in the movement = the mechanical work done (which is transformed into electrical energy in the winding) + the increase of the stored magnetic energy, (neglecting the small losses due to heat production, etc.).

Hence, from (1) and (2) we have the mechanical work done by the electromagnet

$$W = W - W' = \frac{I^2}{2}(L_2 - L_1) \text{ joules.} \qquad \text{... (3)}$$

Now, if F = average force or pull of the magnet in gram weight, during its stroke *l*, the mechanical work

$= Fl$ gram-cm.

$= Flg$ absolute units, g, being the acceleration of gravity ($=981$ cm/sec.^{2})

$= Flg\ 10^{-7}$ joules ... ... (4)

(since 1 joule$=10^7$ absolute units (ergs),

From (3) and (4) we, therefore, have—

$$Fl = \frac{I^2}{2g} (L_2 - L_1)\ 10^7 \text{ gram-cm.} \quad \dots \quad (5)$$

as the mechanical work of the electromagnet,

and,
$$F = \frac{I^2}{2g} \cdot \frac{L_2 - L_1}{l}\ 10^7 \text{ grm.} \quad \dots \quad \dots \quad (6)$$

as the average force or pull of the electromagnet, during its stroke l.

Equation (6) may be put in the form—

$$F = \frac{I^2}{2g} \frac{dL}{dl}\ 10^7 \text{ grms.} \quad \dots \quad \dots \quad (7)$$

where dl represents a motion-element in any position l of the electromagnet.

REDUCTION TO FOOT-POUNDS:—

If l is in feet, and F in pounds, we divide by $454 \times 30.5 = 13{,}850$, and substitute for $g=32$, and thus get

$$Fl = 3.68\ I^2\ (L_2 - L_1) \text{ ft.-lb.} \quad \dots \quad (8)$$

$$F = 3.68\ I^2 \frac{(L_2 - L_1)}{l} \text{ lb.} \quad \dots \quad \dots \quad (9)$$

$$F = 3.68\ I^2 \frac{dL}{dl} \text{ lb.} \quad \dots \quad \dots \quad (10)$$

These equations apply equally to the direct and alternating-current electromagnets.

In the alternating-current electromagnet, if I is the effective value of the current, then F is the effective or average value of the pull, and the pull or force of the electromagnet pulsates with double frequency between 0 and 2F.

OTHER FORMS OF EQUATIONS (5) to (7) :—

In the alternating current electromagnet usually the voltage consumed by the resistance of the winding, IR, can be neglected in comparison with the voltage consumed by the reactance of the winding, IX. The latter is, therefore, practically equal to the terminal voltage, E, of the electromagnet, and hence by the general equation of self-induction,

$$E = 2\,\pi f\,L\,I,$$

where f = frequency in cycles per sec.; and substituting this in equs. (5) to (7), we have :—

In the metric system :

$$F\,l = \frac{I\,(E_2 - E_1)\,10^7}{4\,\pi f g}\ \text{grm.-cm.} \ldots \qquad (11)$$

$$F = \frac{I\,(E_2 - E_1)\,10^7}{4\,\pi f g\,l} = \frac{I}{4\,\pi f g}\,\frac{dE}{dl}\,10^7\ \text{grms.} \ldots \qquad (12)$$

In foot-pounds:

$$F\,l = \frac{.586\,I\,(E_2 - E_1)}{f}\ \text{ft.-lbs.} \ldots \qquad (13)$$

$$F = \frac{.586\,I\,(E_2 - E_1)}{f l} = \frac{.586\,I}{f}\,\frac{dE}{dl}\ \text{lbs.} \ldots \qquad (14)$$

Example 9. In a 50-cycle A. C. lamp magnet, the stroke is 4 cms., the voltage, consumed at the constant alternating current of 5 amps., is 10 in the initial position, and 20 in the final position. Find the average pull of the magnet.

Solution :—

Here, $l = 4$ cms.

$E_1 = 10$ volts.

$E_2 = 20$ volts.

$i = 5$ amps.

$f = 50$

Hence, by eqn. (12), we have

$$F = -\frac{5 \times 10 \times 10^7}{4 \times 3.14 \times 50 \times 981 \times 4.} = 203 \text{ grms.} \quad (= .45 \text{ lbs.}).$$

30. Efficiency of an Electromagnet.—The work done by an electromagnet, and also its pull, can be calculated from eqn. (12). With a given maximum voltampere IE_2, available for the electromagnet, the maximum work would thus be done, in other words, the greatest pull would be produced, if the volt-amperes at the beginning of the stroke were zero, that is, if $E_1 = 0$; and the maximum output of the magnet thus would be

$$F_m l = \frac{I E_2 10^7}{4 \pi f g}. \quad \text{grm.-cm.} \qquad \ldots(15)$$

The ratio of the actual output, to the maximum output, or the efficiency of the electromagnet, is thus :

$$\eta = \frac{F}{F_m} = \frac{E_2 - E_1}{E_2} \qquad \ldots(16)$$

Or, using the more general equation,

$$\eta = \frac{L_2 - L_1}{L_2}, \qquad \ldots(17)$$

Thus, the efficiency* of an electromagnet is:

(1) the difference between the maximum and minimum voltage, divided by the maximum voltage: or,

(2) the difference between the maximum and minimum volt-amp. consumption, divided by the maximum volt-amp. consumption; or,

(3) the difference between the maximum and minimum inductance, divided by the maximum inductance.

Deductions.—From eqn. (15) it is evident that the maximum work that we can derive from a given expenditure of volt-amperes is limited. Thus, for $IE_2 = 1$, or for 1 volt-amp., the maximum work that could be derived from a given alternating-current electromagnet, is—

$$F_m l = \frac{10^7}{4\pi f g} = \frac{810}{f} \text{grm. cm.} \qquad \ldots(18)$$

Thus, referring to the previous example, a 50-cycle electromagnet can never give more than 16.2 grm.-cm., and a 25-cycle electromagnet never more than 32.4 gram-cm. pull per volt-ampere supplied to its terminals.

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* **Cf.—Expression for efficiency in thermo-dynamics**

$$\frac{T_2 - T_1}{T_2}$$

Inversely, for an average pull of 1 gram over a distance of 1 cm., a minimum of 1/16.2 volt-amp. is required at 50-cycles, and a minimum of 1/32.4 volt-amp. at 25-cycles.

Or, reduced to pounds and inches, for an average pull of 1 lb. over a distance of 1 in. at least 71 volt-amps. are required at 50-cycles, and at least 35.5 volt-amps. at 25-cycles.

This is a criterion by which to judge the success of the design of electromagnets.

31. The Constant-Potential Alternating Electromagnet.—

Let E_0 = constant alternating voltage impressed upon an electromagnet;

I = current, in amperes, which varies from a maximum I_1, in the initial position 1, to a minimum I_2 in the final position 2, of the armature;

L_1, L_2 = inductances of the winding in the two cases.

If the voltage consumed by the resistance, IR, be neglected compared with the voltage consumed by the reactance, IX, then the voltage E_0 impressed on the electromagnet is constant and is the terminal voltage E_0. Thus the magnetic flux, ϕ, is also constant during the motion of the armature of the electromagnet.

The e. m. f. induced in the magnet winding by the motion of the armature,

$$e = n \frac{d\phi}{dt} 10^8 \text{ volts}$$

is therefore zero, and hence also the electrical energy expended, $W=0$.

That is, the electric circuit does no work, and the mechanical work of moving the armature is done by the stored magnetic energy.

As in eqn. (4) the increase of stored magnetic energy is,

$$W' = \frac{I_2^2 \cdot L_2 - I_1^2 L_1}{2},$$

and the mechanical energy is by (4),

$$W = Flg\ 10^{-7} \text{ joules.}$$

Hence, the equation of the law of conservation of energy,

$$\overline{W} = W' + W$$

becomes $$0 = \frac{I_2^2 L_2 - I_1^2 L_1}{2} + Flg\ 10^{-7},$$

or, $$Fl = \frac{I_1^2 L_1 - I_2^2 L_2}{2g} 10^7 \text{ gram-cm.} \quad \ldots \quad (19)$$

Substituting from the equation of self-induction,

$$E_0 = 2\pi f L_1 I_1 \quad \ldots \quad \text{(in position 1)}$$
$$E_0 = 2\pi f L_2 I_2 \quad \ldots \quad (\text{,, } \quad \text{,, } \quad 2)$$

in eqn. (19), we have the equation of the constant-potential alternating electromagnet :

$$Fl = \frac{E_0 (I_1 - I_2)}{4\pi f g} 10^7 \text{ grm. cm.} \quad \ldots \quad (20)$$

and $$F = \frac{E_0 (I_1 - I_2)}{4\pi f g l} 10^7,$$

$$= \frac{E_0}{4\pi f g} \frac{dI}{dl} \text{ grms,} \quad \ldots \quad (21)$$

or, reducing to foot-pounds —

$$Fl = \frac{.586 \ E_0 \ (I_1 - I_2)}{f} \text{ ft.-lb.} \quad \dots \quad (22)$$

$$F = \frac{.586 \ E_0 \ (I_1 - I_2)}{f \ l} = \frac{.586 \ E_0}{f} \ \frac{d \ I}{dl} \text{ lb.} \quad (23)$$

Generalisation :—

If we substitute $Q = EI =$ volt-amperes, in eqns. (20) to (23), and eqns. (4) to (7), of the previous section, we have the same expressions of the mechanical work and pull, both for constant-potential and constant-current alternating electromagnets.

In metric system :

$$Fl = \frac{\delta \ Q}{4 \ \pi \ f \ g} 10^7 \text{ grm.-cm.} \quad \dots \quad (24)$$

$$F = \frac{\delta \ Q}{4 \ \pi \ f \ g \ l} 10^7 = \frac{1}{4 \ \pi \ f \ g} \cdot \frac{d \ Q}{d \ l} 10^7 \text{ grms.} \quad \dots \quad (25)$$

In foot-pounds :

$$F \ l = \frac{.586 \ \delta \ Q}{f} \text{ ft.-lb.} \quad \dots \quad (26)$$

$$F = \frac{.586 \ \delta \ Q}{f \ l} = \frac{.586}{f} \ \frac{d \ Q}{d \ l} \text{ lb.} \quad \dots \quad (27)$$

where $\delta \ Q =$ difference in volt-amperes consumed by the magnet in the initial and final positions of the armature.

Both types of alternating-current magnet, then, gives the same expression of efficiency, viz :

$$\eta = \frac{\delta \ Q}{Q_m}$$

where Q_m = the maximum volt-amperes consumed, corresponding to the end-position in the constant-current magnet, and to the initial position in the constant-potential magnet.

32. Law of the Plunger Electromagnet.—The following formula has been found by practice the most accurate and complete for the design of plunger electromagnets.

Let P = pull in pounds,
B = flux density in the working air-gap,
l = length of the air-gap,
IN = ampere-turns in the winding,
A = cross-section of plunger in sq. in,
P_c = pull at 10,000 ampere-turns and 1 sq. in. of plunger,
n = ampere-turn factor,

and, L = length of the winding in inches.

Then, the pull due to an iron-clad solenoid is—

$$P = \frac{AP_c\,(IN - n)}{10{,}000 - n},$$

and, at points along the uniform range of solenoids the pull for the plunger electromagnet will be:

$$P = A\left(\frac{IN^2}{7{,}075{,}600 \times l^2} + \frac{P_c\,(IN - n)}{10{,}000 - n}\right).$$

Here l must include the extra length assumed due to the reluctance outside of the working air-gap.

Pull in Pounds, and Ampere-turn Factor at Different Points along an Electromagnet.

L	P_c	n
1	33.0	3600
2	28.3	3150
3	23.4	2800
4	19.2	2500
5	16.0	2200
6	13.8	1970
7	12.2	1750
8	11.0	1580
9	10.0	1400
10	9.2	1230
11	8.4	1100
12	7.8	960
13	7.2	840
14	6.8	725
15	6.4	625
16	6.0	525
17	5.7	430
18	5.3	350
19	5.0	270
20	4.7	210

To approximate the curve of a Plunger Electromagnet at points between the centre of the winding and the ends of the winding where the plunger enters, assume that the curve is a straight line for the last ·4 of the distance ; then the pull at any point, *la* as measured in inches, back from the end of the winding will be :

$$P = A\left(\frac{I\,N^2}{7075000\,l^2} + \frac{l_a\,P_r\,(I\,N - n)}{.4\,L\,(10000 - n)},\right)$$

It is assumed that the winding is approximately as long as the inside of the frame.

33. The Permissible Heating of Magnet Coil and Surface of Emission.—The passage of a current through a wire is always accompanied by a generation of heat according to Joule's law. As a result, a general rise of temperature is produced in the wire, which goes on until the rate of loss of heat by radiation is equal to the rate of generation. In practice, it is necessary, therefore, to so fix the permissible heating of magnet coils and surface of emission, that no overheating is produced in the coils, by providing a radiating surface proportional to the heat developed, i. e. to the energy wasted in the coils as heat.

Let us term 'the energy dissipated per unit of radiating surface in watts the specific energy loss.' Now, the rise of temperature per unit specific energy loss, θ_m, is found to be constant under ordinary circumstances. Therefore, the actual rise of temperature of any magnet coil, above the atmospheric temperature, θ, is given by—

$$\theta = \theta_m \frac{P_m}{A_m},$$

where P_m = energy in watts wasted in the coils,

A_m = cooling or radiating surface,

so that, $\frac{P_m}{A_m}$ = specific energy loss.

Now, the rise of temperature which is compatible both with efficiency and safety, in practice, is found to vary between limits of 10° C and 50° C. And according to Esson, rate of emission of heat from field magnet coils is 1/355 watt per sq. cm. per degree Centigrade.

Hence, $\theta_m = 355$, for Centigrade degrees and radiating surface in sq. cms.

$= 55$, for Centigrade degrees and radiating surface in sq. inches.

$= 99$, for Farenheit degrees and radiating surface in sq. inches.

Therefore, for a rise of 10° C, $\dfrac{P_m}{A_m} = \dfrac{10}{55} = \dfrac{2}{11}$ watt per sq. inch.

And for a rise of 50° C, $\dfrac{P_m}{A_m} = \dfrac{50}{55} = \dfrac{10}{11}$ watt per sq. inch,

which gives as a mean value $\frac{6}{11}$ watt per sq. in. for a rise of temperature of 30° C. Inversely, the radiating surfaces for the temperature rises mentioned, will respectively be 5.5 and 1.1 sq. inches (or, 35 and 7 sq. cms.) per watt, giving as a mean value 3.3 sq.inches per watt. From this we derive the following rule for winding a coil :—

With a certain coil space permitting only a certain amount of surface, the winding must be so performed

that each 3.3 sq. inches of radiating surface can dissipate energy at the rate of one watt, in order that the temperature shall not rise above 30° C.

It may be mentioned here that Kapp allows 2.5 sq. inches (16.2 sq. cms.) per watt for field magnet coils, and 2 sq. inches per watt for a brush-arc dynamo.

The following is Dr. S. P. Thompson's formula for finding the MAXIMUM PERMISSIBLE CURRENT, I_m, with the maximum rise of temperature permissible—

$$I_m = \sqrt{\frac{t^\circ\ C \times \text{sq. cms.}}{355 \times \text{resistance (hot)}}},$$

or, $$I_m = \sqrt{\frac{t^\circ\ F \times \text{sq. inches}}{99 \times \text{resistance (hot)}}}.$$

If we assume that a safe limit of temperature is 90° F (50° C) higher than the surrounding air, then the largest current which may be used with a given electromagnet is expressed by the formula :—

$$\text{Highest permissible amperage} = 0.95\sqrt{\frac{\text{sq. inches}}{\text{resistance (ohms)}}}$$

Similarly, for shunt coils we have :—

$$\text{Highest permissible voltage} = 0.95\sqrt{(\text{sq. Ins.}) \times (\text{resist.})}$$

34. Loss of Heat in a Coil is Independent of the Size of Wire:—For a coil of given volume the energy wasted as heat is the same for the same magnetising power, irrespective of the gauge of the wire.

If H = amount of heat generated per second,

IN = ampere-turns,

then, $H = I^2 R_m$, ... (1)

and IN = the excitation ampere-turns.

Let now the gauge be so changed that for the same volume we have the same magnetising power or excitation,

i. e. $NI = I'N'$

or, $I' = \frac{N}{N'} I.$

But, for wires of the same material the resistance in ohms varies directly as the square of the number of turns in a coil filling a given space. Therefore,

$$\frac{R'_m}{R_m} = \frac{N'^2}{N^2}$$

or, $R'_m = \frac{N'^2}{N^2} R_m.$

Hence the heat generated in this case,

$$H' = I'^2 R'_m = \left(\frac{N}{N'} I.\right)^2 \times \left(\frac{N'^2}{N^2} . R_m\right) = I^2 R_m$$

which is the same amount as given by (1)

Example 10. The field magnet coil of a $2\frac{1}{2}$ kilowatt series dynamo giving 10 amperes at 250 volts requires 5250 ampere-turns, the winding having a depth of 1.5 inches. The core is 4 inches by 3 inches, and also 7 inches long. Determine the resistance, size, and weight of the copper wire used for the winding.

Solution :—

(1) We assume that the temperature rise does not exceed 30° C. at normal load.

Here, $IN = 5250$, and $I = 10$ amperes,

$$\therefore N = \frac{5250}{10} = 525.$$

The length of a mean turn is

$$l_m = 2\ (4+1.5) + 2\ (3+1.5)$$
$$= 20 \text{ inches.}$$

Therefore, the total length of the wire

$$l = \frac{20 \times 525}{12} = 785 \text{ ft.}$$

The radiating surface of the coil

$$A_m = 7\left\{2 \times (4+3) + 2 \times (3+3)\right\}$$
$$= 182 \text{ sq. inches.}$$

Allowing 3.3 sq. inches for radiating heat at the rate of one watt, we have

$$I_m^2\ R_m = \frac{182}{3.3}.$$

$$\therefore\ R_m = \frac{182}{100 \times 3.3} = .552 \text{ ohms,}$$

which is the resistance of the coil at a temperature 30°C above that of the atmosphere, which we may suppose to be at 15° C.

Taking the temperature coefficient of copper to be .00387, the resistance of the winding at 15^0 C is—

$$R = \frac{R_m}{(1 + .00387 \times 30)} = \frac{.552}{1.1161} = .495 \text{ ohms.}$$

(2) The resistance of 1000 ft. of the wire

$$= \frac{.495 \times 1000}{875} = .566 \text{ ohms.}$$

From the wire table we find that the nearest gauge corresponding to this is No. 10 S. W G.

(N. B.—The size of the wire is also, to some extent, fixed by the current which has to flow through it, and

by the permissible heating. The maximum permissible current for No. 10 S. W. G. is 32 amps. So 10 amps. is quite a safe current to pass through it.)

(3) The wire has a weight of 78.97 lbs. per each ohm of resistance. Therefore, the weight of the winding

$$W = 78.97 \times .496,$$
$$= 39 \text{ lbs.}$$

VERIFICATION.—The diameter of the wire is .128 inch, and allowing .012 inch for insulation, we have .14 inch as the diameter of the insulated wire.

$\therefore$ Number of layers $= \dfrac{1.5}{.14} = 10.7,$

and number of turns per layer $= \dfrac{7}{.14} = 50.$

$\therefore$ Total number of turns, $N = 50 \times 10.7 = 535.$

35. Lamont-Frohlich's Law of the Electromagnet.—The electromotive force E, induced in the armature of a generator is proportional to the magnetic flux linked with the armature coils, and to the speed of the machine.

Thus, a relation can be found between the E. M. F. produced in the armature at a given speed, and the exciting current in the field magnet coils. In other words, E can be expressed as an imperical function of the magnetising current, I (say), with a certain degree of accuracy.

Such an emperical formula which agrees best with

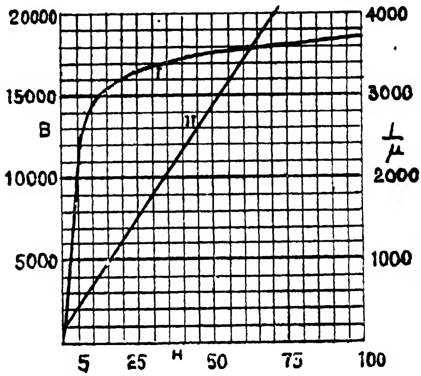


Fig. 2.03.

experimental results is due to Frohlich previously proposed by Lamont, and may be deduced from information supplied by the magnetisation curves of samples of iron and steel employed in dynamo construction. In Fig. 2.03 the magnetisation curve (I), for a sample of steel forging has been constructed from the following data given by Prof. Ewing :—

Magnetizing Force. H		Induction. B		Permeability. μ		Reluctivity. $\frac{1}{\mu}$ or K
5	...	12300	...	2460	...	406
10	...	14920	...	1492	...	670
15	...	15800	...	1054	...	949
20	...	16280	...	814	...	1228

H		B		μ		$\frac{1}{\mu}$ or K
30	...	16810	...	560	...	1785
40	...	17190	...	429	...	2331
50	...	17500	...	350	...	2857
60	...	17750	...	296	...	3375
70	...	17970	...	257	...	3891
80	...	18180	...	225	...	4444
90	...	18390	...	204	...	4902
100	...	18600	...	186	...	5376

The reluctivity $\frac{1}{\mu}$ (reluctance of a centimetre cube, which is always less than unity) given in the 4th column, is expressed in terms of a unit which is the millionth part of the reluctivity of air. Curve (II) represents the eluctivity of the sample of steel forging for dynamo nagnets, and, as will be observed, the reluctivity follows a straight line law. Hence,

$$\frac{1}{\mu}=a_1+b_1H=K \qquad \dots \qquad \dots \qquad (1)$$

where a_1 and b_1 are constants) represents the relation between reluctivity and magnetizing force.

Also, $$H=\frac{4\pi N I}{10l}, \text{ and } B=\mu H. \qquad \dots \qquad (2).$$

From eqns. (1) and (2) we get the following elation between induction B and the number of mpere-turns N I :—

$$\mu=\frac{1}{a_1+b_1H.}$$

$$B=\mu H=\frac{\frac{4\pi}{10l}\,N\,I}{a_1+b_1\frac{4\,\pi}{10\,l}N\,I}=\frac{a\,N\,I}{1+b\,N\,I}=\frac{I}{\alpha+\beta\,I}\quad\cdots\quad(3)$$

where

$$a=\frac{4\,\pi}{10\,\,la_1},\ b=\frac{4\,\pi\,b_1}{10\,la_1},\ \alpha=\frac{a_1}{\frac{4\,\pi\,N}{10\,l}},\ \beta=\frac{b}{a}=b_1.$$

The expression in eqn. (1) represents Frohlich's Law in a simple form.

Eqn. (1) can also be expressed in terms of B. Thus,

since $B=\mu H$, and $\mu=\frac{1}{K}$, we have $H=K\,B$,

and therefore from (1),

$$K=a_1+b_1\,K\,B,$$

or, $$K=\frac{a_1}{1-b_1\,B.}\qquad\ldots\qquad\ldots\qquad(4)$$

This form is sometimes useful in calculations relating to compound magnetic circuits containing air-gaps.

A short table of values of a_1 and b_1 for the relation $K=a_1+b_1H$, due to Messrs. Haustan and Kennelly, is given below for reference.

For ordinary dynamo cast iron	$K= 0.0026+0.000093\ H.$
For dynamo wrought iron	$K= 0.0004+0.000057\ H.$
For soft iron(stoletow)	$K= 0.0002+0.000056\ H.$
For cast iron	$K= 0.0010+0.000129\ H.$
For Norway iron	$K= 0.0001+0.000059\ H.$
For steel	$K=0.00045+0.000051\ H.$

Example 11. A soft iron-ring, 4 sq. cms. cross-section and 12 cms. mean radius, is wound with 870 turns of wire carrying a current of 50/87 amps. Determine the reluctance of the magnetic circuit, the total magnetic flux and the permeability of the iron.

Solution :—

$$\text{Since} \quad H = \frac{.4\,\pi\,N\,I}{l},$$

$$H = \frac{.4\,\pi \times 870 \times 50}{2 \times 12 \times \pi \times 87} = 8.3 \text{ units.}$$

Now, substituting the values of a_1 and b_1 for soft iron, the reluctivity

$$K = .0002 + .000056\ H.$$

$$= .0002 + .000056 \times 8.3 = .00066.$$

$$\therefore \text{Reluctance } R = K.\frac{l}{A} = .00066\ \frac{24\ \pi}{4} = .0124,$$

$$\text{and, permeability } \mu = \frac{1}{K} = \frac{1}{.00066} = 1515.$$

$$\text{Again, magnetic flux, } \phi = \frac{\text{M. M. F.}}{\text{Reluctance}}$$

$$= \frac{1.257\ N\ I}{.0124}$$

$$= \frac{1.257 \times 870 \times 50}{87 \times .0124}$$

$$= 50685.$$

Example. 12. Determine the current required to produce a flux density of 12500 lines in an air-gap 4 mms. wide cut across the iron ring.

Solution :—

$$\text{Reluctance of air-gap } R_a = \frac{.4}{4} = .1$$

$$\text{Reluctance of iron } R_i = K_1 . \frac{l_1}{A},$$

$$\text{where } K_1 = \frac{a_1}{1 - b_1 B} = \frac{.0002}{1 - .000056 \times 12500}$$

$$= \frac{.0002}{.3} = .00067,$$

$$\therefore R_i = .00067 \times \frac{24\pi - .4}{4} = .012556,$$

$$\text{and,} \quad R = R_a + R_i = .1 + .012556 = .112556,$$

$$\text{Now, total flux, } \phi = \frac{\text{M. M. F.}}{\text{R.}}.$$

$$\therefore 4 \times 12500 \times .112556 = 1.257 \times 870 \times I$$

$$\therefore I = \frac{562780}{109359} = 5.11 \text{ amperes.}$$

These results agree fairly well with those of Ex. 13 which verifies the truth of Frohlich's Law.

Example 13. An iron-ring, 4 sq. cms. cross-section and 12 cms. mean radius, has an air-gap 4 mms. wide cut across it. The ring is wound uniformly with a number of turns of insulated wire carrying a current of 5 amperes. If there is no leakage, find the number of turns necessary to produce a field of strength 12500

C. G. S. units in the gap (permeability of iron = 2000, for the excitation used).

Solution :—

Here, we have—

$I=5$ amps., $N=10,000\times 4$, $l_1=.4$, cm., $l_2=(2\pi\times 12-.4)$, $A_1=A_2=4$ cms., $\mu_1=1$, $\mu_2=2000$, so that

$$\phi=\frac{1.257\ N\ I}{\frac{l_1}{\mu_1 A_1}+\frac{2\pi r-l_1}{\mu_2 A_2}}=\frac{1{,}257\times N\times 5}{\frac{.4}{4}+\frac{24\pi-.4}{2000\times 4}},$$

$$\text{or,}\quad 12500\times 4=\frac{1.257\times N\times 5}{.10935}=\frac{1.257\ N}{.02187},$$

$$\therefore N=\frac{12500\times 4\times .02187}{1.257}=870 \text{ turns.}$$

Exercises.

1. A ring-shaped electromagnet has an air-gap 6 mm. long and 20 sq. cm. in area, the mean length of the core being 50 centimetres, and its cross-section 10 sq. cm. Calculate, approximately, the ampere-turns required to produce a field of strength $H=5{,}000$ in the air-gap. (Assume permeability of iron as 1,800). (C. & G., II., 1908).

2. What is an electromagnet ? In what way does its magnetism depend on its core ? What are the reasons which determine, in any case, whether the coil should consist of a few turns of thick wire or of many turns of thin wire ? (C. & G., I., 1908).

3. What rules can you give about winding electro-magnets? What are the circumstances that determine the selection of any particular size of wire for the coil? (C. & G., I., 1906).

4. The air-gap area of each pole of a smooth-core four-pole dynamo is 300 square centimetres, and the distance from pole-face to core is 5 millimetres. How many ampere-turns must be used on each pair of poles for air-gap excitation alone if the magnetic density in the air-gap is 10,000 lines per square centimetre? (C. & G., II., 1910).

5. What is the magnetisation curve of a material? Sketch approximately to scale such curves for air, hard steel, wrought-iron and cast-iron. (C. & G., II., 1913).

6. How does the pull, which an open solenoid exerts on its core, vary with the position of the core in the following cases: (a) when the core itself is much longer than the solenoid; (b) when the core is from one-quarter to one-half as long as the solenoid; (c) when a very short cylinder or a sphere of iron is used instead of the usual core? In the case (a) state also what difference, if any, will result if the solenoid is surrounded by a cylindrical jacket of iron, closed at the bottom by an iron disc. State also how the pull, in any given case, will be affected by a reduction of the excitation to one-half its normal value. (C. & G., II., 1907).

7. A solenoid, 12 in. long, wound on a brass tube $1\frac{1}{4}$ in external diameter, is coiled with a No. 13 S.W.G.

wire, having 10 layers with 115 turns in each layer, and can carry 5 amperes without undue rise of temperature. Calculate the strength of the magnetic field produced, (a) at the centre of the coil, (b) at the open mouth of the coil. What is the maximum pull you would expect it to exert on a cylindrical rod of soft iron half an inch in diameter and 15 in. long? In what position of this rod will the pull be a maximum?

(C. & G., II., 1910).

8. A smooth-core armature, working in a four-pole field magnet, has a gap (from iron to iron) of 0.5 inch. The area of surface of each pole is one square foot. The flux from each pole is 7 megalines. Find (a) the mechanical force with which the pole attracts the armature; (b) the amount of energy expressed in joules that is stored in the four gaps. (N.B.—746 joules = 550 footpounds at London; 1 foot = 30.48 cm.; 1 pound = 453.6 grm.). (C. & G., II., 1909).

9. Explain why the holding-on force of a magnet is proportional to the square of the flux-density at the surfaces in contact. Which will exert the greater hold-on force—a magnet having a flux of 400,000 magnetic lines and a contact surface of 8 square inches, or one having 500,000 lines and a contact surface of $12\frac{1}{2}$ sq. inches? (C. & G., II., 1909).

10. One field coil of a dynamo has to give 10,000 ampere-turns with a potential difference of 110 volts between its terminals. The mean length of one turn of

wire on the coil is 0.4 metre. Find the diameter in millimetres of the copper wire required; also find the number of turns of wire in the coil for a permissible loss of 90 watts. Take the resistance of a copper wire 1 metre long and 1 square millimetre in cross-section at the temperature of the coil as 0.02 ohm.

(C. & G., II., 1914).

11. You have to make a long coil of wire such that when 10 amperes flow through it the magnetic field at the centre shall be 1000 times as strong as the earth's horizontal field (H). How many turns per centimetre must be put on the coil? $H = 0.18$. (C. & G. 91.).

12. You are required to wind over a brass tube 500 centimetres long and 2 centimetres external diameter with one layer of covered wire, 1 mm. diameter over the covering. What length of wire will you require? What will be the strength of the magnetic field at the centre of the axis of such a helix when a current of one ampere flows through the wire?

(C. & G., 87).

13. Calculate approximately the strength of the magnetic field produced at the centre of a solenoid 4 inches long, the coils of which are $1\frac{1}{2}$ inches thick and have a mean diameter of $7\frac{1}{2}$ inches, when the current density taken over wire and insulation together and measured on a plane containing the axis of the solenoid is 750 amperes per square inch. Could a coil so constructed

and working with this current density be used for long periods without UNDUE heating? (C. & G.).

14. A closed soft iron ring, 100 cms. mean circumference and 5 sq. cms. cross section, is uniformly wound with 200 turns of insulated wire. Suppose you have found that the following relations exist in iron of this quality:

B=10200	12000	13700
μ= 2000	1500	1000

Calculate the current C at which the total flux of magnetic lines is 65000 C. G. S. lines. (C & G., 92.).

15. Calculate approximately the strength of the magnetic field produced at the centre of a solenoid 4 inches long, the coils of which are $1\frac{1}{2}$ inches thick, and have a mean diameter of $7\frac{1}{2}$ inches, when the current density taken over wire and insulation together and measured on a plane containing the axis of the solenoid is 750 amperes per sq. inch. (C. & G.).

16. An electromagnet has to be designed to produce an approximately uniform field of 10,000 lines per square centimetre over an area of 12 sq. centimetres in an air gap half a centimetre long. Sketch an electromagnet, approximately to scale, with which this result can be obtained without undue heating of the coils. State the gauge of wire, number of convolutions, and current that may be used, and describe in detail how your results are obtained. (C. & G.).

17. Two electromagnet bobbins are fully wound with the same weight of wire, the latter being 4 mils gauge in the one case and 2 mils gauge in the other. What will be the relative resistances of the two bobbins? (C. and G.).

18. An electromagnet is wound to a resistance of 320 ohms, with wire 20 mils in diameter. What diameter would the wire have to be in order that with the same weight of wire on the electromagnet the resistance may be 20 ohms? (C. and G.).

19. The resistance of the wire on a bobbin fully wound with silk-covered wire 7 mils diameter is found to be 120 ohms. What will be the resistance if the same bobbin be equally wound with 10 mils wire? The thickness of covering to be taken as 1 mil in each case. (C. and G.).

20. An electromagnet bobbin, 2 inches long, 1 inch external, and $\frac{1}{2}$ inch internal diameter, is to be filled with wire, 20 mils in diameter; what length of wire will be required? (C. & G.).

21. Calculate the size, resistance, and weight of copper wire such that, if wound on a magnet core, 7 inches by $3\frac{1}{2}$ inches, and having a potential difference of 25 volts maintained between the terminals, 5000 ampere turns will be produced. Length inside former is 8 inches. (C. & G.).

22. When fully excited, the values of the induction B, in the various parts of the magnetic circuit

of a dynamo, and the mean lengths of path l of the magnetic flux, are as given below :—

Magnet cores, high permeability cast steel (two in a circuit) ...	$l=30$ cms.:	$B=13000$
Magnet yokes, high permeability cast steel ...	$l=40$,, :	$B=10000$
Air-gap (flux crosses twice in a circuit) ...	$l=7$ mm.:	$B=7000$
Armature Teeth (flux passes through two groups each) ...	$l=25$,, :	$B=20000$
Armature core, charcoal iron sheets ...	$l=30$ cms.:	$B=10000$

The mean length of a turn of shunt winding is 80 cms., and the E.M.F. measured at the ends of the coils on a pair of limbs is 100 volts. What would be the size of wire required for a shunt winding neglecting any allowances for armature reaction ? (C. and G.).

23. The limb of a dynamo magnet is of square section, each side measuring 8 inches the length of the portion occupied by shunt winding is 7 inches ; what size and quantity of wire will be required to form a shunt coil which will give 12000 ampere-turns when a pressure of 100 volts is maintained between its two ends, and allowing a cooling surface of 2 square inches per watt wasted? The cooling surface may be taken as the area of the external surface of the coil, plus that of the two ends. The resistance of a cubic inch of copper is approximately 0.66 microhm at 60° F., and

the coefficient for increase of resistance with rise of temperature 0.21 per cent. per degree F. The working temperature of the coil is to be taken at 110°F. (C.&G.).

24. A horse-shoe electromagnet with a core and keeper forged from 1-inch square iron is excited by 300 ampere-turns. Let the joints between the pole faces and keeper be scraped so as to make a perfect fit, and assume that the permeability of the metal at the junction produced by 300 ampere-turns is 1500. Length of magnetic circuit 16 inches. Find the total flux through core and keeper, and the force required to tear the keeper off. (C. & G., 93.).

25. A drum armature in a 2-pole field contains 150 external conductors, and runs at 550 revolutions per minute. Find the total flux passing through the armature, which is required to produce an electromotive force of 115 volts on open circuit. (C. & G., 93).

26. Calculate the resistance of a Gramme armature wound with 144 turns of rectangular wire 0.2×0.21 inch, length of armature core without insulation 12 inches, radial depth 2.5 inches. The resistance of 100 yards of copper rod, one square inch in cross section, is .0025 ohm. (C. & G.).

27. You have a cylindrical armature core 12 inches diameter, 15 inches long, and two-pole field, bore of pole field 13.4 inches, polar angle 112°. The armature has 150 external conductors, and carries a current of 250 amperes. Determine the following:

(1) total flux of lines to produce an armature E. M. F. of 115 volts at 550 revolutions per minute; (2) exciting power required for air-space; (3) average induction in air-space; (4) induction under the polar edges at full load. (C. & G.).

28. Determine the winding of a Gramme armature, the core of which has a length of 12 inches, (not including the thickness of the insulation), a diameter of 7 inches, and a radial depth of iron of 2 inches. The bore of the pole-pieces is not to exceed 13 inches. The dynamo is to produce 100 volts at 1000 revolutions per minute, and the largest current permissible so as not to raise the temperature of the armature more than 75° F. above that of the air. Assume such an induction as you think desirable. Calculate the resistance of the armature cold and hot. (C. & G.).

29. A firm has in stock a field magnet wound with 1540 turns on each of its two limbs, and having a total resistance of 14 ohms warm. The length of the mean line is 80 centimetres, and the polar bore 14½ inches in diameter. Consider whether it is possible to construct an armature for this field to give 400 amperes at 110 volts at a speed of 540 r.p.m. If it be possible, give the diameter of the smooth core you would use, the number and size of the armature bars, and state whether the machine would have any faults which would not have existed had you designed an entirely new machine. The length of the magnet parallel with the shaft is 20 inches and its smallest

width 9 inches. The materials used in the armature and field cores have the following qualities :—

For a density	= 11000	C.G.S. lines	in the	armature	$H = 5.6$
,,	= 12000	,,	,,	,,	,,	$H = 7.6$
,,	= 13000	,,	,,	,,	,,	$H = 11.3$
,,	= 14000	,,	,,	,,	,,	$H = 16.9$
,,	= 12000	,,	,,	in the field magnets.		$H = 10$
,,	= 13000	,,	,,	,,	,,	$H = 14$
,,	= 14000	,,	,,	,,	,,	$H = 18$

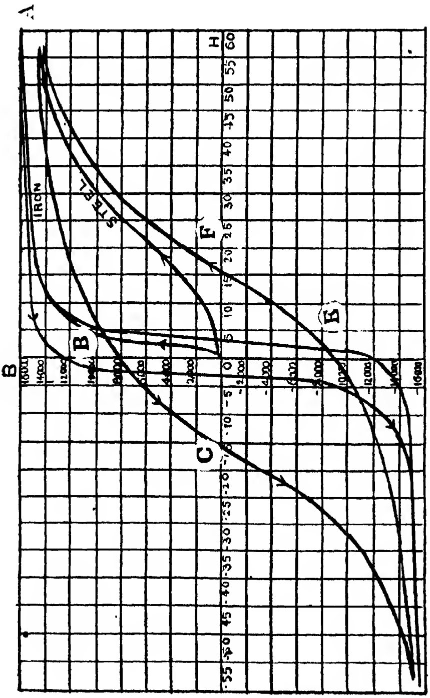
(C. & G.).

CHAPTER III

HYSTERESIS AND EDDY CURRENT.

36. If a magnetic substance is magnetized in a strong magnetic field it retains a considerable portion of magnetism after the magnetic force has been withdrawn.

The phenomenon of the lagging of the induction behind the magnetizing force is known as MAGNETIC HYSTERESIS. It involves dissipation of energy in the form of heat in overcoming the molecular friction of iron and the work done is proportional to the area of the hysteresis loop.



HYSTERESIS CURVE.

Fig. 3.01

Apply a gradually increasing magnetizing force to a sample of annealed wrought iron which has been previously demagnetised, and measure the induction B. We find that when the magnetizing force is first applied the induction B varies with H in a manner which may be represented by the curve OA. After having reached the point A on the ascending curve let the magnetizing force be gradually reduced from A to O again. It will be found that the descending curve is not identical with the ascending curve, but is considerably higher, as may be shown by the part A B. When the magnetising force is zero, the induction B is not zero, but has a value represented by OB. The height OB represents the residual magnetic induction of the iron, i. e., the lines of force, per square centimetre, passing through the iron when the magnetizing force has been entirely withdrawn. Now reverse the magnetising force by reversing the direction of the current through the magnetising coil. The iron rapidly loses its magnetism, and a negative force, represented by OC, is necessary to deprive the iron of all its magnetism. When the negative magnetising force is increased beyond OC the iron becomes negatively magnetised and reaches a maximum at the point D. At this point the induction B is of the same numerical value as at the point A, but of the opposite sign. After the magnetising force is again reversed it requires a positive force equal to OF to deprive the iron of its negative magnetism. Lastly, by increasing the magnetising force from O to the same positive maximum as at first, the

curve EFA is obtained. A loop ABCDEFA has then been described, and is known as a complete magnetic cycle. If the cycle of operations be repeated a curve indential with ABCDEFA will be obtained.

We find that in the curve (1) when H is reduced to 0,B still has a considerable value, OB; this is termed the *retentivity*. (2) To reduce the induction to zero a negative or demagnetising force, represented by O C, has to be applied; this is called the *coercive force*. (3) The return half DEFA of the curve is a repetition of the first half ABCD, and OE=OD, and OF=OC.

37. The two losses occuring in iron subjected to an alternating magnetic field are (1) the hysteresis loss (2) the eddy current loss.

Hysteresis loss may be produced in two ways: (1) when the magnetic force acting upon the iron passes through zero when changing from a positive to a negative—this is known as an alternating field, having a fixed orientation in the iron and takes place in the core of transformers. (2) When the magnetic force remains constant in value but varies in direction, this is termed a revolving or rotating magnetic field, in which the orientation of the induction rotates continuously. It is the field, an armature is subjected to, when it revolves between the pole-pieces. The resultant hysteresis loss from these two causes differs greatly.

HYSTERESIS LOSS.

38. The hysteresis loop indicates by its area the work done on the electromagnet per cycle of change of current.

Let T = the time of the cycle variation of current,
W = work performed during the cycle,
S = number of turns,
e = the induced E. M. F.,
ϕ = total flux = $A\ B$.
i = the current in amperes,
H = the magnetizing force,
L = the length of the magnetic circuit,
A = cross-sectional area of the magnetic circuit in square centimeters.
V = volume of the magnet = $A.l$

The work done in an electric circuit is $\int eidt$,

then, $$W = \int_0^T eidt,$$

$$e = -\frac{S}{10^8}\frac{d\phi}{dt},$$ where $d\phi / dt$ is the rate of change of flux.

$$\therefore W = -\int_0^T \frac{Si}{10^8}\frac{d\phi}{dt}dt.$$ But $\phi = AB$.

$$\therefore W = -\int_0^T \frac{ASi}{10^8}\frac{dB}{dt}dt.$$

But $$H = \frac{0.4\pi iS}{l}$$

Thus, $$iS = \frac{lH}{0.4\pi},$$

Substituting,

$$W = -\int_0^T \frac{AlH\,dt}{0.4\pi \times 10^8}\,\frac{dB}{dt} = \frac{Al}{10^7 \times 4\pi}\int_0^T HdB.$$

But Al is equal to V. Thus,

$$W = -\frac{V}{10^7 \times 4\pi}\int_0^T HdB \text{ joules per cycle.}$$

$\int_0^T HdB$ is the area of the hysteresis loop corresponding to maximum density, B, as seen from the loop. The area is obtained from measurement with a planimeter and is approximately equal to twice the retentivity into twice the coercive force, that is, $2\,OC \times 2\,OB = BE \times CF$. The work is given in joules per cycle.

39. The Form and Area of a hysteresis loop depend upon the kind of material, and the harder the physical state of the material the larger will be the area of the loop, and consequently the greater will be the hysteresis loss. This may be seen from the figure 3·01.

It is convenient to have the loss of energy due to hysteresis expressed in watts per kilogramme of material.

Ergs per cubic centimetre per cycle

$$= \frac{\text{area of hysteresis loop}}{4\pi},$$

Watts per cubic centimetre per cycle per second

$$= \frac{\text{area}}{4\pi} \times 10^{-7}.$$

Now, 1 cubic, centimetre of sheet iron weighs 7.8 grammes. Hence watts absorbed per kilogramme per cycle per second

$$=\frac{\text{area}}{4\pi} \times 10^{-7} \times \frac{1}{7.8} \times 1000,$$

$$=0.000001 \times \text{ergs per cubic centimetre per cycle.}$$

Since 1 erg per second $= 10^{-7}$ watts, the watts lost per cubic centimetre per cycle per second $= 10^{-7}.\eta.B^{1.6}$. Therefore, watts lost per kilogramme per cycle per second $= 10^{-7} \times \eta \times B^{1.6} \times \frac{1}{7.8} \times 1000$,

$$=0.000013\ \eta.\ B^{1.6}.$$

So that, when w kilogrammes of iron are subjected to n magnetic cycles per second,

the total loss in watts $=0.000013\ \eta \times B^{1.6} \times w \times n$.

Example 1. Find the rise of temperature in degrees Centigrade of a mass of iron having a volume of 20000 cubic centimetres. The iron revolves for one hour in a magnetic field the induction density of which is 10,000 lines per square centimetre and has a frequency of 50 cycles. Assume 20 per cent of the heat to be lost by radiation. When B=10000 the ergs lost per cycle per cubic centimetre equals 11000.

Solution:—

Let T be the rise of temperature.

Specific heat of iron $=0.11$.

One cubic centimetre of iron weighs 7.8 grams.

One calorie $=4.2 \times 10^7$ ergs.

The total work done against hysteresis,

$= 11000 \times \text{Volume} \times \text{cycles per second} \times \text{time},$

$= 11000 \times 20000 \times 50 \times 60 \times 60$ ergs.

The work done in heating the iron

$= \text{weight} \times \text{temperature rise} \times \text{specific heat} \times 4.2 \times 10^7$

80 percent of the work done against hysteresis

= work done in heating the iron

That is $\frac{80}{100} \times 11000 \times 20000 \times 50 \times 60 \times 60$

$= 7.8 \times 20000 \times T \times 0.11 \times 4.2 \times 10^7$

$\therefore$ the rise of temperature of the iron

$$T = \frac{80 \times 11000 \times 50 \times 60 \times 60}{100 \times 7.8 \times 0.11 \times 4.2 \times 10^7} = 44^\circ \text{ C nearly.}$$

40. The Hysteretic Loss is proportional to the 1.6 power of the maximum density and directly proportional to the frequency.

If V = the volume,

η = the constant which depends upon the quality of the iron, $W = \frac{\eta V f B_m^{1.6}}{10^7}$ watts (Steinmetz). In centimetre measurements η varies from 0.001 to 0.002 in ordinary sheet iron and may be 10 times as great in tempered steel. In the best silicon steel, it is 0.0006, which corresponds to 0.54 watt per lb. at 60 cycles and a density of 64,500 lines per sq. in. or 10,000 lines per sq. cm.

For higher flux-densities one must use a curve derived from experiment. Fig 3.02 (next page) when used in conjunction with the hysteretic constant, is useful in giving the hysteresis loss up to any flux-density ordinarily

employed in dynamos. In what follows, K_h is a function of B, such that $K_h \times \eta$ = the hysteresis loss in joules per cycle.

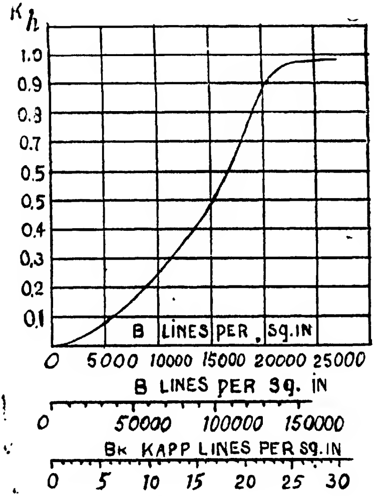


Fig. 3·02

Shows how the hysteresis loss in iron increases with the flux-density.

Table I (next page) gives the value of the hysteretic constant for different kinds of iron and steel.

Fig. 3·02 has been arranged so that, whichever of the three commonly used systems of units is employed, the loss per cu. cm., the loss per cu. in. or the loss per lb. can be readily arrived at. The following are the constants to be used in conjunction with K_h given in the figure :

$K_h \times \eta$ = joules per cu. cm. per cycle.

$K_h \times \eta \times n$ = watts per cu. cm. at frequency n.

$16.4 \times K_h \times \eta \times n$ = watts per cu. in. at frequency n.

$59 \times K_h \times \eta \times n$ = watts per lb. at frequency n.

Table I Hysteretic Constants.

Material.	Hysteretic constant = η	Material.	Hysteretic constant = η
Good dynamo Sheet steel.	0.002	Silicon steel (Si) = 1.8 %	0.004
Fair dynamo Steel.	0.003	Silicon steel (Si) = 0.2 %	0.0021
Silicon steel (Si) = 4.8 %.	0.00076	Very soft iron	0.002
Silicon steel = 4 %	0.001	Cast iron	0·016 0.011 to
Silicon steel = 3.5 %	0.0013	Cast steel	0.003 to 0.012
Silicon steel = 3 %	0.0016	Hardened cast steel	0.028
Silicon steel = 2.5 %	0.0022	Barrett's aluminium iron	0.00068

In direct current armatures hysteresis loss usually amounts to 2.8 watts per pound at f = 60 and B = 64500.

Example 2. Find the loss due to hysteresis in the armature iron behind the slots of a 25-cycle generator, the maximum flux-density in the iron being 15,000 lines

* Vide Miles Walkers Specification And Disegn of Dynamo-Electric Machinery.

per square cm. and the volume of iron (which is of ordinary quality) being 300,000 cu. cm.

Solution :—

From Fig. 3.02, for $B=15{,}000$, $K_h=0.43$. We will take the hysteretic constant as being 0.003.

$\therefore$ Hysteretic $\text{loss}=0.43\times0.003\times25\times300{,}000$,

$=9675$ watts.

Example 3. Find the hysteretic loss in the teeth of the same generator, the total volume of the teeth being 2000 cu. in. and the average flux-density in the teeth being 125,000 lines per square inch.

Solution :—

From Fig. 3.02, $K_h=.85$,

$\therefore$ Hysteretic $\text{loss}=16.4\times.85\times0.003\times25\times2000$

$=2091$ watts.

Example 4. If we were to work the iron behind the slots of this generator at 20 kapp lines per sq. inch, how much extra loss would we have and how many lbs. of iron would we save ?

Solution :—

300,000 cu. cms. of iron weigh 5184 lbs.,

150,000 lines per sq. cm. $=16$ kapp lines per sq. in.,

$\frac{5184}{1}\times\frac{16}{20}=4147$ lb. giving saving of 1037 lb.

From Fig. 3.02. for $B_k=20$, $K_h=.8$.

$\therefore$ Hysteretic $\text{loss}=59\times.8\times0.003\times25\times4147$

$=14680$ watts,

and hence, $14680-9675=5005$ watts extra loss at the higher flux-density.

Example 5. The hysteresis loss for a particular quality of iron is 1.2 watt per lb. at 50 cycles per second, and 40,000 lines per sq. in.

(a) Calculate the loss for 20 lb. of iron at 40 cycles per second, and 70,000 lines per sq. in.

(b) If the specific gravity of the iron is 7.7, calculate the hysteretic constant.

Solution :—

(a) $\text{Loss} = 20 \times 1.2 \times \frac{40}{50} \times \left(\frac{70,000}{40,000}\right)^{1.6}$

$= 20 \times 1.2 \times .8 \times (1.75)^{1.6} \quad = 47.4$ watts.

(b) 40,000 lines per sq. in. $= \frac{40,000}{6.45}$ lines/sq. cm.

$= 6202$ lines/sq. cm.

1 lb. of iron $= \frac{453.6}{7.7}$ c.c. $= 58.9$ c.c.

1 watt $= 10^7$ ergs per sec.

$\therefore$ Loss per cycle per c. c. $= \frac{1.2 \times 10^7}{50 \times 58.9} = 4074.7$ ergs at 6,202 lines/sq. cm.

$\therefore \ 4074.7 = h(6,202)^{1.6}$

$\therefore \ h = \frac{4074.7}{(6202)^{1.6}}$

$= .00354$

41. Nature of Hysteresis.—In stationary machines, e. g. transformers, this loss due to hysteresis is supplied electrically in the form of an additional component of the magnetising current.

In generators, hysteresis causes an opposing torque between the armature and the field, and the energy

lost for overcoming this torque is supplied by the prime mover.

In motor, this loss usually reduces the useful torque available on the shaft, or may be supplied from the mains.

(2) It is important to note that the COUNTER-TORQUE CAUSED BY HYSTERESIS IS INDEPENDENT OF THE SPEED OF THE MACHINE. This may be shown as follows:

Let T be this torque in foot-pounds, and n the speed of the machine (r.p.m.)

The power-required to overcome this torque is proportional to the product "torque × speed,"

i.e. $W = K\, n\, T$ watts, ...(2)

where K is a constant.

Therefore, from (1) and (2), we have

$$K\, n\, T = \eta\, V\, f.\, B^{1\cdot 6} \qquad \ldots \qquad \ldots (3)$$

Now, f the frequency of magnetisation is proportional to the speed n of the machine, so that (3) reduces to

$$T \doteq K'\, \eta\, V\, B^{1\cdot 6} \qquad \ldots \qquad \ldots (4)$$

where K′ is a new constant.

Equation (4) being independent of n proves that the hysteresis torque is independent of the speed of the machine.

Again, the power lost by hysteresis is proportional to the number of cycles per second,

i.e. $W = k \times n \times$ loss per cycle,

k being a constant.

Equating this exprassion to (2),

$$T = k' \times \text{loss per cycle},$$

(k' being another constant) which shows that hysteresis loss also does not depend on the speed of the machine.

42. Ewing's Hysteresis Tester.—By this instrument a quick determination may be made of the energy lost by hysteresis in a sample of iron. The specimen is in the form of a bundle of thin laminations, each 3″ by $\frac{5}{8}$″, a sufficient number being taken to give a thickness of about $\frac{1}{4}$″.

This bundle is held in the clamps S (Fig. 3.03), and is rapidly rotated between the poles of a C-shaped permanent magnet M by means of the hand-wheel H. The magnet is supported on a knife-edge K, and carries a pointer P which moves over a graduated scale D. The lower end of the magnet carries a control weight W, and a vane V, which, moving in an oil bath B, serves to damp the vibrations.

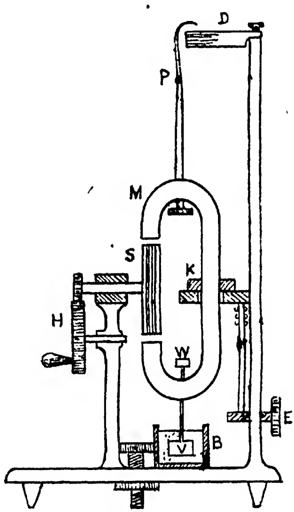


Fig. 3.03

By turning the milled head E the magnet can be raised off its knife-edge and clamped when not in use.

As the specimen rotates it is magnetised in alternate directions, passing through a complete cycle in each revolution. Thus hysteresis loss occurs. The lag in magnetisation causes the sample to drag by attraction the respective poles adjacent to its ends through a certain angle whose value depends upon the hysteresis of the sample. This angle is given by the pointer and is independent of the speed of rotation. This is proved in the previous article, and may also be shown as follows :—

The loss of energy due to hysteresis per cycle for a certain fixed flux density is constant. The loss of POWER due to hysteresis depends, therefore, on the number of cycles per second (and increases with the maximum flux density to which magnetisation is carried); so that,

$$\text{Power lost} = K \times n \times \text{loss per cycle}, \quad \ldots \qquad (1)$$

K being a constant.

Now, the power necessary for rotating the specimen is proportional to the product "driving-torque × revolutions per second." Hence, the extra power necessary due to hysteresis is proportional to

the torque due to hysteresis × n,

i.e., $\text{Power lost} = n \times \text{hysteresis torque} \quad \ldots \qquad (2)$

From (1) and (2) we have—

$$\text{Hysteresis torque} \propto \text{loss per cycle},$$

which shows that the torque caused by hysteresis is independent of the speed.

The angle through which the magnet is deflected depends on the average torque on the magnet which is

merely that due to hysteresis. Hence, the deflection of the magnet measures the hysteresis loss per cycle, independent of speed. It is, however, not directly proportional to the loss, but is connected with the latter by a straight line relation.

A number of standard specimens, having different but known hysteresis losses, are supplied with the instrument. Deflections corresponding to three of these, at least, are plotted against their hysteresis losses, and a straight line is drawn through the three points. The hysteresis loss of the sample under test may then be read off as soon as its deflection is obtained.

To eliminate any zero error the iron should be rotated in both directions, and the mean of the two deflections may be taken as the correct value. The tests are made at the flux density of about 4000 lines per sq. cm. The loss at any other value of B may be easily calculated from the spacing hysteresis loss $= \eta B^{1.6}$ (see p. 87).

The accuracy of the result is not affected by any change in the strength of the magnet, for all the deflections are then altered in the same ratio. Also, the exact cross-section of the sample is not so important, if it is very nearly equal to that of the standard, for any such alteration is largely compensated by an opposite change in the flux density. The object of using three standards instead of only two, is to detect and avoid any error due to changes in the standards with lapse of time. The

reading of the third standard may also serve as a check on the accuracy of the other two standard readings.

For use, the level of the instrument and the control weight should be adjusted to bring the pointer to the zero mark, and the magnet should be placed with its plane coinciding with the plane of the magnetic meridian so as to avoid the effect of the earth's field on its deflection.

43. Ewing's Permeability Bridge.—This is an instrument for quickly measuring the permeability of a sample of iron or steel in the form of a rod by comparison with a standard test-piece.

The construction of the instrument is simply depicted in Fig. 3.04. The standard rod E, whose B-H curve has been determined beforehand by some other method, and the test rod F, turned truly to the diameter of E, are clamped securely in two soft iron yokes PP′ having two vertical projecting horns YY′ also of soft iron. In the gap between the two horns is placed a magnetic needle M,

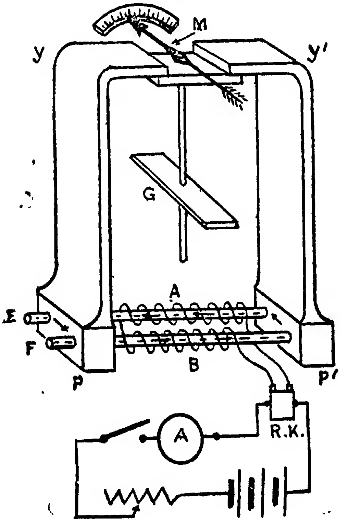


Fig. 3.04

the sensibility of which depends on the position of the controlling magnet G. A and B are two parallel solenoids each of length 4 π cms., coiling the two rods in series ; so that, when a current passes through the coils the rods with the soft-iron yokes form a closed magnetic circuit as shown by the arrows.

The number of turns in A can be either 50 or 100, while that in B can be varied by means of three dial switches from 1 to 210. There is also another switch for stopping or reversing the current in the whole circuit.

The test is made by passing a current I ampere through the two coils with S_A and S_B turns respectively and operating the reversing switch repeatedly as S_B is increased or diminished, watching the movement of the needle M during the operation.

When the needle takes up its zero position, that is, when no magnetic flux passes from P to P′ through the horns YY′, the magnetic induction of the one rod exactly balances that of the other; in other words, the yokes are then at the same magnetic potential.

Now, the magnetising field H_A for the standard rod is given by—

$$H_A = \frac{S_A I}{10} \text{ C. G. S. units,} \quad \ldots \quad (1)$$

and that for the test-rod, H_B, is given by—

$$H_B = \frac{S_B I}{10} \text{ C. G. S. units.} \quad \ldots \quad (2)$$

If we denote the induction or flux density, which is the

same for the two rods (the cross-section and total flux being the same) by B, we have—

$$B=\mu_A H_A = \mu_B H_B, \qquad \ldots \qquad (3)$$

and, from eqns. (1) and (2),

$$H_B=\frac{S_B}{S_A} H_A. \qquad \ldots \qquad (4)$$

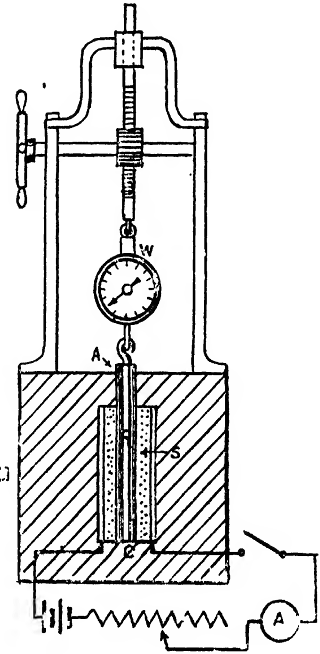
Clearly, H_A is either 5 or 10 C. G. S. units per ampere (according as $S_A=50$ or 100), whereas H_B ranges from 0 up to 21 C. G. S. units per ampere (for $S_B=0$ up to 210).

Knowing the balancing values for different currents, a series of values for H_B of the test-rod can be found from (4); the corresponding B's for the test-rod are noted on the B-H curve of the standard rod for values of H_A producing the same induction as H_B (equation 3). From these two sets of values a complete B-H curve of the sample may be obtained.

44. Thompson's Permeameter.—This provides ready and simple method of finding the relation between B and H of iron and steel in the form of a short rod.

It consists of a massive block of iron M (Fig. 3.05) with a rectangular aparture cut through it, so as to hold a magnetising coil S. The test-rod R, placed as shown is a good fit at A, and makes good contact with the block at C.

The coil is excited by a current, and the pull P, just necessary to break the contact at C, is read on the spring balance W.



If A is the cross-sectional area of the rod in sq. cms.,

$$P = \frac{B^2}{8\pi} . A \text{ dynes,}$$

(Chap. VII)

$$\text{or } B = \sqrt{8\pi \frac{P}{A}} . \quad (1)$$

where B is the flux density in lines per sq. cm.

Again, the magnetising field H of the coil is given by—

$$H = \frac{4\pi \, S \, I}{10 \, l}, \quad (2)$$

where S is the number of turns, I the current in amperes, and l the length of the coil in cms.

Fig 3.05

By passing different currents two series of values for B and H can be obtained from (1) and (2), and a complete B-H curve can then be drawn for the sample rod.

In formula (1), B–H should be used instead of B in cases of iron of very inferior quality, but for

ordinary grades of iron, H may be neglected in comparison with B.

If P is in lb. and A in sq. cms., B in eqn. (1), is given by—

$$B = 3340\sqrt{\frac{P}{A}}.$$

45. The Grassot Fluxmeter.—This is an instrument for direct measurements of magnetic flux with the help of a search or test coil used in conjunction with the fluxmeter. It has the great advantage over the ballistic galvanometer in that it gives instead of a throw, a deflection, which remains practically constant for an appreciable time, some ten seconds or more, and which is moreover, independent of the time taken by the change of magnetic flux.

The construction is essentially that of a moving coil ballistic galvanometer, having a suspended

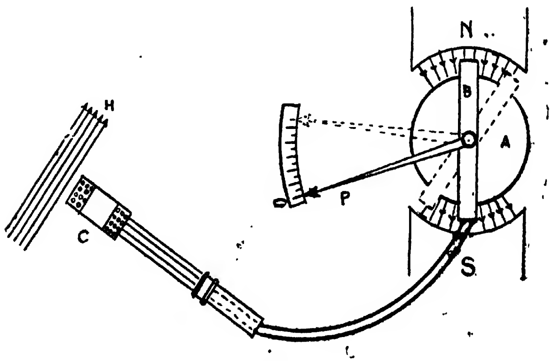


Fig 3.06

coil B, (Figs. 3.06, 3.07) enclosing a fixed soft iron cylinder A, which can move freely in the interspaces between the cylinder and the poles N S of a permanent magnet. The suspension is by a single cocoon or silk fibre attached to the spiral spring R of negligible torsional stiffness to prevent damage from shocks. The stiff wire frame E fixed upon the coil B allows of a central attachment of two fine silver strips s,s′ by which the current enters and leaves the coil. The mechanical control being extremely small the coil takes a considerable time to return to zero from a deflected position. The damping due to air resistance is usually insignificant. The only effective damping, in this case, is the electromagnetic damping, i.e., the retarding force due to currents in the coil induced by the coil's motion. In fact, the period of oscillation of the coil on open circuit is of the order of a minute.

The suspended coil is in series with the search coil which, in testing, is moved into the space in which the flux is required. In performing this action the coil cuts the flux to be measured, and as a result, the former coil is deflected. A pointer P on it, moving over a scale, gives the angle of displacement.

A mirror on the moving coil and a lamp and scale may be used for more accurate work.

The principle of the instrument is as follows :—

In Fig. 3.06, B is the suspended coil in its initial

zero position, C the search coil, and the arrow heads H denote the field of which the strength is to be measured.

Suppose the coil B is displaced through a *small* angle θ into the dotted position. In doing so it cuts a number of lines of force proportional to θ. Thus a quantity of electricity q, proportional to $\frac{\theta}{R}$, will flow through the circuit of B, where R is the resistance of the circuit. Conversely, if the coil B is initially in its zero position, and the same quantity of electricity, q, is passed through it, the displacement of A will be θ as before.

Hence, supposing the search coil C is moved across a magnetic field of strength H when the coil B is in its zero position, a quantity of electricity, proportional to

$$\frac{HSa}{R},$$

will flow through the circuit of the two coils, causing the coil B to deflect through an angle θ, such that

$$\theta \propto \frac{HSa}{R},$$

S being the number of turns of C, a the mean area of the turns of C, and R the total resistance of the circuit containing B and C. For a given search coil, therefore, S, a and R being constant, the strength of the field under test is proportional to the deflection, that is,

$$H = K.\theta$$

K being a constant. This result is independent of the time during which the change in flux takes place.

By using search coils with various number of turns and suitable sizes, a wide range of magnetic fields may be measured. The readings of the fluxmeter, multiplied by the constant for a given search coil, will thus give the value of the flux density under test.

46. Eddy Currents.—Eddy or Foucault current means the current which does not flow along a particular prescribed path, but follows the line of least resistance, and flows, generally, at right-angles both to the magnetic field and to the direction of motion.

Eddy current is induced when a massive conductor is cut by a magnetic field, but its exact path and intensity cannot generally be accurately determined.

Eddy current represents a loss of energy which is ultimately converted into heat, and which raises the temperature of the iron or copper sometimes to a prohibitive degree.

USE.—Sometimes eddy current is turned to account with great advantage, as in some forms of tram car brake, for damping galvanometers and ammeters, and various kinds of electrical measuring instruments.

47. Eddy Current in Electrical Machines.—Eddy currents also occur in the copper bars or wires of the armature winding itself, specially if these be very massive. When the revolving part of an electrical machine revolves in its field, there is a tendency to set up eddy currents. But there is also a tendency towards

setting up of eddy currents in the stationary parts as the flux passes through or changes in them.

48. Method of Minimizing Eddy-Current Loss.—Eddy current tends to flow at right angles to the direction of the flux; hence the resistance of its path can be increased, and the intensity of its e.m.f. decreased, by laminating the metals in which it tends to flow. It may be reduced as much as desired by making the laminations of the armature core sufficiently thin. A satisfactory value foi this law may be obtained by assuming it equal to the hysteresis loss. The laminations 0.01 to 0.03 in. thickness should be parallel to the direction of the flux, and at right angles to the axis of rotation. The plates are insulated from one another by means of varnish, or thin paper, or by the oxide on the surface.

TO ELIMINATE EDDY CURRENTS IN THE COPPER BARS (1) the edges of the poles are rounded off, (2) the air gap is made larger towards the edges than at the centre, causing a gradual change in the field strength.

To almost ENTIRELY AVOID EDDY CURRENTS toothed armatures are used in which the lines pass almost entirely through the teeth, and the conductor is cut uniformly at the same moment in a similar manner preventing eddy currents. In cases where the teeth are highly saturated the width of any individual conductor measured across the slot should be kept as small as possible.

49. Method of Computing Eddy-Current Loss.—The eddy-current loss in any volume of metal that

cuts, or is cut by, a flux must depend (1) on the specific resistance or resistivity of the metal, and (2) the frequency with which the flux cuts the metal—that is, on the rate of cutting. Lamination increases the resistance; hence, the thinner the laminations the less the eddy current loss.

If P_e = eddy-current loss in watts,

j = a coefficient varying with the quality and kind of metal in which the eddy currents are induced (which for silicon sheet steel varies from .000043 to .000098 with an average of .000065 ; for ordinary electrical sheet steel varies from .00012 to .00025 with an average of .00022),

V = the volume of the metal in which the loss occurs in cubic inches,

t = thickness of the sheets in inches,

f = frequency in cycles per second,

B = the maximum flux density in kilolines (thousands of lines) per square inch,

then, $P_e = 0.254 \times j \times V\ (t \times f \times B)^2$ watts..............(1)

Prof. Sheldon gives the following formulæ for the calculation of hysteresis and eddy-current losses in iron:—

$P_h = 8.3\ \eta\ f\ VB^{1.6}\ 10^{-8}$ watts,

$P_e = 4.07\ V\ (f\ t\ B)^2\ 10^{-17}$ watts,

where η = the magnetic (hysteretic) constant,

f = the number of magetic reversals per second $\left(=\dfrac{\text{number of poles} \times \text{r. p. s.}}{2}\right)$,

t = thickness of laminations (in mils),

V = the volume of iron (in cubic inches),

B = the flux density in the iron (maxwells per sq. inch).

If we had a simple alternating magnetic flux through the sheet steel of an armature, the direction of the flux being strictly parallel to the plane of the laminations, and if the individual sheets were perfectly insulated from one another, the eddy-current loss in watts per cubic centimetre of iron would be

$$W_e = \frac{\pi^2}{6} \times \frac{1}{\rho} \times t^2 \times n^2 \times B^2_{max} \times 10^{-16}, \qquad \dots (2)$$

where, ρ = the specific resistance of the iron,

t = the thickness of the sheet in centimetres,

n = the frequency, and

B_{max} = the maximum flux-density in lines per sq. cm.

Dr. Steinmetz has developed a formula for calculating the losses due to eddy currents in laminated iron, which may be stated thus:

$$W_e = (t\ f\ B)^2 \times 10^{-16}, \qquad \dots \quad \dots \quad (3)$$

where, W_e = the Foucault loss per cubic centimetre,

t = the thickness in mils of the laminations or plates,

f = the frequency, and

B = the maximum induction (as in the hysteresis formula).

The total loss in V cubic centimetres is therefore

$$W_e = V\ (t\ f\ B)^2 \times 10^{-16}. \qquad \dots \quad \dots \quad (4)$$

To express in watts per pound of iron we have

$$W_e = 6\,V\,(t\,f\,B)^2 \times 10^{-15}, \qquad \ldots \qquad (5)$$

where V is the volume in cubic inches. The eddy current losses are only 15°/₀ to 25°/₀ of the total core losses.

Note.—Using formula (1) and proceeding as in §41, the torque caused by eddy currents is given by

$$\tau = K'\,Vn\,(t\,B).^2 10^{-16},$$

where K′ is a constant.

Thus we find that WHILE THE HYSTERESIS TORQUE IS INDEPENDENT OF THE FREQUENCY OF MAGNETISATION AND IS THE SAME AT ALL SPEEDS, THE EDDY-CURRENT TORQUE IS PROPORTIONAL TO THE SPEED OF ROTATION OR TO THE FREQUENCY OF MAGNETISATION. This difference is utilized in practice for separating the losses caused by hysteresis and eddy currents.

50. Separation of Eddy and Hysteresis Losses.—In certain cases it is desired to investigate the iron of a transformer by separating the hysteresis from the eddy loss.

Let W_1 = the core loss at normal voltage and frequency,

W_2 = the core loss at half the voltage and half the frequency,

W_e = eddy current loss at normal voltage and frequency,

W_h = hysteresis loss at normal voltage and frequency,

then, $W_e = 2\,W_1 - 4W_2.$

$W_h = 4\,W_2 - W_1.$

Example 6 Calculate the eddy-current power loss in a mass of ordinary laminated electrical steel having a volume of 70 cu. in. if the iron is acted upon by a flux (maximum) of 16,000 lines per sq. in. at a frequency of 50 cycles per section. The laminations are 0.015 in. thick.

Solution :—

Substitute in the formula (1) :—

$P_e = 0{\cdot}254 \times j \times V\ (t \times f \times B)^2 = 0{\cdot}254 \times 0{\cdot}00022 \times 70$ $({\cdot}015 \times 50 \times 16)^2 = 0.0039 \times (12)^2 = 0{\cdot}0039 \times 14{\cdot}4 = {\cdot}562$ watts.

Example 7. Find the watts lost due to (a) hysteresis, (b) eddy current in a transformer of 5000 cubic centimetres, in which $B = 4000$ gausses, $f = 50$, and the hysteretic constant $h = 20 \times 10^{-11}$ watts per cubic centimetre per cycle; the laminae are 12 mils thick.

Solution :—

$$\text{(a)}\quad W_h = Vh\ f\ B^{1\cdot6}$$

$$\therefore\quad W_h = 5000 \times 20 \times 10^{-11} \times 50 \times 4000^{1\cdot6} = 29 \text{ watts.}$$

$$\text{(b)}\quad W_e = V\ (t\ f\ B)^2 \times 10^{-16}$$

$$\therefore\quad W_e = 5000 \times (12 \times 50 \times 4000)^2 \times 10^{-16} = 2.88 \text{ watts.}$$

Exercises.

(1) The eddy-current loss in a transformer is 200 watts when operated on 50 cycle mains. Calculate the eddy-current loss when the transformer is operated on a 25-cycle circuit, the voltage of which is equal to that of the 50-cycle circuit.

(2) 25 per cent of the iron loss in a 50-cycle transformer, when operated at rated frequency and voltage, is due to eddy currents. Calculate the percentage increase or decrease in the total iron loss when the transformer is operated at 25 cycles and rated voltage.

(3) The hysteresis loss in a 50-cycle, 2200-volt transformer is 400 watts. Calculate the hysteresis loss when operated on a 50-cycle, 1100 volt circuit.

(4) The eddy-current loss in a 50-cycle, 2200-volt transformer is 400 watts. Calculate the eddy-current loss when operated on a 50-cycle, 1150-volt circuit.

(5) The hysteresis loss in a transformer is 130 watts when operated on 50-cycle mains. Calculate the hysteresis loss when the transformer is operated on a 25 cycle circuit, the voltage of which is equal to that of the 50-cycle circuit.

CHAPTER IV.

DIRECT CURRENT GENERATORS.

51. Direction of Induced E. M. F.—Hold the thumb, the index, and the middle finger of the right hand at right angles to each other, to represent the three rectangular axes in space. If the thumb points, to the direction of the motion, the index finger points along the direction of the magnetic lines of force, then the middle finger will point

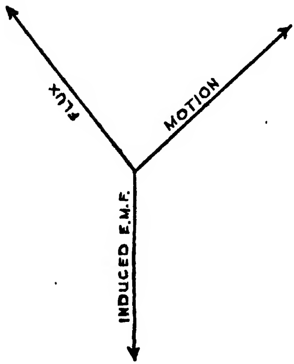


Fig. 4.01

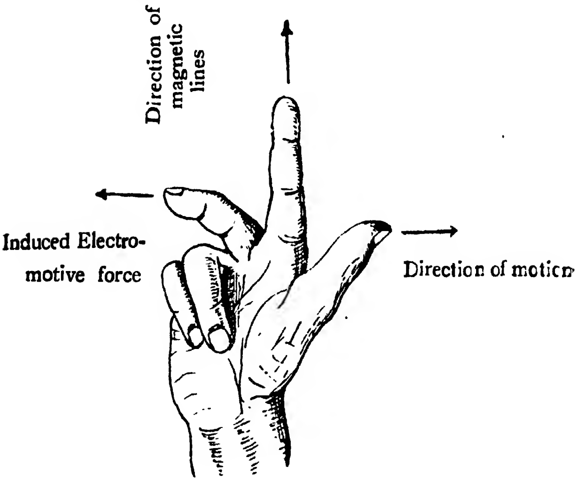


Fig. 4.02 Right hand

in the direction of the induced electromotive force and current.

52. Alternating-Current Generators.—If a loop of a conductor ending in two slip rings A and D is revolved in a magnetic field about an axis perpendicular to the lines of force, then each side of the loop being a conductor moving across the lines of force, will have an E.M.F. induced in it. Now if the circuit is completed by joining A and D to an external

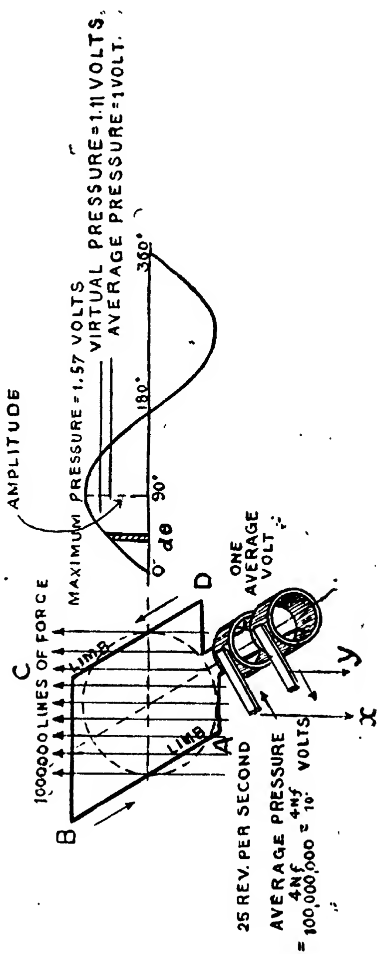


Fig. 4.03

circuit, through the brushes x and y, a current flows through the brush x to the external circuit, and enters the loop through the brush y while the conductor A moves up and the other end D moves down. During the other half of the revolution the end A goes down, the current reverses its direction and enters the loop at x and goes out to the external circuit at y. Now, while the motion of one conductor ending in A is up and that of D is down, the directions of the induced E. M. F.'s in the two sides would be opposite to each other. But as they form the opposite sides of the loop, the pressure will be cumulative, i. e., instead of neutralizing each other, the two pressures will be added to each other.

When the loop reaches a position such that conductor A begins to move downwards, the direction of the induced E. M. F., and consequently of the current, is changed in both the conductors of the loop. Thus in each complete revolution the current changes its direction twice. The brushes x and y always being in contact with the ends, A and D alternately send out currents of opposite sign. The strength of the current however varies throughout the CYCLE.

53. Direct-Current Generators.—If instead of connecting the terminals A and D to two separate slip rings, the connection is made to a single ring which is divided, the segments being insulated from each other, and the loop is revolved in a magnetic field, we get a unidirectional current in the external circuit. The brushes are

so placed that at the instant the induced E.M.F. in the loop changes in direction, the brushes will slide across from one segment to the other. The

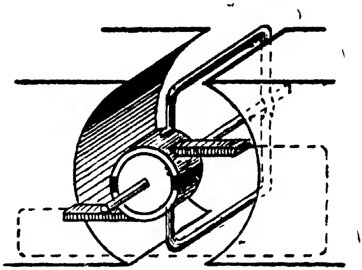


Diagram of Reversing Action of Commutator.
Fig. 4.04.

current following the E. M. F., while reversed in the loop, will flow in the same direction in the external circuit, as the brushes pass from one segment to the other. Further, when the loop is in the neutral position it is generating no electromotive force. As before, the magnitude of the current and electromotive force varies with the position of the loop.

If a number of loops is symmetrically placed and connected in series all round the armature core, the E.M.F. of the system is the sum of the instantaneous E. M. F.'s induced in the individual loops, and the potential difference between the brushes x and y is approximately constant, and so is the current. Hence, it should be noted that in order to get a continuous current we must replace the two-part commutator by a larger number of segments connected with a larger number of coils, which are so arranged that one set comes into action while the other is going out of action.

If the number of coils used is very large, the current curves will overlap completely, and the row of summits will form practically a straight line, or the whole current will be practically constant (Figs. 4.05 and 4.051).

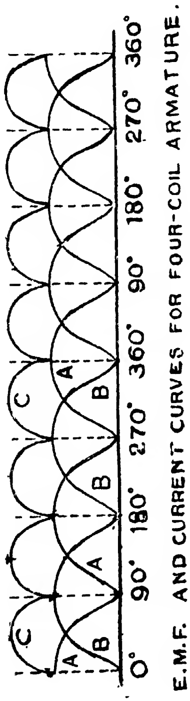


Fig. 4·05

It will be noticed that the E. M. F. and current generated in the armature coils are essentially of an alternating nature. It is only by the device of commutation that they are made unidirectional or direct with reference to the external circuit.

The best method of commutation requires the use of auxiliary poles, which are commonly known as COMMUTATING POLES or INTERPOLES; being small auxiliary poles put in addition to the main poles. The commutating poles are always series wound. As they are in series with the armature, the strength of the poles will always be in proportion to the strength of the current in the armature, and thus gives correct commutation.

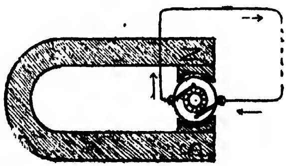
54. A direct current is a unidirectional current.

A continuous current is a steady non-pulsating direct current.

The field magnets of a dynamo may be : (1) PERMANENT MAGNETS, (2) ELECTRO-MAGNETS.

The electromagnets again may be (a) SEPARATELY EXCITED, and (b) SELF EXCITED.

55. Magnetic Machines or Magneto-Dynamos.—In these machines (Fig. 4·06) permanent magnets are used as field magnets. These are now generally used in medical batteries.



Magneto Machine.
Fig. 4.06.

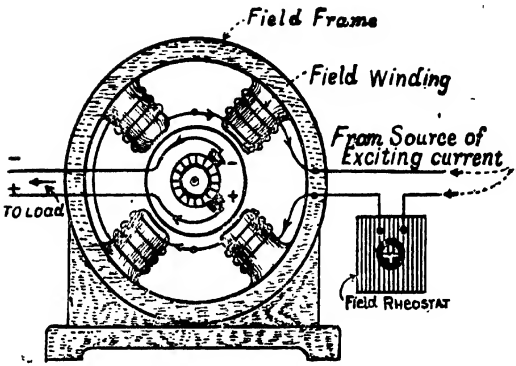
Disadvantages:—

(1) Greater weight for a given flux.

(2) Liability to lose their magnetism owing to imperfect aging.

(3) Difficulty of alternating the useful flux.

56. Separately Excited Dynamos.—In these (Fig. 4·07) the current which excites the field magnet coils is obtained from some other source.



Separately Excited Dynamo.

Fig. 4.07.

Use:—

(1) Where close regulation of field strength is necessary.

(2) For electrolytic and electroplating works.

(3) Where it is essential that the polarity of a machine be not reversed.

N.B.—Alternating current generators are practically all separately excited.

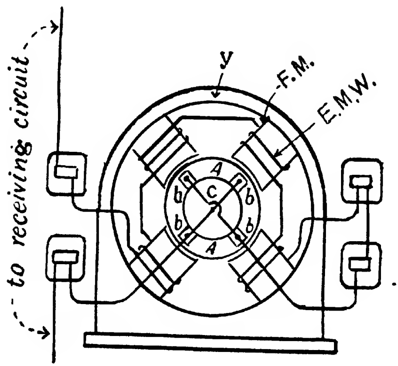
57. Self Excitation may be Secured in Three Ways.—(1) The entire current of the generator may be passed through the field magnet coils that are in series with the main circuit.

(2) A portion of the current from the armature may be utilized to excite the field magnet coils consisting of many turns of wire of thinner

cross section and higher resistance, connected as a Shunt.

(3) The exciting current may be used from a second armature revolving in the same field, or from some of the coils of the armature that may be separately joined up for that purpose.

58. The Series Generator.—Series-wound machines (Fig. 4·08) have the field coils in series with the armature, and have only one circuit. The same current flows through each part of the circuit (Fig. 4.09). As the load changes, the voltage also changes; so that the current remains unchanged. The field magnets become demagnetized immediately the external circuit is broken.

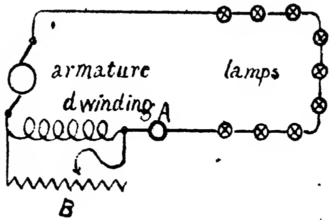


Series-wound Dynamo.
Fig. 4.08.

Disadvantages :—

(1) It does not start action until a certain speed has been attained, or unless the resistance of the circuit is below a certain limit.

(2) It is liable to become reversed in polarity.



Connection of Series Generator and its Receiving circui

Fig. 4.09

(3) Its power is diminished by increasing the resistance in series, and there is the risk of getting too great a current when the resistance is diminished.

59. Series Generator is used almost exclusively for supplying constant current to series and arc lamp systems, where the advantage lies in saving of conductor, and for long distance direct current power transmission. The line can be made of much smaller wire than in the case of constant pressure circuit, for as the load increases the power or energy transmitted is increased by raising the potential, the current remaining unaltered. The size of the wire is determined by the strength of the current; hence a current large enough to supply one lamp of a constant pressure circuit can supply all the powers of a constant current circuit. The generator is provided with a regulator which maintains the current constant when the load changes.

The resistance in the external circuit of a dynamo seldom remains constant. In a series dynamo if the lamps or load, in the external circuit, are joined up in series, any addition to the number will increase the external resistance and the current will be reduced; thus the field will decrease and so the current, is further decreased. If the lamps are joined in parallel, any addition to the number in the external circuit decreases the total resistance and increases the current, and thus the field is further strengthened and the current is still further increased. Hence in the series dynamo considerable difficulties arise when there is variation of resistance in the external circuit.

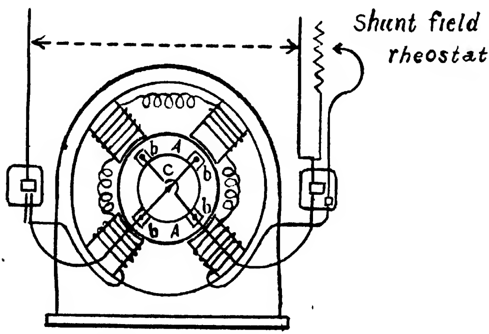
Further, th variation of the current does not produce proportional variation in the magnetic field, and in order to attain sparkless commutation at the brushes, the angle of lead must be altered for every change in the current.

A series dynamo can be used most conveniently when a constant current strength is required, for if the resistance of the circuit remains constant the current also remains constant, provided the dynamo be driven at a regular speed. If the resitsance vary, the current can still be kept constant by providing an arrangement to automatically vary the strength of the field in which the armature rotates, e. g., by shunting the field magnet coils by a resistance.

60. A Series Generator cannot be used for charging a secondary battery. For if by any cause,

such as engine troubles, the voltage of the machine becomes lower than the voltage of the battery, a current from the battery will at once flow to the dynamo. The excitation of the dynamo will thus be reduced, and the voltage of the dynamo will fall further, and ultimately the current in the dynamo will be reversed. Consequently the original magnetization will be reduced to zero, and finally built up in the opposite direction. The dynamo will however be driven by the prime mover in the original direction, and will now generate current in the same way as the battery, resulting in what is like a heavy short circuit across the battery. Thus either the battery or the dynamo will be heavily damaged unless prevented by proper automatic cut-outs and fuses.

61. The Shunt Generator.—In shunt machines (Fig. 4.10) the field coils consisting of many turns of thin wire form a shunt across the armature terminals. Thus



Shunt-wound Dynamo
Fig. 4.10.

the field winding and the external receiving circuit are in parallel with each other between the brushes, so that if the external circuit is broken, there is an alternative path left for the current through the field magnet coils. The current supplied to the field is equal to the terminal E. M. F. of the machine divided by the resistance of the field circuit. If the terminal voltage is constant the magnetic flux is constant. This will, therefore, give a substantially constant voltage and is less liable to reverse the polarity than series machines. An adjustable resistance (rheostat) is usually connected

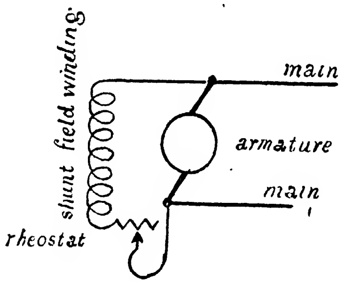


Fig. 4.11.

in the field circuit for controlling the exciting current, and thereby controlling the voltage of the machine. The arrangement is shown in Fig. 4.11. No harm would result if a shunt machine were short circuited and then put in motion. The field would fail to build up, and all the current that would flow would be that due to the small voltage generated by the residual magnetism. But if a big machine in operation is short

circuited, the magnetism would not die down instantly when short circuit is established, and so, if the condition last only for a few seconds it may be seriously damaged. Use :—

(1) Where the load is practically constant.

(2) Where the load changes slowly and frequently.

(3) Where the load consists exclusively of motors which do not require close voltage regulation.

(4) Where an attendant is employed to manipulate the field rheostat and maintain constant voltage.

(5) Where the voltage is controlled by a Terrill Regulator or other automatic device.

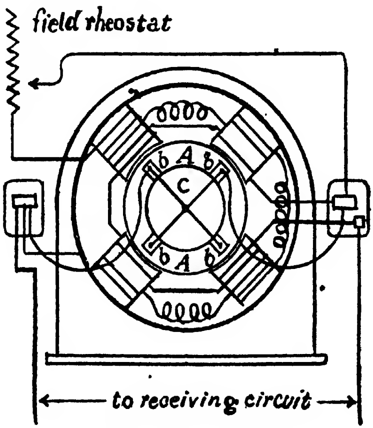
The shunt dynamo is specially suitable for ordinary factory, power and lighting purposes, and for electroplating where large currents at low pressure are essential, and it is the only type for charging accumulators.

62. A Shunt Dynamo is always used for charging a secondary battery.—For if by any cause the voltage of the machine becomes lower than the voltage of the battery, a current from the battery will pass to the dynamo, but a part of this current will pass through the field circuit which will be further excited, causing an increase in the voltage of the machine within a certain limit. Thus the dynamo voltage will run higher than the voltage of the battery which will therefore be charged.

63. The Compound Generator.—In this a combination of the shunt and the series windings are used

(Fig. 4.12), and in general the series turns help the shunt turns. Its voltage will somewhat increase with the load. The voltage will depend upon the degree of excitation of the magnetic circuit when the shunt is acting alone on open circuit. The dynamo will keep a practically steady voltage at all loads, and if any slight adjustment is necessary, it is performed by means of the shunt field regulator.

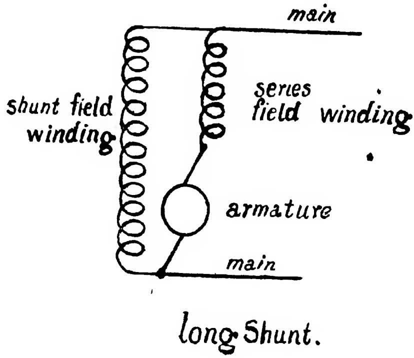
A COMPOUND DYNAMO containing both shunt and series field coils CANNOT be used in charging a battery. If it is to be used for charging a battery, the series coil in the field should be cut off, so that the dynamo, for the time being works as a Shunt Dynamo.



Compound-wound Dynamo.

Fig 4·12

64. In Long Shunt Compound Dynamos the shunt coils are connected outside of the series coils (Fig. 4·13).



Long Shunt Compound Dynamo

Fig 4·13

56. In Short Shunt Compound Dynamos the shunt coils are connected inside of the series coils (Fig. 4·14). Generators are more commonly connected in this way, for it tends to maintain the shunt field current more nearly constant on variable loads, as the drop in the series winding does not directly affect the voltage on the shunt field with this arrangement.

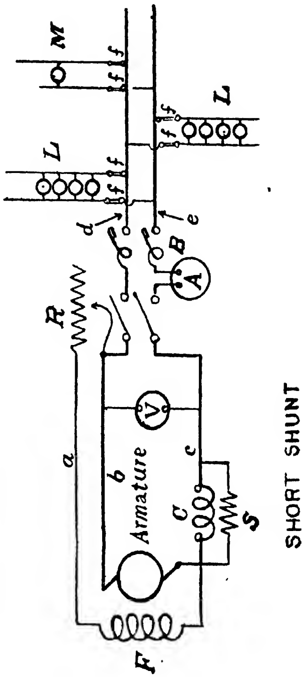
Use:—

(1) Where the terminal voltage is to increase with the load.

(2) Where an approximate compensation is to be automatically made for the internal drop of the generator.

(3) Where the drop in a long transmission line or a long feeder is to be automatically compensated for.

The compound dynamo is thus specially suitable for traction and power circuits, where the load often suddenly varies within wide limits. The field current is more nearly constant on variable loads, as the drop in the series winding does not directly affect the voltage on the shunt field with this arrangement.



Short Shunt Compound Dynamo and its Receiving Circuit.

Fig 4·14

66. A Flat Compound Generator has its series coils so proportioned that the voltage remains practically constant at all loads within its range.

67. An Over Compound Generator has its series windings so proportioned that the full load voltage is generally 5 to 10 percent greater than its no load

voltage. It is used when it is desirable to maintain a practically constant voltage at a point at some distance from the generator.

68. The Fundamental Equation of the Direct-Current Dynamo —

Let ϕ = magnetic flux passing through the armature from one pole to the other,

p = number of pairs of field poles,

N = number of conductors on the outside of the armature,

Q = number of electrical paths in parallel between the brushes,

E = electromotive force induced in the armature,

n = speed of armature in revolutions per second.

Then, in $\frac{1}{2pn}$th of a second a conductor cuts ϕ lines of force.

Therefore the average rate of cutting of lines of force

$$= \frac{\phi}{\frac{1}{2pn}} = 2\phi pn \text{ lines of force per second,}$$

which is the average E. M. F. in the given conductor while it is moving from one brush to the other. There are N/Q conductors in series in each path between the brushes.

Therefore, $\frac{N}{Q} \times 2\phi pn$ is the E. M. F. between the brushes in each coductor;

$$\text{i.e. } E = 2p\phi Nn/Q \quad \text{C. G. S. units,}$$

$$= 2p\phi Nn/Q \times 10^{-8} \text{ volts.}$$

This equation gives the E. M. F. induced in the windings between the brushes, when the brushes are at the neutral points in any kind of windings except the open coil windings.

Example 1. A four-pole machine with a four-circuit armature winding, has the flux of 200000 lines of force per pole, the total number of inductions in the armature is 600, and the speed of the machine is 1200 r. p. m. Determine the voltage generated.

Solution :—

$$E = \frac{2p\phi nN}{Q \times 10^8}$$

Substituting,

$$E = \frac{4 \times 200000 \times 1200 \times 600}{4 \times 10^8 \times 60} = 240 \text{ volts.}$$???

69. The Essential Parts of C.C. Generators.—

I. THE FRAME :—This is the supporting structure including the base and the supports for the bearings in which the armature shaft rests, as well as the yoke.

II. THE FIELD MAGNETS, which produce the field in which the conductors move, consist of three parts:—(1) the field core, magnet core, or pole terminating to (2) a pole shoe, and (3) the field windings—conductors which carry the currents to excite the field magnets. These are not necessary in magneto- generators.

III. THE YOKE—from which the poles project radially inwards and which joins the two poles.

IV. THE ARMATURE which consists of :—

(1) The armature windings, a connected system of

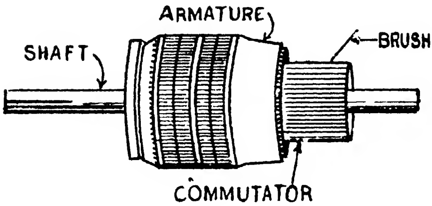


FIG. 4.15

conductors, which cut the magnetic lines of force and thereby generate the E. M. F.

(2) The armature core, which supports the armature conductors, causes them to rotate, and provides a path of low reluctance through the armature for the flux from the north poles to the south poles of the field magnets (See Fig. 4.15).

V. THE COMMUTATOR (or the slip rings or collector rings in alternate current machines), which are rotating conductors connected to the armature windings on which the brushes rest to conduct the current to or from the external circuit (Fig. 4.15).

VI. THE BRUSHES which connect the external circuit and the armature conductors through the commutator.

VII. THE BRUSH-HOLDER AND THE ROCKER which hold the brushes and provide the means to shift the position of the brushes.

VIII. The Armature Spider :—In big generators this is provided in order to connect the armature core to the shaft.

IX. The Shaft and the Bearings which permit of the necessary rotation of the armature.

X. The Coupling—the mechanical connection between the shaft of the generator and that of the prime mover when they are coupled together.

Note.—The armature cores are of two types :—

(1) Ring cores were used in early dynamos. These were iron rings through which the conductors were threaded. This is seldom used in modern machines owing to its electrical and mechanical differences.

(2) The drum core consists of an iron cylinder, and the armature conductors are placed upon its surface.

70. Function of a Commutator.—The movement of the armature coils past the magnet poles generates alternating currents. In all continuous current machines the commutator commutates these into one direction. Imperfect commutation at once makes itself shown by the sparks which appear between the commutator and the brushes, and this sparking is intimately connected with the various reactions of the armature. Sparks caused by imperfect commutation are observed just under the tips of the brushes.

All the conductors in the armature have their currents reversed and re-reversed at every revolution. In

bipolar machines the reversal cocurs twice in each revolution. Reversal occurs at the moment when the conductor, or the section of which it forms a part, passes the brush or undergoes commutation. The production or non-production of sparks depends on the conditions under which the reversal of current takes place, and is a consequence of the property of self-induction—the property, in virtue of which (owing to the current in a conductor setting up a magnetic field of its own in the surrounding space) it is impossible instantaneously to start, stop, or reverse a current.

71. Commutation.—Fig. 4·16, shows part of a machine with a ring winding having two turns per coil and with the current in the coil M undergoing commutation. The brush B being of copper, the resistance of the contact between the brush and the commutator is negligible.

As the armature moves, and the brush changes from one segment to the next segment, the current in the coil M is automatically reversed. During this change, for a short period while the brush is in contact with both the segments, the coil M is short-circuited.

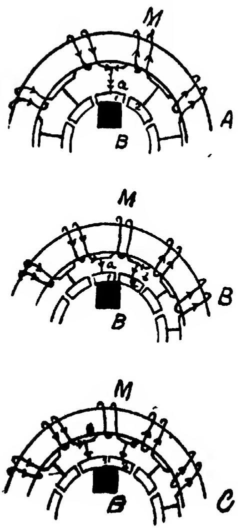


Fig 4.16

But no e. m. f. is generated in the coil since it is not cutting lines of force, so that no current passes through the short circuit.

The ideal arrangement is attained if the brushes be shifted just so far beyond the point of maximum electromotive force, that while the sections pass under the brush and are short-circuited, they should actually have a small reverse electromotive force induced in them ; and this action should last just so long in each successive section as to stop the current that was circulating, start a current in an opposite direction, and let it grow exactly equal in strength to that which is circulating in the other half of the armature, while it is then ready to join. At this set of conditions there should be no sparks.

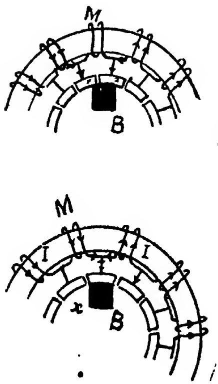


Fig. 4.17

In diagram A, Fig. 4.16, the currents enter the brush through the commutator lead a.

In diagram B, the brush makes contact with two segments, and the currrnt flowing to the brush through the coils under the S pole no longer requires to flow round coïl M, because it has an easier path through the lead b. The current in coil M therefore dies down to zero because, being in the

neutral position, the coil M is not cutting lines of force, so that no e.m.f. is generated in it to maintain the current.

When in position C, the coil is said to be in the neutral position.

In diagram D, Fig. 4·17, segment 1 of the commutator is about to break contact with the brush, and the coil M carrying no current is about to be thrown in series with the coils under the N pole. At the instant the contact is broken, as shown in diagram E, the current in M tries to increase suddenly from zero to a value I. But this change of current is opposed by the self-induction of the coil M, so that the current prefers to pass to the brush across the air space between the segment of the commutator just broken contact with the brush, and the edge of the brush.

If the current in the coil M is reduced to zero, and then, by some means or other, raised to a value I in the opposite direction during the time the coil is short-circuited by the brush, so that when contact is broken from the last segment, there is no sudden change of current in the coil, then the commutation becomes sparkless.

If the brushes are shifted forward in the direction of motion, while the coil M is short-circuited, it is in a magnetic field, and an e. m. f. is generated in it which will produce the required growth of current. This magnetic field is called the reversing field. At this position of the brushes the commutation is sparkless.

For sparkless commutation, the strength of the reversing field must be increased as the current taken from the generator is increased; so that the brushes must be moved nearer to the pole tips and further from the no-load position. Thus, the brush position is changed with the change of load.

Interpoles are supplied in the interpole machines with series field coils, to increase the strength of the reversing field, as the current drawn from the armature is increased.

The object in using the commutating poles is to produce within the armature coil under commutation, an e. m. f. of the proper value and sign, to reverse the current in the coil while it is yet under the brush.

72. Neutral and Commutating Planes.—The NEUTRAL PLANE (Figs. 4.18, 4.19) through an armature is a plane at which the e. m. f. induced in the inductors is zero. This is because at such a plane the inductors move parallel to the flux, and hence do not cut it. The NORMAL NEUTRAL PLANE (Fig. 4.20) is the plane which is the neutral one when there is no current in the armature inductors. This plane lies midway between the adjacent poles of opposite polarity.

The COMMUTATING PLANE is that imaginary plane, passing longitudinally through the armature and brushes, at which commutation occurs. The commutating plane may not coincide with the neutral plane, because it may be necessary to shift the brushes ahead, in the direction

of rotation of the neutral plane, in order to ensure sparkless commutation.

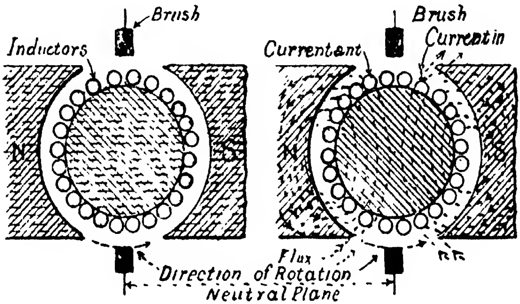
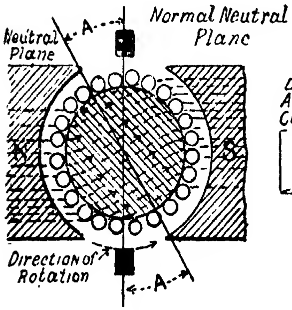


Fig. 4·18 Fig. 4·19



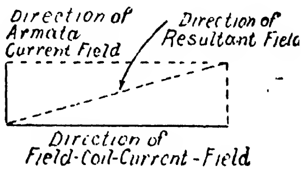


Fig. 4·20 Fig. 4·21

73. Armature Reaction.—The current circulating in the armature winding produces magnetising effects, which interfere with those of the exciting currents of the field-magnets. The reactions of the running armature

manifest themselves in several ways, the more important of which are (a) a tendency to CROSS-MAGNETIZE the armature, (b) a tendency to spark at the brushes (hence, the necessity of shifting brushes through a certain angle to such a point that sparking disappears), (c) a consequent tendency for the armature current to DEMAGNETIZE, (d) variations of sparking, and consequently of the neutral points when the amount of current drawn from the machine is altered, (e) heating of armature cores and coils, (f) heating of the pole pieces of the field magnets, (g) a consequent discrepancy between the quantity of mechanical horse-power imparted to the shaft, and the electric horse-power furnished in the electric circuit. The nature of these reactions demands careful attention.

74. The Cross Magnetising Effect.—When the field coils are excited but no current is flowing in the armature winding, the distribution of the magnetic flux in a two-pole machine is as shown in Fig. 4.18. The direction of flux will be straight across from pole to pole,

When the armature is carrying current, the brushes being in the neutral position and the field coils not excited, the distribution of magnetic flux is as shown in Fig. 4.19. The current passing downward in the conductors under the S. pole of the machine, and up in those which are under the N. pole, causes the armature to become an electromagnet with lines of force, which pass through the armature in a direction determined by the cork-screw law, and which return

across the pole faces to complete the circuit. When a generator is in operation, there are two fields which tend to react on its armature. One due to the field current, is as shown in Fig. 4.18; and the other, due to current in the armature inductors, has a direction almost at right angles to the first as shown in Fig. 4.19. In an operating generator, the effect of these two fields are simultaneous and superimposed. They combine to produce a distorted field as shown in Fig. 4·20. The flux across the air-gap, in the pole pieces and armature core, is now no longer uniform, but becomes dense towards the toes of the pole shoes in the direction of rotation. Furthermore, the neutral plane is no longer coincident with the normal neutral plane, but is shifted through some angle as A, Fig. 4·20, in the direction of rotation. The field-coil-current field is usually much stronger than the armature-current as shown graphically in Fig 4·21.

With no current in the armature circuit, the neutral plane coincides with the normal neutral plane as in Fig. 4·18. But with the current in the armature inductors, the neutral plane will shift in the direction of rotation as in Fig. 4·20. The greater the armature current, the greater will be the effect of armature magnetization, and the greater will be the shifting of the neutral plane, i. e., the greater the load on a generator the greater is the armature reaction.

75. Demagnetising Effect —When the brushes

are shifted from the no-load neutral in the direction of motion so as to improve commutation, the distribution of magnetic flux, when the armature is carrying current and the field coils are not excited, will then be changed as follows:— The armature field is no longer at right angles to that produced by the field magnets, but acts as the resultant of the two magnetic fields, as shown in the figure ; one of which oy is called the CROSS-MAGNETIZING COMPONENT, and the other ox is called the DEMAGNETIZNG COMPONENT, because it is directly opposed to the field produced by the field magnets. Figs. 4·21, 4·211 show these two components clearly.

76. Effect of Armature Reaction on Commutation.—When there are no interpoles in the machines, then in order to make the commutation take place in a reversing magnetic field under pole-tips a and c (Fig .4·15), the brushes are shifted forward in the direction of motion. But Fig. 4·15 C, shows that the effect of armature reaction is to weaken the magnetic field under these pole-tips, and so impair the commutation. This effect is minimized by making the air-gap clearance as large as possible, so that there is a large reluctance in the path of the cross field. The reluctance of the main magnetic path is also increased as the air gap increases. It is then necessary to increase the number of exciting ampere-turns on the poles, in order to produce the required main flux. Such machines are said to have a stiff magnetic field, for it is not exactly affected by armature reaction.

77. Limitation of Output of a Dynamo as a Generator or as a Motor.—The output of a dynamo is limited by three considerations: (1) heating and consequent destruction of insulating materials used in the machine, (2) excessive sparking at the brushes, hence the rapid wear, unsatisfactory running, and excessive heating of the commutator, (3) excessive drop of voltage in a dynamo, or of speed in a motor, at excessive load.

78. Characteristic Curves of a dynamo give us the best means of studying the operation of the machine. It shows at a glance the relation existing between the current generated or supplied by a machine, and the voltage under which it operates. The curves are plotted with the current strengths as abscissae and voltages as ordinates.

79. The Internal Characteristic, of a dynamo is the curve which shows the variation of the induced E. M. F. with the field ampere-turns.

The Total Characteristic represents graphically the relation between the generated E. M. F. of the machine and the armature current.

The TOTAL CHARACTERISTIC is obtained from the external characteristic by adding the value of the IR drop to the terminal E. M. F., and adding the field current to the external current, if this last is available, as in a shunt dynamo.

80. The External Characteristic of a dynamo is the curve which shows relations between the electromotive force at the armature terminals, and the current output of the machine when the latter is varied.

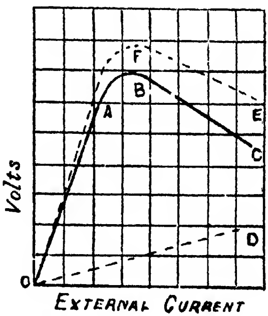
81. Experimental Determination.—Run the machine on open circuit at normal speed, and separately excite the field coils. A set of readings of the exciting current is obtained by placing a regulating resistance in series with the field coil. The corresponding values of the induced E. M. F. are read in a voltmeter connected across the armature terminals.

The values of the E. M. F., s are marked off along the ordinate, and the corresponding values of the field amperes, along the abscissa. (See Figs. 4·22—4·24.)

82. The External Characteristic of a Series Dynamo.—Run the machine at a constant speed, vary the current by means of an adjustable resistance in the external circuit, and observe the corresponding values of the current output and terminal voltage. Plot voltage against current. A curve such as O A B C (fig. 4.22) is obtained, which is called the External Characteristic.

Determine the values of $I\ (R_a + R_s)$, and plot the straight line OD. OD shows the variation of pressure drop (total RI drop) with the load.

Add the corresponding ordinatés of OC and OD, and get another curve OE.



The characteristic curves of a Series Generator.

Fig 4·22

OE shows the variation of the total induced E. M. F. in the armature with the external current.

The curve OFE is called the Total Characteristic. The total characteristic curve is plotted between total armature current and generated E. M. F. If there were no demagnetizing action of the armature ampere-turns, the curve OE would have been identical with its MAGNETIZING CURVE. As the parts of the magnetic circuit, with increase of load, approach saturation, the reactions of the armature and IR drops become of relatively greater importance. The result is, that the curve flattens out and finally bends downwards.

Observe.—(1) When the current is increased beyond a certain limit, the P. D. drops owing to (i) the drop of pressure in the armature and series coil, (ii) the demagnetising effect of the armature ampere-turns.

(2) The field current is the same as the main current. Hence the voltage rises in value with the increase in the current; but the rise is proportional to begin with, and less rapid afterwards.

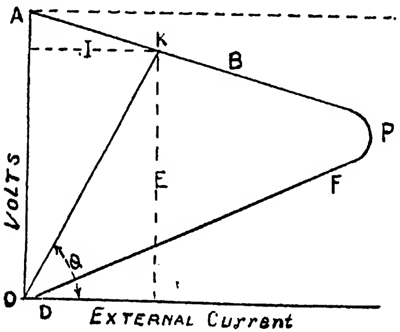
(3) While the total characteristic rises more and more slowly, the external characteristic rises to a maximum at the point B and then falls. The dynamos must be worked beyond the point B to obtain satisfactory stability of the voltage.

83. Determination of the External Characteristic at a speed n′ when that at a known speed n is given —

In a series generator the voltage is proportional

to the speed for a given value of the output current. The external characteristic curve corresponding to the speed n' (final speed) may be derived from the external characteristic curve corresponding to the speed n (the initial speed) as follows :—Add IR to each ordinate of a given characteristic, thus finding the total characteristic for the same speed n. Then multiply the ordinate of this characteristic curve by n'/n thus finding the total characteristic curve for the speed n'. Subtract IR from each ordinate of this curve, thus obtaining the external characteristic curve for the speed n'.

84. The Characteristics of a Shunt Dynamo.— Run the machine at normal speed, and read the terminal voltage and the current output.



The Characteristic Curves of a Shunt Generator.

Fig. 4.23

The resistance of the field winding is kept constant, and when the external circuit is closed and armature

current is increased from the no-load value, the P. D. on the armature terminals gradually decreases as shown in Fig. 4·23, owing to (1) the drop in pressure due to IR_a, (2) the demganetizing effect of the armature current, (3) the lowering of voltage at the armature terminals decreasing the strength of the current in the field coils, i. e., the exciting current. The flux being thus reduced causes further reduction of the terminal P. D.

OBSERVE.—(1) The E. M. F. across the armature terminals is a maximum on open circuit.

(2) The effect of the increase of the demagnetising ampere-turns of the armature, and the pressure drop owing to armature resistance, is that the voltage continuously decreases as the load is increased up to the point P where the maximum current is reached, beyond which it is impossible to go. Here the flux set up by the demagnetising ampere-turns, and that set up by the field magnets, are equal, and so the voltage decreases to zero. The curve P D bends back and cuts the abscissa at D a little to the right of the origin, for the voltage, generated in the armature, due to the residual magnetism of the field magnets. Owing to thermal considerations, the full load current is often made half of this maximum value.

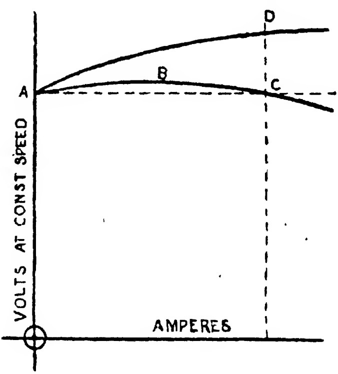
AK shows the change in E. M. F. between no load and full load, and is known as THE REGULATION or LOAD CURVE.

(3) There are two values of the terminal E. M. F. for every value of the current, excepting the maximum, and the resistance of the external circuit determines the value that is obtained.

$$R = \frac{\text{Terminal PD}}{\text{External Current.}} = \frac{V}{I} = \tan \theta,$$

and is given by the slope of CK, where K is any point determined by V and I. The greater part of the curve FD is a straight line, which shows that slight changes in the external resistance will make great changes in pressure and current. F D defines the CRITICAL RESISTANCE of the external circuit. In this part the magnetic flux of the field magnet is unstable, and the machine fails to excite.

85. The Characteristics of a Compound Dynamo.—This is the resultant of the shunt and series Characteristics. The increase of load diminishes the voltage of a shunt dynamo, but increases the voltage of a series-wound one, except with a very large current. Hence, the terminal voltage at full load



The Characteristic Curves of a Compound Generator. ABC, Flat Compounded. AP, Over Compounded.

Fig. 4.24

may be made the same as that at no load. The external characteristic is shown in Fig. 4.24.

At the point, B, the generator is flat-compounded, i. e. it has the same P. D. as at no load. This point can be made to occur at any one load. If this point is at about 2/3 full load, the fluctuations in the P. D. are made small over the range of full load. It is sometimes desirable to make the P. D. rise when the load increases. This is secured by increasing the number of series turns. The generator is then over-compounded, and has an external characteristic such as AP.

EFFICIENCY.

86. Efficiency of Direct Current Machines.—

The efficiency of a machine $= \dfrac{\text{Output}}{\text{Input}}$,

$$= \frac{\text{Output}}{\text{Output} + \text{Losses}}.$$

The losses are:

(1) Stray loss. (2) Copper loss.

(1) Stray loss:
- (a). Mechanical losses:
  - Windage.
  - Bearing Fraction.
  - Brush Friction.
- (b). Iron losses:
  - Hysteresis loss.
  - Eddy current loss.

(2) Copper loss .... I^2R losses.

The resistance R to be used, is the resistance of the winding at the steady running temperature of the

machine, which is the maximum temperature reached after running for 6 hours or more if necessary. It is also called the hot resistance.

Example 2. If a 100 KW, 220 volt generator requires an armature current of 25 amperes when run as a motor at no-load and normal speed and voltage, find the stray loss, the resistance of the armature circuit being .008 ohms.

Solution:—

The armature input at no-load $= 220 \times 25$ watts

$= 5500,$

$=$ stray loss $+ I^2R_a,$

$=$ stray loos $+ (25^2 \times .008) = 5$ watts.

$\therefore$ stray loss $= 5500 - 5 = 5495$ watts.

87. The Electrical Efficiency is also called the ECONOMIC COEFFICIENT.

The MECHANICAL EFFICIENCY is called the COEFFICIENT OF CONVERSION, or EFFICIENCY OF CONVERSION.

The efficiency, which is defined as the output of power divided by the intake of power, is called the TRUE EFFICIENCY, or THE COMMERCIAL EFFICIENCY.

The commercial efficiency of a generator is the product of the conversion and economic co-efficients, or the product of the electrical and the mechanical efficiency, for both generators and motors.

The commercial efficiency, or simply, the efficiency

of a generator $= \dfrac{VI}{B.H.P. \times 746},$

where V = the terminal P. D. of the generator in volts,
I = the current in amperes in the external circuit,
B. H. P. = the horse power actually supplied to the generator.

THE PLANT EFFICIENCY: This includes all the sources of loss, and is equal to the ratio

$$\frac{\text{The useful work done in a given time}}{\text{The energy supplied to the prime mover for the same time}}.$$

THE COMBINED EFFICIENCY of a generator

$$=\frac{\text{Output of the generator in watts.}}{\text{Indicated H. P. steam Engine} \times 746},$$

= Commercial Efficiency of Generator × Mechanical Efficiency of Steam Engine.

The Commercial Efficiency or simply, the Efficiency of a motor $=\dfrac{\text{B.H.P.} \times 746}{VI}$,

where V = the supply P. D. in volts,
I = the total amperes supplied,
B. H. P. = the useful horse power of the motor.

The Electrical Efficiency of a generator

$$=\frac{VI}{VI+I^2R}.$$

[Compare the Electrical Efficiency of a motor

$$=\frac{VI-I^2R}{VI}.]$$

88. Efficiency of Different types of Dynamos.

Let E = the E.M.F. of the machine,
V = the potential difference at the terminals or the terminal P. D.,

R_a = the resistance of the armature winding,
R_s = the resistance of the series field magnet winding,
R_{sh} = the resistance of the shunt magnet winding,
R = the resistance of the electrical circuit,
I = the current flowing through the line,
W = Stray loss.

To determine the efficiencies, we consider the case of each machine separately as follows:

89. (1) **Magneto-Dynamo.**—Here the circuit is a simple series circuit, and the only loss is in the armature conductor and brush contacts, etc.

The power lost in the armature $= I^2R_a$,

$$\therefore \text{Electrical Efficiency} = \frac{VI}{VI + I^2R_a},$$

$$= \frac{V}{E}. \quad \ldots \quad (1)$$

Now, the total mechanical loss W takes into consideration the losses due to, M the mechanical friction loss, P_h the hysteresis loss, P_e the eddy-current loss.

$$\text{Mechanical Efficiency} = \frac{EI}{VI + I^2 R_a + W}.$$

90. (2) **Series Dynamo.**—

$$\text{Electrical Efficiency} = \frac{VI}{VI + I^2 (R_a + R_s)}.$$

$$\text{Mechanical Efficiency} = \frac{EI}{VI + I^2(R_a + R_s) + W}.$$

Commercial Efficiency

$$=\frac{VI}{VI+I^2(R_a+R_s)}\times\frac{EI}{VI+I^2(R_a+R_s)+W}$$

$$=\frac{VI}{VI+I^2(R_a+R_s)+W},$$

for $EI=VI+I^2(R_a+R_s)$.

Example 3. A series generator delivers 25 amperes at 220 volts between its terminals;

$$R_a=.16,$$
$$R_s=.04.$$

Stray power loss at given speed and voltage $=600$ watts. Determine the efficiency of the generator

Solution :—

(a) Power output $=25\times 220=5500$ watts.

(b) Series field loss $=25\times 25\times .04=25$ watts.

(c) Armature loss $=25\times 25\times .16=100$ watts.

(d) Efficiency $=\dfrac{5500}{5500+25+100+600}=.88.$

Note that if only efficiency is to be determined it is the commercial efficiency which is to be found.

91 (3) **Shunt Dynamo.**—

(a) Current in the shunt coil $=\dfrac{V}{R_{sh}}$ amperes.

Waste in the shunt coil $=\dfrac{V^2}{R_{sh}}$ volts.

(b) The current in the armature $= I + \dfrac{V}{R_{sh}}$.

Waste in the armature $= \left(I + \dfrac{V}{R_{sh}}\right)^2 R_a$ watts.

$\therefore$ Electrical Efficiency $= \dfrac{VI}{VI + \dfrac{V^2}{R_{sh}} + \left(I + \dfrac{V}{R_{sh}}\right)^2 R_a}$.

Mechanical Efficiency =

$$\frac{EI}{VI + \dfrac{V^2}{R_{sh}} + \left(I + \dfrac{V}{R_{sh}}\right)^2 R_a + W}.$$

Example 4. A shunt generator delivers 25 amperes of current at 220 volts between its terminals.

$R_{sh} = 80$ ohms (hot) including the portion of the field rheostat which must be in circuit to bring the voltage between the terminals of the machine to the specified value.

$R_a = 0.28$ ohm (hot)

Stray power loss at given speed and voltage = 600 watts.

Determine the efficiency of the generator.

Solution :—

(a) Power output $= 220 \times 25 = 5500$ watts.

(b) Field loss $= \dfrac{220 \times 220}{80} = 605$ watts.

(c) Armature loss $= \left(25 + \dfrac{220}{80}\right)^2 \times .28 = 216$ watts.

(d) Efficiency $= \dfrac{5500}{5500+605+216+600} = .794.$

92. Long Shunt Compound Dynamo.—The Copper loss occurs in three parts of the machine.

The output $= VI$.

(a) The current in the shunt circuit $= \dfrac{V}{R_{sh}}$,

$\therefore$ the loss in the shunt coil $= \dfrac{V^2}{R_{sh}}$.

(b) The current in the series field circuit is

$$I + \frac{V}{R_{sh}},$$

$\therefore$ loss in the series field circuit $= \left(I + \dfrac{V}{R_{sh}}\right)^2 R_s$.

(c) The current in the armature $= I + \dfrac{V}{R_{sh}}$,

$\therefore$ loss in the armature $= \left(I + \dfrac{V}{R_{sh}}\right)^2 R_a$.

$\therefore$ Electrical Efficiency $= \dfrac{VI}{VI + \dfrac{V^2}{R_{sh}} + \left(I + \dfrac{V}{R_{sh}}\right)^2 (R_s + R_a)}$.

Mechanical Efficiency

$$= \frac{EI}{VI + \dfrac{V^2}{R_{sh}} + \left(I + \dfrac{V}{R_{sh}}\right)^2 (R_s + R_a) + W}.$$

Example 5. A long shunt compound generator delivers 25 amperes of current at 220 volts between its terminals.

$R_{sh} = 90$ ohms (hot),
$R_s = 0.04$ ohm (hot),
$R_a = 0.14$ ohm (hot).

Stray power loss at given speed and voltage = 600 watts.

Calculate the efficiency of the generator.

Solution :—

(a) Power output $= 220 \times 25 = 5500$ watts.

(b) Shunt field loss $= \dfrac{90 \times 220 \times 220}{90 \times 90} = 538$ watts.

(c) Series field loss $= (25 + \frac{220}{90})^2 \times .04$
(d) Armature loss $= (25 + \frac{220}{90})^2 \times .14$ $\Big\} = 494$ watts.

(e) Efficiency $= .77$

93. Short Shunt Compound Dynamo.—

The output $= VI$,

V being the P. D. at dynamo terminals.

(a) Current in the series coil $= I$,

$\therefore$ loss in the series coil $= I^2 R_s$.

(b) Current in the shunt coil $= \dfrac{V + IR_s}{R_{sh}}$,

$\therefore$ loss in the shunt coil $= \left(\dfrac{V + IR_s}{R_{sh}}\right)^2 R_{sh}$.

(c) Current in armature coil $= I + \dfrac{V + IR_s}{R_{sh}}$,

$\therefore$ loss in the armature coil $= \left(I + \dfrac{V + IR_s}{R_{sh}}\right)^2 .R_a$.

$\therefore$ Electrical Efficiency $=$

$$\frac{VI}{VI+I^2R_s+\left(\frac{V+IR_s}{R_{sh}}\right)^2 R_{sh}+\left(I+\frac{V+IR_s}{R_{sh}}\right)^2 . R_a}.$$

Mechanical Efficiency $=$

$$\frac{EI.}{VI+I^2R_s+\frac{(V+IR_s)^2}{R_{sh}}+\left(I+\frac{V+IR_s}{R_{sh}}\right)^2 . R_a+W}.$$

Example 5. Calculate the efficiency of a short shunt compound generator assuming the data as in the case of the long shunt generator of Ex. 4.

Solution :—

(a) Power output $=220\times 25=5500$ watts.

(b) Shunt field loss $=\left(\frac{220+25\times .04}{90}\right)^2\times 90$,

$=543$ watts.

(c) Series field loss $=25\times 25\times .04=25$ watts.

(d) Armature loss $=\left(25+\frac{220+25\times \cdot 04}{90}\right)^2\times 1.6$,

$=120$ watts,

(e) Efficiency $=\frac{5500}{5500+543+25+120+600}=.81.$

94. Minimum and Maximum Efficiencies.— The efficiency of an electrical machine, whether generator or motor, is ZERO at no-load, and increases with the load; for the constant losses at smaller load are large in comparison with the power output.

The MAXIMUM EFFICIENCY IS ATTAINED when the constant losses are equal to the variable losses. This is proved as follows:

Let

W = the constant losses,

V = the terminal voltage,

R = resistance of the armature series field winding and brush contact,

I = current output of a dynamo or current input of a motor. Consider the current flowing through the armature (since the shunt current is very small excepting in very small machines).

In a dynamo:

$$\text{Power output} = IV.$$

$$\text{Variable losses} = I^2R.$$

$$\therefore \text{Efficiency} = \frac{IV}{IV + I^2R + W}.$$

Differentiating this with respect to I the current, and equating the expression to zero, we get the condition for maximum efficiency. Thus,

$$\frac{dx}{dI} = \frac{(IV + RI^2 + W)\,V - IV\,(V + 2RI)}{(IV + RI^2 + W)^2},$$

$$= \frac{IV^2 + RVI^2 + WV - IV^2 - 2\,I^2RV}{(IV + RI^2 + W)^2},$$

$$= \frac{WV - I^2RV}{(IV + RI^2 + W)^2} = \frac{V\,(W - I^2R)}{(IV + RI^2 + W)^2} = 0.$$

$$\therefore W = I^2R,$$

which is the condition required.

That is, the machine has its highest efficiency when the armature current is such as to make the variable losses just equal to the constant losses.

95. The Voltage Regulation of a Generator.—When the speed of the armature, the resistance of the armature winding and the resistance of the field circuit are constant, the voltage regulation is the ratio of the maximum deviation of the full load voltage from the no-load voltage to the full load voltage.

$$\text{Voltage regulation} = \frac{(\text{No-load voltage}) - (\text{Full-load voltage})}{\text{Full-load voltage}},$$

and has reference to the changes in voltage that occur when the load on it changes, because of conditions within and brought about by the machine itself.

Example 7. A generator has a no-load voltage of 230 and a full-load voltage of 225. Find the voltage regulation.

Solution :—

$$\text{Voltage Regulation} = \frac{230-225}{225} = \frac{1}{45},$$

$$= 0.022 = 2.2 \text{ per cent.}$$

Note (1) that a flat compound generator has practically a perfect regulation, and a shunt generator has poor regulation; (2) that the word REGULATION is applied to changes of voltage inherent in the machine itself, while the word CONTROL is applied to changes of voltage due to deliberate adjustments by an attendant.

To obtain the regulation of a SHUNT, or SEPARATELY EXCITED MACHINE, we put full-load upon the dynamo,

and adjust the field until the voltage is the rated voltage of the machine. The speed is also adjusted to the rated value. The main switch is then opened so as to remove all of the load from the machine. The speed will now increase, and it will be necessary to adjust it to normal again, or else make a correction for the changed speed. The voltage is again read. The rise in voltage divided by the full-load voltage is the per cent. regulation.

In the case of a SERIES MACHINE the term regulation, as used above, has no particular value, and is not employed.

The regulation of a COMPOUND-WOUND MACHINE is not of great importance, and is rarely required. It is obtained by taking the characteristic curve, and obtaining a maximum deviation of the curve from a straight line connecting the full-load voltage and the no-load voltage.

When the armature speed, field resistance and armature resistance are constant, the COMPOUNDING OF A GENERATOR is the ratio of the increase in voltage, between no-load and full-load to the no-load voltage.

The Percentage of Over Compounding

$$= \frac{\text{Full-load E. M. F.—No-load E. M. F.}}{\text{No-load E. M. F.}}.$$

Example 8. A machine gave 220 volts at no-load, and 260 volts at full-load at the same speed. Find the percentage of over compounding of the machine.

Solution :—

$$\text{Percentage over compounding} = \frac{260-220}{220},$$

$$= .181 \text{ or } 18.2 \text{ per cent.}$$

STARTING, STOPPING AND RUNNING GENERATORS IN SERIES AND PARALLEL.

96. Carefully note the following before starting a machine :—

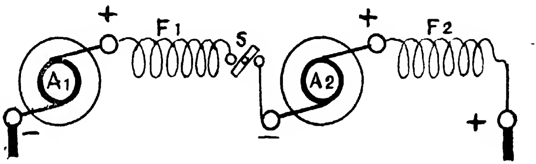
(a) Clean the machine thoroughly, specially the commutator, brushes, etc.

(b) Examine the machine to ascertain that no parts are loose or out of place, the brushes make good contact with the commutator at the proper point, the oil cups have sufficient oil and the oil rings etc. work freely, and the belt, if there is any, has the proper tension.

(c) If for the first time, run the machine a few turns, or turn it very slowly with hand, to determine that the belt runs on the centre of the pulley, and the shaft revolves freely. Bring the machine up to speed with care, and watch that everything is all right. The attendant should be ready to throw the apparatus out of circuit, and stop the machine, if anything is wrong.

97. To run generators in series, the positive terminal of the one must be connected to the negative terminal of the next, and each must have a

current capacity equal to the maximum current in the circuit, but the E. M. F. may differ. The voltages of the machines are added together causing an increased danger to persons and insulation.

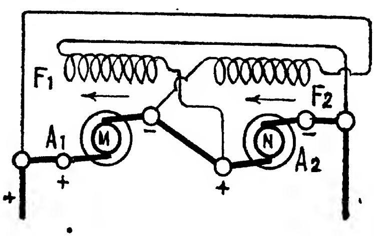


Series Dynamos in Series.

Fig. 4.25

SERIES GENERATORS:—As a rule, series generators are run in series (Fig. 4·25), and the current is kept constant by using a regulator or diverter as already mentioned. The regulators do not generally work well together, as they are apt to seesaw with each other. They are so connected as to work together, or one is used to give full E. M. F. and the other to control the current.

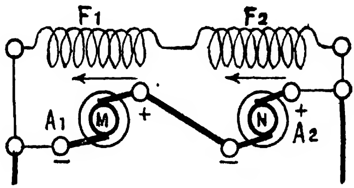
SHUNT OR COMPOUND GENERATORS:—Shunt or compound generators run well in series. Join the shunt field coils together to form one shunt across both machines (Fig. 4.26).



(a) Shunt Dynamos in Series.

Fig. 4.26

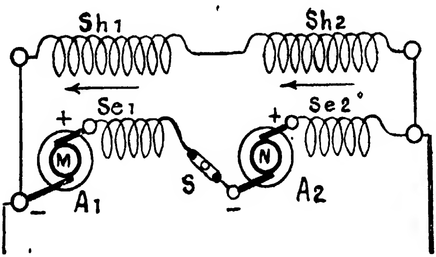
Or, connect each shunt field so that it is fed by the armature of the other machine (Fig. 4.27).



(b) Shunt Dynamos in Series.

Fig. 4.27

Or, connect both the shunt coils so as to be fed by one armature. In the case of compound dynamos a further precaution is to be taken—all the series coils must be connected in series with the main circuit (Fig. 4.28).

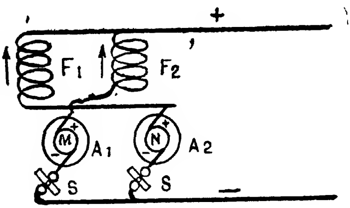


Compound Dynamos in Series.

Fig 4.28

98. Parallel Openation of Series Dynamos.—When two series dynamos M and N (Fig. 4.29) are run in parallel, the current taken from the armature of M is sent round the field of N, and the current of N is

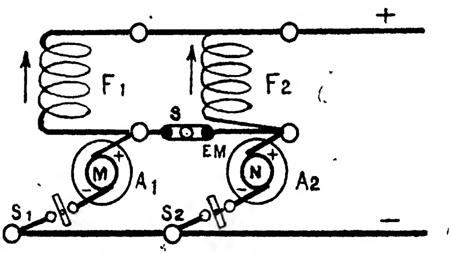
sent round the field of M. Otherwise the current from one dynamo may reverse the polarity of the other. Any variation of current in M correspondingly alters the field, and the current of the other, and the two thus tend to balance each other. Both the machines are started before they are switched in the circuit.



(a) Series Dynamos in Parallel.

Fig. 4.29

A second method which is not so good as the one



(b) Series Dynamos in Parallel.

Fig. 4.30

just described is as follows:—The terminal between the armature and field of M is connected with the corresponding terminal of N (Fig. 4.30) by a thick conductor which is called the equalizing main, and the reversal of the field is thus prevented. The switch S is closed first, M is started and its switch is closed, then N is started

and switch is closed. In this case the dynamos do not exercise much controlling effect on each other.

Series dynamos are seldom run in parallel, and shunt and compound dynamos in series.

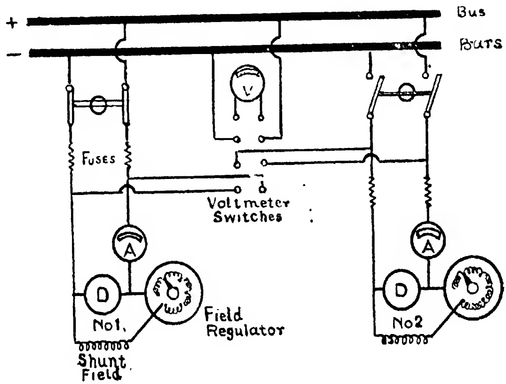
In order to run direct current generators in parallel their voltages must be equal, but their currents may differ.

99. (1) To Start a Shunt-Wound Generator. Bring the machine up to speed. (a) Insert the entire field resistance in circuit. (b) When the armature has got good speed, cut out field resistance gradually until the voltage of the machine is normal or equal to that on the bus-bars. (c) Close the main or line switch and watch the ammeter and voltmeter, making further adjustment with the field rheostat if necessary.

100. (2) To Shunt down a Shunt-Wound Generator.—(a) Reduce the load by throwing in resistance with the field rheostat. (b) Throw off the load by opening the circuit-breaker, if there is any, otherwise open the feeder switches and finally the main generator switches. (c) Shut down the driving machine.

101. (3) Parallel Operation of Shunt Generators.—If a shunt generator has to be put in parallel with other machines which are already running (a) bring it up to speed on open circuit, (b) adjust the field current until its E. M. F. is equal to the P. D. between the bus-bars, (c) then close the switch and connect the dynamo terminals to the main terminals or bus-bars of the same sign.

The voltage of one is apt to rise above that of another and drive it as a motor. When it is running as a motor its direction of rotation will be the same as when it was generating. Hence the attendant must watch



Parallel Connection of Shunt Dynamos.

Fig 4.31

the ammeters closely for an indication of this trouble. Shunt generators are now seldom installed, and are seldom operated in parallel, although they will work that way. Two or more generators having similar characteristics and after proper regulation will automatically divide the load as it varies.

Suppose that two shunt generators D_1 and D_2 (Fig. 4.31) equally loaded, are running in parallel. Now if the prime mover driving D_1 slightly slows down in speed, the E. M. F. generated by it will decrease; D_2 will

have to take a greater portion of the load, and thus a larger portion of the current will be shifted on to D_2. The loss in the armature is decreased owing to the smaller current now passing through the armature $I^2R > I_1^2R$. Where $I > I_1$, the armature reaction is also decreased, and the speed is thus slightly increased. Consequently, the E. M. F. and P. D. of the dynamo are increased.

Again as D_2 takes a larger portion of the current, the P.D. at the terminal is reduced. Thus the electrical interactions of the two machines exert an inherent tendency to equalize the speeds and loads of the two machines.

Shunt Generators running in parallel do not divide the load very well between themselves, and do not operate well in parallel, because if the voltage of one machine rises high above the others it will run them as motors.

102. (4) To Shut down a Shunt Generator working in Parallel with others.—A shunt generator operating in parallel with other generators should be disconnected from the load circuit by opening the lines, switch or circuit breakers, after reducing either its speed or its field excitation, and increasing the field of the others, so as to keep the load distributed, and volts steady, until the current in the armature circuits is zero.

103. (5) To Start a Compound Generator:—(a) See that all field resistance is cut in. (b) Slowly

bring the prime mover to speed. (c) When the machine has come up to normal speed, cut out the field resistance until the voltage of the machine is normal, or equal to, or a trifle above that on the bus-bars. (d) Throw on the load.

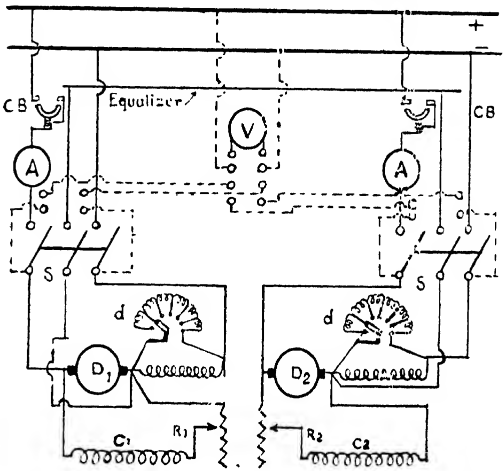
IF IT IS TO RUN IN PARALLEL (Fig. 4.32) with others, and if three separate switches are used, CLOSE THE EQUALIZER SWITCH FIRST, the series coil line switch second, and the other line switch third. If a three-pole switch is used all the three poles are, of course, closed at the same time. (e) Watch the voltmeter and ammeter, and adjust the field rheostat so that the load is properly divided between them as shown by their ammeters. A machine generating the higher voltage will take more than its share of the load, and if its voltage is too high, it will run the other as a motor.

Parallel Operation of Compound Generators is effected if the machines are of the same make and voltage, or are designed with similar electrical characteristics. The only change usually required is the addition of an equalizer connection between the machines.

THE EFFECT OF THE EQUALIZER is to divide the load properly between the several generators, and without equalizer connection the generators are in unstable equilibrium.

IF THERE IS NO EQUALIZER connection, and the speed of one generator is slightly greater than that of the other, its voltage increases, and it takes greater part

of the total load. If the total load is constant, the increased load on one generator causes a larger current to flow in its series field winding, and reduces



Parallel connection of Compound Dynamos.

Fig 4·32

the current flowing in the series field winding of the other generator. The equilibrium of the generators is thus disturbed, and as a result, one generator tends to take up the entire load and to operate the other as a motor.

IF THERE IS EQUALIZER CONNECTION the increased speed in one increases its voltage and current output. But the increased current output instead of flowing through the series field winding of one generator, divides itself at

the brush, part flowing through the series field winding of each generator. Thus the voltage of each dynamo is equally increased, and the equilibrium is not disturbed. The equilizer bars should be large, and must be of negligible resistance as compared with the series coils and fuses, and no other circuit-interrupting devices should be placed in an equalizer circuit.

104. (6) To Shut down a Compound-wound Generator Operating in Parallel with others.—(a) Reduce the load as much as possible by throwing in resistance with the field rheostat. (b) Throw off the load by opening the circuit-breaker, if there is any; otherwise open the main generator switches. (c) Shut down the driving machine.

If the generators have different compounding ratios, it may be necessary to readjust the series field shunts to obtain uniform conditions.

105. Design of Shunt Rheostat.—

(1) In general, dynamo shunt rheostats are divided into sections each of which has the same resistance.

(2) The number of steps is determined by the permissible percentage variation in the total induced E. M. F., when one section is cut out or put in the circuit. This variation, allowed per step, is generally four per cent of the total range of induced E. M. F. between no-load and full-load.

(3) The value of the shunt regulating resistance is such that at full-load 5 to 10 per cent of the terminal pressure is absorbed.

(4) The section of the resistance coils must be such that when the maximum current flows through the wire, the steady temperature attained by the wire will not exceed a certain safe limit which is generally taken about 100°C.

Now, $d^3 = 2I^2\sigma$ (P. 204, Vol. I).

$\therefore\ d = \sqrt[3]{2I^2\sigma}.$

(5) The length of the wire required to give any particular value of resistance is easily obtained, for

$$R = \frac{\rho L \times 4}{\pi d^2}, \text{ or } L = \frac{\pi d^2 R}{4\rho}$$ (P. 140, Vol.I).

106. I. To find the size of the Engine required to Drive a Direct Current Generator.–Determine the horse-power of the generator (when running at full load) from the relation:

$$H.P. = \frac{E \times I}{746 \times \text{efficiency}}.$$

Consider the overload capacity (1) of the Generator, and (2) of the Engine.

If the generator is rated on the maximum basis, due allowance must be made for this.

If the generator is belt driven, some power is wasted, and an allowance of from 2 to 5 per cent in the belt driven generator should be added.

II. To find the Size of a Direct Current Generator when the B. H. P. of the Engine is given.—We make use of the following relations:

B. H. P. × 0.746 = Kw. Capacity of the Engine.

Kw. Capacity of the Engine $\times$ Efficiency of the Generator
$=$ Kw. output of the generator.

Where the generator efficiency is not known, it may be assumed to be 90 per cent.

Example 9. Determine the size of the (1) internal combustion i. e. gas, gasoline or oil engine, (2) steam engine, to be used in a set which can take a lighting load of 100 Kw. normally rated direct current generator. The efficiency of the generator is 84 per cent.

Solution:—

(1) Normally rated generators usually have an over-load capacity of 25 per cent. for two hours.

The maximum power that this machine could develop, for any considerable length of time, would be $100 \times 1.25 = 125$ Kw.

$$\text{H.P. of the Engine} = \frac{125}{0.84 \times 0.746} = 199.2 \text{ H.P.}$$

That is, 199.2 H.P. would be required in mechanical power at the generator pulley, to produce 125 Kw. of electrical power at the generator terminals.

Assuming a belt loss of 5 per cent, there would be required at the engine fly-wheel $199.2 \times 1.05 = 209.2$ H.P.

Hence, the engine should have a rating of at least 209.2, if the full-capacity of the generator is to be developed. Normally, combustion engines have little, if any, overload capacity. In practice, 200 H.P. internal combustion engine would probably be used for the application in the discussion.

(2) The overload capacity of steam engines, as engines are usually rated, is about equivalent to the overload capacity of a normally rated, direct current generator.

Hence, if the generator is driven by a steam engine, the overload capacity of the generator is not taken into consideration.

Thus for a normal full-load rating of 100 Kw., and an efficiency of 84 per cent., there would be required to drive at full-load, 100 Kw.$\div$0.84$=$119 Kw.

The horse power equivalent of 119 Kw. is 119$\div$0.746$=$159.5 H.P.

Then, if a belt loss of 3 per cent. be assumed, the engine would have to develop, when the generator is fully loaded,

$$159.5 \times 1.03 = 164 \text{ H.P.}$$

That is, an engine rated at about 164 H.P. should be used to drive the machine.

Exercises.

1. Show by means of sketches, the methods used for connecting compound-wound dynamos in parallel. (C. & G. 1897).

2. Give sketches showing the way in which (a) two shunt dynamos, (b) two compound-wound dynamos should be connected so as to work in parallel, and explain your object in connecting them as shown. (C. & G. 1901).

Convert a 550 volt Generator to a 220 volt machine.—(1) Short circuit the series operating coil by a switch in the base of the machine. (2) Open another switch on the generator panel and introduce a pre-arranged fixed resistance into the field circuit reducing the excitation to the value where 220 volts will be developed. Make closer adjustment with the field rheostat. Place disconnecting switches at the rear of the board for throwing the machine leads from 500 volt to 220 volt buses.

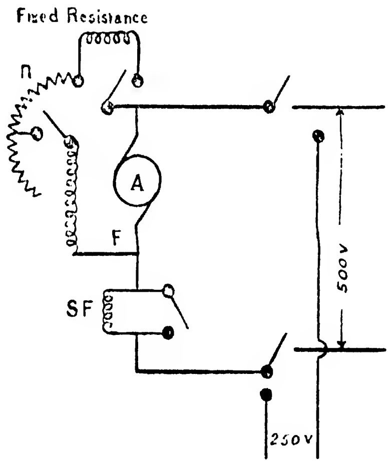


Fig. 4.33.

To face page 165.

3. Sketch diagrams of connections you would employ to do the following :—

(a) Couple two equal series dynamos in parallel.

(b) Couple two equal shunt dynamos in series.

(C. & G. 1893).

4. Why is it dangerous to lift the brushes of a shunt-wound dynamo when shutting down ? State what would occur if you were to lift the brushes of a running shunt-wound dynamo (a) while a fair load of lamps are being supplied ; (b) when no current is being supplied to the external circuit. (C. & G. 1896).

5. Explain why it is generally found impossible to do any of the following things :—

(a) Charge a battery from a plain series dynamo.

(b) Couple two plain series dynamos in parallel.

(c) Couple two plain shunt dynamos in series.

(d) Operate several ordinary electric bells in series. (C. & G. 1892).

6. A drum armature in a 2-pole field contains 150 external conductors, it runs at 550 revolutions per minute. Find the total flux passing through the armature which is required to produce an electromotive force of 115 volts on open circuit. (C. & G. 1893).

7. What difficulties are likely to occur in charging batteries with a compound-wound machine, and what precautions would you take to avoid them.

(C. & G. 1898).

8. How does the current in the armature bars under the pole face in a continuous current dynamo affect the field distribution in the airgap? How does varying the position of the plane of commutation in the interpolar space with a given load affect the field excitation required for a constant voltage with constant speed? (C. & G. 1900).

9. Name the several causes of waste of energy in a continuous current dynamo machine, and state what devices are necessary, and what precautions should be taken to ensure these being a minimum in any given design. (C. & G. 1902).

10. A dynamo takes 98 horse-power to drive it, and runs at 500 revolutions per minute. If it is driven by a belt, and the pulley is 24 ins. in diameter, what will be the difference in tension between the driving and the slack sides of the belt? (C. & G. 1902).

11. A rectangular frame is wound with 100 turns of wire, the average area enclosed by the turns of wire is 500 sq. cms., and the frame is revolved at 1000 revolutions per minute about an axis in a magnetic field of 750 C. G. S. units, the axis of rotation being at right angles to the direction of the field. What will be the value of the alternating E. M. F. generated? (C. & G. 1903).

12. Discuss the merits and demerits of "resistance" commutation and "E. M. F." commutation in D. C. dynamos, and point out the conditions under which one or the other is to be preferred. (C. & G 1903).

NOTE—Since the current in a loop periodically reverses in direction, the current in the conductor must be reduced to zero, and a current of like value established in the opposite directoin, during the short time the brush is in contact with the commutator segments to which the terminals of the armature coils are connected. The inductance of the circuit tends to maintain the current in the original direction at the time when the brush short circuits the coil.

The positive or negative current may be reduce to zero and a current in the opposite direction may be established by (1) changing the relative resistance of the paths through which the current may flow, this is called the RESISTANCE COMMUTATION, (2) causing the conducters to generate an electromotive force opposite to that which established the original current. This is called the E. M. F. COMMUTATION. This is secured by advancing the brushes in the direction of armature rotation, as by this the commtation is delayed until an opposing E. M. F. is induced in the coil which just passes through the brush. This opposing E. M. F. tends to reduce the current flowing in the coil and to establish a current in the opposite direction.

In recent machines a combination of (1) and (2) are used. High resistance carbon brushes are used in connection with an angular advance of the brushes or with inter-poles.

13. If in Question 11 all the conditions remain the same, but instead of the 100 turns of wire being

in one or parallel planes, they are distributed around the surface of a cylinder of non-magnetic and non-conducting material (to form a drum or barrel winding), and provided with a commutator sector at each turn, and brushes as in a dynamo, what will be the steady E. M. F. generated, neglecting any consequences that might result from some coils being short circuited at the brushes? (C. & G. 1903).

14. Enumerate the essential parts of a C. C. dynamo, writing them down in a table, and opposite each briefly explain the object and action of each part. (C. & G., I., 1907).

15. What is the object of equalising connections on a C. C. generator, and to what class of windings are they applied? Give a diagram to show the way in which these equalisers are connected up to the armature winding. (Lond. Univ., El. Mach., 1909).

16. A continuous current series generator is driven by a motor, and connected to a resistance of 50 ohms. It fails to give an appreciable current. State two possible causes for this failure, and how you would overcome the difficulty. (C. & G., II., 1914).

17. The electromotive force of a shunt generator is 225 volts when operated without load, and the rated output is 20 kw. at 220 volts. Calculate:— (a) the voltage regulation, (b) the resistance of the armature circuit, if one-half the drop in terminal voltage between no load and full load is due to armature resistance.

18. A 1000-kw. generator has a terminal voltage of 550 at no-load and is 5 per cent overcompounded. Find (a) the size of wire required to transmit 100 amperes at a distance of 1000 feet, the voltage between the terminals of the load apparatus to be 550, (b) the watts lost in the line, (c) the total output of the generator.

19. The rated output (line) of a compound generator = 200 amperes, and its no-load voltage is 220. The current is transmitted over a line, the resistance of which is 0.15 ohm, and the voltage at the load terminals is the same at full load as at no-load. Find (a) the terminal voltage of the generator at full load, (b) percentage overcompounding, (c) watts lost in line, (d) watts output of generator.

CHAPTER V

DIRECT CURRENT MOTORS

107. The Advantages of the Electric Drive.–Some of the advantages of the electric drive are:—(1) An increase in production. (2) A greater flexibility of arrangement of machines. (3) A clear headroom is provided. This permits the free use of overhead cranes, and results in better illumination and greater freedom from accidents. (4) Because the power can be transmitted electrically for considerable distances with small loss, the power plant can be centralized where the power can be generated most efficiently. Tests of shafting drives indicate that the loss varies from 25 to 75 per cent., and in most cases is about 50 per cent. of the power generated. Furthermore, the loss is nearly constant, regardless of the load. With the electric drive the loss depends upon the amount of power transmitted, and the total loss including that in generators, feeders and motors, is only from 20 to 30 per cent. (5) The electric service is more reliable, since a breakdown usually puts out of commission only a small part of the motors. (6) It is possible to operate sections of the factory at high efficiency when the other sections are shut down, whereas, whithout the electric drive, possibly all the shafting in a large mill might have to be operated for the purpose of driving one or two machines. (7) Speed control is easily secured by means of the electric motor. Instead

of speed changes in large steps by changing gears or shifting belts, a very large number of small speed changes can be easily obtained. This results in keeping the operating speed closer to its economical limit, and thus increasing the production. (8) A study of machine performance can easily be made by introducing meters in the motor circuit. By this means, an accurate record may be obtained of the power requirements for a particular operation, and any excessive demand for power due to the faulty working of the machine is easily detected.

108. Dynamo and Motor.—Every dynamo can be used as a motor as they are identical in structure. The dynamo is used to transform mechanical energy into electrical energy, while the same machine used as a motor transforms electrical energy into machanical energy.

109 Direction of Running of a Motor.—Hold the thumb, the index finger and the middle finger of the left hand at right angles to represent the three rectangular axes in space. If the index finger points to the direction of the field, the middle finger to the direction of the current, then the thumb points to the direction of the force exerted, and therefore to the direction of motion.

110 Driving Force of a Motor.—If a voltage is impressed upon the terminals of a motor, a current is forced through the armature conductors lying across a magnetic field. The conductors thus carrying currents in

a magnetic field are acted on by forces perpendicular to themselves, and to the direction of the field (Principle of the Motor).

The currents are so distributed by the commutator, that while all the conductors under one pole carry currents in one direction, all the conductors under the next or opposite pole carry currents in the opposite direction. And thus while they are repelled by one pole, they are attracted by the opposite pole, and a continuous torque is maintained.

111. The Counter or Back E. M. F. of a Motor—When the armature of a motor rotates, an E. M. F. is generated in the inductors as they cut the lines of force due to the field magnets. By Lenz's law the E. M. F. thus induced opposes the direction of the main current, and is therefore known as a counter E. M. F. or back E. M. F.

Let S = number of armature conductors,
n = number of revolutions of the armature per minute,
p = number of pairs of poles,
ϕ = the flux entering the armature.

In a bipolar machine, when a conductor on an armature core rotates through one revolution, it cuts the lines of force passing from one pole to the other twice. Hence, the number of lines of force cut by one conductor during one revoluion is $2\, p\, \phi$, or $2\, p\, \phi \frac{n}{60}$ per second.

Therefore, the E. M. F. induced

$$e = \frac{2\, p\, \phi}{60 \times 10^8} \text{ volts.} \quad \ldots \quad \ldots \quad (1)$$

In a multipolar machine the armature conductors S are generally arranged in q parallel circuits, and S/q gives the number in series in each circuit. The total E. M. F., e, induced in the armature revolving at n revolutions per minute is given by—

$$e = \frac{2\, p\, \phi\, n}{60 \times 10^8} \times \frac{S}{q}, \quad \ldots \quad (2)$$

$$= \frac{2\, p\, n\, S\, \phi}{q \times 60 \times 10^8} \text{ volts.} \quad \ldots \quad (3)$$

If a voltmeter is connected across the armature terminals, and the main current is switched off, the voltmeter which shows the counter E. M. F. takes several seconds to fall to zero, and it reaches zero when the armature comes to rest.

112. Fundamental Equation of a Direct Current Motor.—Connection between the voltage, current, flux, speed, etc. of a Motor :—

Let r = resistance of the armature circuit including the contact resistance between the brushes and the commutator,

and, I = the armature current.

Then the voltage drop in the armature when stationary is Ir.

When the armature is rotating in a magnetic field, the counter E. M. F. is generated in the motor. The voltage applied to the terminals of a motor must be sufficient to neutralise this counter E. M. F., and also to send the necessary current against the resistance of the armature circuit.

If e be the counter E. M. F., and V the voltage applied to the terminals of the motor,

$$V = e + Ir, \qquad \ldots \qquad (4)$$

$$\text{or,} \quad I = \frac{V - e}{r}. \qquad \ldots \qquad (5)$$

V must always be greater than e; but if the motor be so designed that the no-load losses are small, then when the motor is running unloaded, e will be very nearly equal to V. The product $(V-e) \times I$ represents the power required to overcome the mechanical and iron losses of the machine; and if e were equal to V, it would mean that no power was required to drive it.

The formula (3), when used for a dynamo, relates to the E. M. F. induced therein; and when applied to a motor, it relates to the induced counter E. M. F. Formulae (3) and (4) are of fundamental impotrance in considering the behaviour of motors.

Example 1. The armature of a 220 volt, 8-pole motor is wave-wound with 888 conductors, the resistance of the armature circuit being 0.3 ohm. If the flux per pole entering the armature is 12,00,000 lines, find the speed of the motor when taking a current of 60 amperes.

Solution:—

By formula (4) we have—

$$220 = e + 60 \times 0.3$$

$$\therefore e = 220 - 18 = 202 \text{ volts.}$$

Using formula (3), and putting

$$e = 202,$$

$$p = 4,$$

$$S = 888,$$

$$\phi = 12{,}00{,}000,$$

and $q = 2$,

we have.

$$202 = \frac{2 \times 4 \times 888 \times n \times 12{,}00{,}000}{2 \times 60 \times 10^8},$$

$$= \frac{888 \times 12 \times n}{15 \times 10^3}.$$

$$\therefore n = \frac{15 \times 10^3 \times 202}{888 \times 12} = 284.$$

113. Theory of Operation of Motors.—

Let I_f = the exciting current through the field coil,

I_a = the current through the armature,

E = the impressed voltage,

R_a = resistance of the armature.

Suppose that (1) the applied voltage, (2) the exciting current, and (3) the magnetic flux per pole, are constant.

At first the motor is at rest. Close the switch to start the motor. A large current I_a flows through the armature, which is given by

$$I_a = \frac{E}{R_a}, \quad \text{for } e = o.$$

The armature conductors carrying current in a magnetic field, are acted on by forces, which ultimately overcome the resisting forces of friction, and of any load the motor is under, and the armature begins to rotate.

As the speed increases, e increases, and the current $I_a = \frac{E - e}{R_a}$ diminishes. The acceleration of the motor ceases, when the current is reduced to such a value that the total force developed is just sufficient to overcome the retarding force.

If more load is now put upon the motor, so that the driving force due to the armature current is not sufficient to overcome the increased resisting force, the speed diminishes, and with it, e also decreases. But as $I_a = \frac{E - e}{R_a}$, a large current flows through the armature. Hence, the speed is so adjusted that the increased current in the armature produces a driving force, which is just sufficient to overcome the increased retarding force.

Similarly, if the load on the motor is decreased, the driving force due to the armature current is more than sufficient to overcome the decreased resisting force, and the motor must accelerate. As the speed is increased, e also increases, and the armature current $I_a = \frac{E - e}{R_a}$ decreases.

The motor stops accelerating, and the speed and current remain constant, when the driving force due to the current has dropped to such a value that it is just

sufficient to overcome the decreased retarding force. The electrical power taken by the motor from the mains, therefore, changes automatically to suit the mechanical load on the motor. Thus, the counter E. M. F. of the motor regulates the flow of the current, in the same way as the governor regulates the flow of steam in a steam engine.

Example 2. A 220 volt, 20 H. P. direct current shunt motor has an efficiency of 88%. The exciting current is 2 amperes, and the armature resistance is 0.08 ohms. Find :—

(a) The motor input.

(b) The current taken from the mains.

(c) The armature current.

(d) The counter E. M. F.

Solution :—

(a) The motor output = 20 H. P.

$$\text{The motor input} = \frac{\text{output}}{\text{efficiency}},$$

$$= \frac{20}{0.08} = 22.727 \text{ H. P.}$$

$$= 16954 \text{ watts.}$$

(b) I_t the current from the mains

$$= \frac{\text{watts input}}{\text{applied voltage}},$$

$$= \frac{16954}{220} = 77 \text{ amps.}$$

(c) The armature current

$=$ total current—exciting current

$=77-2=75$ amps:

(d) The applied voltage $=220$.

The voltage to overcome the resistance drop

$=I_aR_a=75\times0.08=6$ volts.

$\therefore$ The counter E.M.F. $=220-6=214$ volts.

Example 3. A 20 H. P., 220 volt, 900 R. P. M. direct current shunt motor has, an armature resistance of 0.08 ohms, and takes an armature current of 75 amps. at full load. Find:—

(a) The torque at full load.

(b) The counter E. M. F. at full load.

If the flux per pole is suddenly reduced to 90 per cent of normal, find:—

(c) The armature current at the same instant.

(d) The torque at the same instant.

Solution:—

(a) The torque $=\dfrac{20\times33000}{2\pi\times900}=116.6$ lb. at 1 ft. radius.

(b) The back E. M. F. $=E-I_aR_a$,

$=220-(75\times0.08)$,

$=214$ volts.

At the instant the flux is reduced to 90%,

$$e=214\times\frac{90}{100}=192.6 \text{ volts.}$$

(c) The armature current $= \dfrac{220-192.6}{0.08} = \dfrac{27.40}{.08}$

$= 342.5$ amperes.

or 4.55 times the full-load current.

(d) The torque $= \dfrac{117 \times 90 \times 342.5}{100 \times 75}$

(since it is proportional to the flux and to the armature current)

$= 480.87$ lb. at 1 ft radius.

or the torque is 4.1 times the full-load torque.

Note that at the instant the flux per pole is reduced to 90 per cent of its normal value, the armature current increases to 4.55 times the normal, and the torque to 4.1 times the normal value. The driving torque being then larger than the retarding torque of the load, the motor must accellerate.

114. Torque, Speed and Power of a Motor.—

If l = the length of the armature condutor in cms.,

S = the total number of inductors,

n = the number of inductors per circuit,

i = the current passing through the armature for each circuit, in C. G. S. units,

i_t = total current passing through the armature circuit in C. G. S. units,

I_t = current in practical units $= i_t \times 10^{-1}$,

F = the force acting on the armature in dynes,

N = speed in revolution per minute,

r = radius of the armature in centimetre,

Q = number of circuits,

T = torque, $= F.\ r.$

e = counter E.M.F. in practical units $= E_1 \times 10^{-8}$, where E_1 is in C.G.S. units,

ϕ = flux entering the armature per pole,

P = number of pairs of poles

H = intensity of the field or flux per unit area of pole.

The work done in one revolution $= F \times 2\pi r$ dyne cms.

The power given out by the motor P

$= \text{torque} \times \text{angular velocity}$

$= F \times 2\pi r \times \dfrac{N}{60}$ dyne cm. per second.

But $F = H \times l \times S \times I$ dynes.

Multiply both numerator and denominator by 2 p.

Then, $P = HlSI. \dfrac{2p}{2p} \times 2\pi r \times \dfrac{N}{60}$ dyne cm. per sec..

and. $Hl\ 2\pi r \times \dfrac{1}{2p} = \phi$. the flux entering the armature per pole approximately.

Also, $S = n \times Q$, and $I = \dfrac{i_t}{Q}$ C. G. S. units.

Substituting these values.

$$P = \phi \times \frac{2Np}{60} \times \frac{i_t}{Q} \times nQ \text{ dyne cms. per second.}$$

$$= 2n\ \varphi \times \frac{Np}{60} \times i_t \text{ dyne cms. per second.}$$

Now, $\dfrac{2n \times Np}{60} \times \phi$ is the value of the counter E. M. F. in C G. S. units, $= E_1$.

$\therefore P = E_1\ i_t$ C. G. S. units,

$$= \frac{E_1}{10^8} \cdot i_t \cdot 10 \text{ Practical units}$$

$$= E_1 i_t \times 10^{-7} \text{ watts.} = eI_t \text{ watts.}$$

If T be the torque in dyne centimetres with which the field magnet acts upon the armature, then the mechanical work done by the motor per second.

$$= T \times \frac{2\pi N}{60} \text{ dyne centimetres.}$$

$$=T.\frac{2\pi N}{60}\times 10^{-7}\text{ watts.}$$

But the power developed by the motor $=e\,I_t$ watts.

$$\therefore\ T=\frac{e\,I_t}{N}\times\frac{10^{-7}\times 60}{2\pi}\text{ dyne centimetres,}$$

$$=\frac{e\,I_t}{N}\times\frac{10^{-7}\times 60}{2\pi}\times\frac{1}{981}\times\frac{1}{1000}\times\frac{1}{100},$$

$$=\frac{e\,I_t}{N}\times 0.96\text{ kilogram metres.}$$

But, $e=\frac{2nNp\phi}{60}\times 10^{-8}$ volts.

$$\therefore\ T=2n\,\frac{N\,p}{60}\times\phi\times 10^{-8}\times\frac{I_t}{N}\times 0.96,$$

$$=0.32\times P\times n\times\phi\times I_t\times 10^{-9}\text{ kilogram metres.}$$

The effective torque at the pulley of the motor will be less than that given by the above equation, by the amount required to overcome the mechanical and iron losses.

If R = radius of armature in feet,

F' = force in pounds on the armature conductor,

$2\pi N = w$ = angular velocity,

T = torque $= F'R$ pound foot,

we have—

$$\frac{e\,I_t}{746}=\frac{2\pi N T}{33000}.$$

$$\therefore T = \frac{33000 \times e\, I_t}{2\,\pi\, N \times 746} \text{ pound foot,}$$

$$= \frac{H.P. \times 33000}{2\,\pi\, N} \text{ pound foot.}$$

Example 4. A 50-B.-H. P., 220-volt motor has to be designed to run at a speed of 500 R. P. M., and have an efficiency at full load of 92 per cent. The motor has 4 poles, and the flux entering the armature per pole equals 4.2 megalines. Calculate the number of inductors if the armature be wave-wound, and the watts absorbed due to armature resistance be limited to 300 watts.

Solution :—

$$\text{Full-load curent of motor } (I) = \frac{50 \times 746}{220} \times \frac{100}{92},$$

$$= 184 \text{ amperes.}$$

Armature resistance (R_a) not to exceed $300/(184)^2$

$$= .00886 \text{ ohms.}$$

Flux per pole (ϕ) $= 4.\ 2 \times 10^6$ lines.

Impressed E. M. F. (E) $= 220$ volts.

And, $\dfrac{R.P}{60} = \dfrac{500}{60} \times 2 = 16.7.$

From the fundamental equation, turns per circuit through the armature

$$= \frac{E - R_a\, I}{4 \times \dfrac{RP}{60} \times \phi \times 10^{-8}} = \frac{220 - (0.00886 \times 184)}{4 \times 16.7 \times 4.2 \times 10^6 \times 10^{-8}},$$

$$= 77.8 \text{ or } 78.$$

Therefore, the total number of armature inductors
$=78\times2\times2=312.$

115. Speed Variation of Motors.—
We have $V=e+Ir$.

$$\therefore\ e=V-Ir.$$

Substituting this value for e in formula (4), we have :

$$V-Ir=\frac{2\,p\,N\,S\,\phi}{q\times60\times10^8}\quad\text{volts.}$$

$$\therefore\quad N=\frac{q\times60\times10^8}{2\,p\,S\,\phi}\times(V-Ir).\qquad\ldots\quad(5)$$

Now, for any particular motor, the quantities q, p, and N, are constant, and 60×10^8 is always so. Representing the whole of these constants by k as follows :

$$k=\frac{q\times60\times10^8}{2\,p\,S},\qquad\ldots\quad(6)$$

we may write :

$$N=\frac{k\,(V-Ir)}{\phi}.\qquad\ldots\quad(7)$$

In a motor of average size, the voltage-drop Ir in the armature at full load is only about 1/20th of the terminal voltage V, so that Ir is almost negligible in comparison with V. We can therefore assume that the speed

$$N=\frac{k\,V}{\phi}\ \text{(approximately)}.\qquad\ldots\quad(7'a)$$

Hence we find that the speed of a given motor is directly proportional to the armature terminal voltage, and inversely proportional to the flux entering the

armature, or, the speed of a motor with a given load can only be varied by altering either its armature terminal voltage, or its magnetic flux, or both. Thus increasing the voltage, or decreasing the excitation will increase the speed, and vice versa.

In a constant pressure system, the voltage across the armature terminals of a motor can only be varied either (a) by inserting a resistance in series with the armature, or (b) by feeding it from a motor-dynamo set, the voltage of the dynamo of which can be varied at will.

116. Regulation of Motors—In the regulation of a motor we are concerned with the speed, whereas in the regulation of a dynamo with the terminal voltage.

The SPEED REGULATION of a motor refers to changes in speed caused by the interactions inherent to and within the motor itself, as the load driven by it decreases or increases. It is numerically equal to

$$\frac{\text{No-load R. P. M.—Full load R. P. M.}}{\text{Full load R. P. M.}}$$

Example 5. If the speed of a motor at no-load is 1400 R. P. M., and its speed at full load is 1350 R. P. M., find its speed regulation.

Solution :—

$$\text{Speed Regulation} = \frac{1400-1350}{1350} = \cdot 037,$$

$$= 3\cdot 7 \text{ percent.}$$

Note that while SPEED REGULATION is an inherent property of a motor, SPEED CONTROL is the variation in speed obtained by external means.

To convert a small 110 volt shunt motor on a 220 volt circuit for occasional service, at low speed (John Burns). Fig, 5.011. The current in the armature of small motors would not much exceed that taken by the field and thus these two circuits can be connected in series across the 110–220 volt three-wire system with the neutral tapped in without much unbalancing.

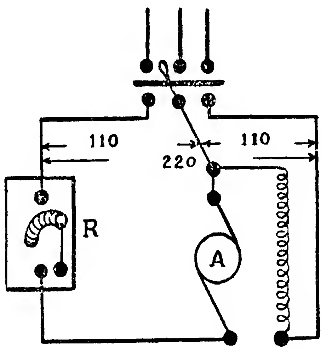


Fig. 5.081.

To face page 185.

117. Speed of a Shunt Dynamo used as a Motor.—A shunt dynamo is used as a motor on a line of the same voltage. The speed of the motor is found as follows :

Suppose as a generator it generates current at E volts, but supplies I amperes to the line at E′ volts, and R is the resistance of the armature. Then,

$$E = E' + IR.$$

The E. M. F. generated in the same machine as a motor is

$$e = E - IR.$$

Since the field strength is the same in the two cases, both being supplied at E volts, the armature E. M. F.'s will be directly proportional to the speed ; or,

$$\frac{\text{Motor speed}}{\text{Dynamo speed}} = \frac{e}{E} = \frac{E' - IR}{E' + IR},$$

$$= 1 - \frac{2IR}{E} \text{ (approx).} \qquad \ldots \qquad (8)$$

$\frac{IR}{E}$ lies ordinarily between 0.03 and 0.05, as we have seen; it follows that the speed of a given machine as a motor will be generally from 6 per cent to 10 per cent less than its speed as a dynamo. This relation does not hold good if the field strength is varied, either because of armature reaction or changes in the field current.

Example 6. A certain dynamo is rated at 20 kilowatts at 220 volts and 500 revolutions per minute.

Its armature resistance is 0.08 ohm. What is its full-load speed as a motor ?

Solution :—

$$I = \frac{20000}{220} = 91 \text{ amperes.}$$

$$\therefore \text{ Motor Speed} = 500 \times \left(1 - \frac{2 \times 91 \times \cdot 08}{22}\right)$$

$$= 464. \text{ R. P. M.}$$

118. Comparison of Shunt, Series and Compound Motors.—

(a) SHUNT MOTOR.—This runs at a practically constant speed at all loads within its capacity, and has a torque almost directly proportional to the armature current with a starting torque 50 to 100 per cent greater than full load running torque. It is therefore suitable for driving shafting, or a machine or group of machines, requiring steady speed. The speed of such a motor increases slowly as the field coils heat up and the excitation is decreased.

Its speed can be easily and economically altered by a shunt ragulator; as such, it is the most suitable motor where a fair amount of speed variation is required without frequently starting and stopping the motor under load. It is a constant speed motor, and is suitable for driving shafting, machine tools, lathes and wood working machines requiring a steady speed.

(b) SERIES MOTOR.—It runs at greatly decreasing speed for increasing loads. This motor exerts a greater

torque than either of the other two types, when it takes a larger current than the full load value. It is the best motor for heavy starting duty. It has a powerful torque. Its speed decreases rapidly with increase of load, and becomes dangerously high at light loads. Hence, such motors should be directly connected to the load. In the series motor as the load increases, the magnetic field strengthens, and the speed decreases ; consequently, commutation is generally very good. The non-reversible series motor gives less commutation trouble than any other types. It has a torque that increases almost as the square of the armature current.

It is suitable for cranes, trains, trams, etc. which start under heavy loads, and for cases providing a steady load requiring a constant speed, such as fans, pumps blowers, etc. It is a variable speed motor which adjusts its speed to load put upon it.

(c) Compound Motor.—Where a larger starting torque is required for a given current than that obtained from the shunt machine, and where the load varies between zero and the maximum, the compound motor is to be selected. Its speed can be varied by inserting a resistance in the shunt circuit as with the shunt motor.

It is better than the shunt motor for heavy starting duty, but is not so good for this purpose as the series motor. The speed drops somewhat with the loads, but the motor runs at a maximum speed even at no load. It approximates the shunt or the series type in its

characteristics, according to the preponderance of the shunt or series winding.

EFFICIENCY.

119. Efficiency of D. C. Motors.—The efficiency of modern D.C. motors is very high, and in many large motors it is often 95 to 99 per cent.

$$\text{Efficiency of a motor} = \frac{\text{Input} - \text{Losses}}{\text{Input}}.$$

(Compare efficiency of dynamo.)

Efficiencies of Different Types of Motors.—

120. (1) Series Motor :—

Let e = the counter E. M. F. of the motor,

V = the impressed or line voltage,

I = the total current taken by the motor.

Then, $e = V - I\ (R_a + R_s)$.

Now, the output of the motor or the rate of doing work mechanically is eI.

$$\therefore \text{ Electrical Efficiency} = \frac{Ie}{VI} = \frac{e}{V}.$$

$$\text{Mechanical Efficiency} = \frac{IV - I^2(R_a + R_s) - W}{eI} = \frac{Ie - W}{Ie}$$

$$\text{Commercial Efficiency} = \frac{Ie - W}{VI}.$$

Example 7. A series motor takes 25 amperes of current from 220-volt mains. Calculate the electrical, mechanical and commercial efficiencies of the motor, having given the following data :—

$R_s = .12$ ohm (hot).

$R_a = .15$ ohm (hot).

Stray power loss (W) = 700 watts at given voltage and speed.

Solution :—

Counter E. M. F., $e = V - I\,(R_a + R_s)$

$= 220 - 25\,(.15 + .12)$

$= 213.25$ volts.

$$\therefore \text{Electrical Efficiency} = \frac{e}{V} = \frac{213.25}{220} = .969.$$

$$\text{Mechanical Efficiency} = \frac{eI - w}{e\,I} = \frac{5331.25 - 700}{5331.25} = .868$$

Commercial Efficiency $= .969 \times .868 = .841$ or 84.1 per cent.

121. (2) Shunt Motor :—

$$\text{The current in the shunt coil} = \frac{V}{R_{sh}}.$$

$$\text{The current in the armature} = I - \frac{V}{R_{sh}}.$$

$$\therefore \text{the loss in the armature} = \left(I - \frac{V}{R_{sh}}\right) R_a \text{ volts.}$$

$$\therefore \text{the back E.M.F} = e = V - \left(I - \frac{V}{R_{sh}}\right) R_a.$$

$$\therefore \text{ Electrical Efficiency} = \frac{\left\{V - \left(I - \frac{V}{R_{sh}}\right) R_a\right\}\left(I - \frac{V}{R_{sh}}\right)}{VI}.$$

$$\text{Mechanical Efficiency} = \frac{\left\{V - \left(I - \frac{V}{R_{sh}}\right) R_a\right\}\left(I - \frac{V}{R_{sh}}\right) - W}{e\left(I - \frac{V}{R_{sh}}\right)}.$$

Example 8. A shunt motor is supplied with 25 amps. of current from 220-volt mains. Calculate the electrical and mechanical efficiencies of the motor, having given the following data :—

$R_{sh} = 44$ ohms (hot),

$R_a = .14$ ohm (hot),

$W = 700$ watts at given voltage and speed.

Solution:—

Current in shunt coil $= \frac{V}{R_{sh}} = 5$ amps.

Current in armature $= I - \frac{V}{R_{sh}} = 20$ amps.

$\therefore$ Armature loss $= 20 \times .14 = 2.8$ volts.

$\therefore$ Back e. m. f. $= e = V - \left(I - \frac{V}{R_{sh}}\right) R_a;$

$= (220 - 2.8) = 217.2$ volts.

$$\therefore \text{ Electrical Efficiency} = \frac{\left[V - \left(I - \frac{V}{R_{sh}}\right) R_a\right]\left[I - \frac{V}{R_{sh}}\right]}{VI},$$

$$= \frac{\left[220 - \left(25 - \frac{220}{44}\right) \times .14\right]\left[.25 - \frac{220}{44}\right]}{220 \times 25},$$

$$=\frac{(220-2.8)\times 20}{5500}=\frac{4344}{44}=.79$$

Mechanical Efficiency

$$=\frac{\left[V-\left(I-\frac{V}{R_{sh}}\right)R_a\right]\left[I-\frac{V}{R_{sh}}\right]-W}{e\left(I-\frac{V}{R_{sh}}\right)}.$$

$$=\frac{4344-700}{4344}=.838.$$

122. (3) Long Shunt Compoud Motor :—

Shunt current $=\frac{V}{R_{sh}}$.

Armature current $=I-\frac{V}{R_{sh}}$.

$$e=V-\left(I-\frac{V}{R_{sh}}\right)R_s-\left(I-\frac{V}{R_{sh}}\right)R_a.$$

∴ The Electrical Efficiency

$$=\frac{\left[V-\left\{\left(I-\frac{V}{R_{sh}}\right)(R_s+R_a)\right\}\right]\left(I-\frac{V}{R_{sh}}\right)}{VI}.$$

Mechanical Efficiency

$$=\frac{\left[V-\left\{\left(I-\frac{V}{R_{sh}}\right)(R_s+R_a)\right\}\right]\left(I-\frac{V}{R_{sh}}\right)-W}{e\left(I-\frac{V}{R_{sh}}\right)}.$$

Example 9. A long-shunt compound motor takes a current of 25 amps. from 220-volt mains. Calculate the electrical and mechanical efficiencies of the motor, having given :—

$R_{sh} = 55$ ohms (hot),

$R_s = .078$ ohm (hot),

$R_a = .09$ ohm (hot),

$W = 700$ watts at given voltage and speed.

Solution:—

$$\text{Shunt current} = \frac{V}{R_{sh}} = \frac{220}{55} = 4 \text{ amps.}$$

$$\text{Armature current} = I - \frac{V}{R_{sh}} = 25 - 4 = 21 \text{ amps.}$$

$$\therefore e = V - \left(I - \frac{V}{R_{sh}}\right)R_s - \left(I - \frac{V}{R_{sh}}\right)R_a$$

$$= 220 - 21 \times .078 - 21 \times .09 = 216.5 \text{ volts.}$$

$\therefore$ Electrical Efficiency

$$= \frac{\left[V - \left\{\left(I - \frac{V}{R_{sh}}\right)\left(R_a + R_s\right)\right\}\right]\left[I - \frac{V}{R_{sh}}\right]}{VI}.$$

$$= \frac{\left[220 - \left\{\left(25 - \frac{220}{55}\right)\left(.078 \times .09\right)\right\}\right]\left[25 - \frac{220}{55}\right]}{220 \times 25}.$$

$$= \frac{[220 - 21 \times .168] \times 21}{5500} = \frac{4545.87}{5500} = .827.$$

$$\text{Mechanical Efficiency} = \frac{4545.87 - 700}{4545.87} = .846.$$

123. (4) Short Shunt Compound Motor.—

Shunt current $= \dfrac{V - IR_s}{R_{sh}}$.

Armature current $= I - \dfrac{V - IR_s}{R_{sh}}$.

Loss of volts in the armature $= \left(I - \dfrac{V - IR_s}{R_{sh}}\right)R_a$.

Loss of volts in series field winding $= IR_s$.

The back E. M. F. $= e = V - IR_s - \left(I - \dfrac{V - IR_s}{R_{sh}}\right)R_a$.

$\therefore$ Electrical Efficiency

$$= \frac{\left\{V - IR_s - R_a\left(I - \dfrac{V - IR_s}{R_{sh}}\right)\right\}\left(I - \dfrac{V - IR_s}{R_{sh}}\right)}{VI}.$$

Mechanical Efficiency

$$= \frac{\left\{V - IR_s - R_a\left(I - \dfrac{V - IR_s}{R_{sh}}\right)\right\}\left(I - \dfrac{V - IR_s}{R_{sh}}\right) - W}{e\left(I - \dfrac{V - R_s I}{R_{sh}}\right)}.$$

Example 10. A short shunt compound motor is supplied with a current of 25 amperes from 220-volt mains. Calculate the electrical and mechanical efficiencies of the motor, having given :—

$R_{sh} = 55$ ohm (hot),

$R_s = .078$ ohm (hot),

$R_a = .09$ ohm (hot),

$W = 700$ watts at given voltage and speed.

Solution:—

Shunt current $= \frac{V - IR_s}{R_{sh}} = \frac{22 - 025 \times .078}{55} = 3.96$ amps.

Armature current $= I - \frac{V—IR_s}{R_{sh}} = 25 - 3.96 = 21.04$ amps.

Loss of volts in series winding $= IR_s = 25 \times .078,$
$= 1.95$ volts.

Loss of volts in armature $= \left(I - \frac{V - IR_s}{R_{sh}}\right)R_a,$

$= 21.04 \times .09,$
$= 1.89$ volts.

$$e = V - IR_s—\left(I— \frac{V—IR_s}{R_{sh}}\right) R_a = 220—1.95—1.89,$$
$$= 216.16 \text{ volts.}$$

∴ Electrical Efficiency

$$= \frac{\left[V - IR_s - R_a\left(I - \frac{V—IR_s}{R_{sh}}\right)\right]\left[I - \frac{V - IR_s}{R_{sh}}\right]}{VI},$$

$$= \frac{\left(220 - (25 \times .078) - .09\left(25 - \frac{220 - 25 \times .078}{55}\right)\right)}{220 \times 25}$$

$$\times \left(25 - \frac{220 - 25 \times .078}{55}\right).$$

$$= \frac{(220 - 1.95 - 1.89) \times 21.04}{5500} = \frac{4548}{5500} = .827.$$

Mechanical Efficiency $= \frac{4548 - 700}{4548} = .846.$

Example 11. A series motor has 240 wires all round with a resistance of 0.08 ohm, and 22 million lines through it. The magnet winding is 0.12 ohm, and the machine is to give 100 B. H. P. when taking a current of 150 amperes. If n be the revolutions per second, and the mechanical friction loss is 100n, the hysteresis loss 80n, and the eddy current loss is $8n^2$, find the E. M. F., the speed, the mechanical and electrical efficiencies. (Joyce).

Solution:—

$Z=240$; $\phi=22\times10^6$; $R_a=.08$ ohm; $R_s=.12$ ohm;

B.H.P.$=100$; $I=150$; Speed$=$n rev. per second.

Mechanical friction loss$=100n$. Hysteresis loss$=80n$.

Eddy current loss$=3\,n^2$.

Armature copper loss$=I^2\,R_a=150^2\times.08$,

$=1800$ watts.

Field loss $I^2\,R_s=(150)^2\times.12=2700$ watts.

$\therefore$ Total loss$=(3n^2+180n+4500)$ watts.

$e=\phi\,Z\,n\times10^{-8}=22\times10^6\times240\times10^{-8}\times n$,

$=52.8n$ volts.

$IR_a=150\times.08=12$ volts.

If $E=$applied E.M.F.,

$E=e+IR_a=(52.8n+30)$ volts.

Power supplied to the motor

$=(52.8n+30)\;150$

$=(7920n+4500)$ watts.

And since total losses$=(3n^2+180n+4500)$ watts.

Useful Power$=7920n+4500-3n^2-180n-4500$,

$=100$ B.H.P.$=746\times100$ watts.

$\therefore \quad 7740n - 3n^2 = 74600,$

or $\quad 3n^2 - 7740n + 74600 = 0.$

$$\therefore \quad n = \frac{7740 \pm \sqrt{7740^2 - 12 \times 74600}}{6},$$

$$= \frac{3870 \pm 3840.97}{6} = 9.67 \text{ revolutions per second.}$$

taking the negative sign. as the + gives an impossible answer.

$$\therefore \text{ Applied voltage} = 52.8n + 30$$
$$= 50.8 \times 9.67 + 30,$$
$$= 510.576 + 30,$$
$$= 540.576 \text{ volts.}$$

$$\text{Power supplied} = 540 \times 150,$$
$$= 81000 \text{ watts.}$$

$$\text{Electrical Efficiency} = \frac{510.576}{540.576} = 94.4\%$$

Mechanical Efficiency =

$$\frac{510.576 \times 150 - \left\{ 180 \times 9.67 + 3(9.67)^2 \right\}}{540.576 \times 150} = 92\%$$

Example 12. A mechanical contrivance requires 100 H. P. and wastes 15 H. P. in friction. If run by a series motor having armature resistance .06 ohm, magnet coil .08 ohm, 360 wires all round the armature and 18 million lines through it, $1\frac{1}{4}$ H. P. lost in friction, 600 watts in iron, and supplied with a P. D. of 700 volts, find the speed, the current taken, the electrical and

mechanical efficiencies of the motor, and the plant efficiency. (Joyce).

Solution :—

$E_b = 18 \times 10^6 \times 360 \times n \times 10^{-8} = 64.8n.$

$E = E_b + IR = 64.8n + .14\ I = 700.$

$$\therefore \quad n = \frac{700 - .14\ I}{64.8}$$

Power supplied to the motor $= (64.8n + .14 \times I)\ I.$

Since total loss $= \frac{5}{4} \times 746 + 600 + I^2\ .14, = 3265.2$ watts.

$\therefore$ useful power $= 64.8n\ I + .14\ I^2 - 932.5 - 600 - .14\ I^2$

$= 64.8\ n\ I - 1532.5 = 74600$ watts.

$$\therefore \quad 64.8\ I.\frac{(700 - .14\ I)}{64.8} - 76132.5 = 0.$$

or $.14\ I^2 - 700\ I - 76132.5 = 0$

or $I^2 - 5000\ I + 543804 = 0.$

$$\therefore \quad I = \frac{5000 \pm \sqrt{25000000 - 4 \times 543804}}{2},$$

$$= \frac{5000 \pm \sqrt{22824784}}{2},$$

$$= \frac{5000 \pm 4777.5}{2} = \frac{222.5}{2} = 111.25 \text{ amperes.}$$

$$\therefore \quad n = \frac{700 - .14 \times 111.25}{64.8} = \frac{700 - 15.575}{64.8},$$

$$= \frac{684.425}{64.8} = 10.56 \text{ revolutions per second.}$$

$E_b = 64.8 \times 10.56 = 684.288$

$\therefore$ Electrical Efficiency $= \dfrac{684.288}{700} = 97.7\ \%$

Mechanical Efficiency $= \dfrac{700 \times 111.25 - 3265.2}{684.288 \times 111.25}$

$= \dfrac{77875 - 3265.2}{76107} = 98\ \%$

Plant Efficiency $= \dfrac{85 \times 746}{77875} = 81.5\ \%$

Example 13. A 20-h.p. 220-volt, 950 r.p.m. shunt-motor has an efficiency of 90 per cent., an armature resistance of .06 ohm, and a shunt field current of 2 amperes. If the speed of the motor is reduced to 475 r. p. m. by inserting a resistance in the armature circuit, the torque of the load remaining constant, find (1) the motor output, (2) the armature current, (3) the external resistance, and (4) the overall efficiency.

Solution :—

At normal load :

The motor output $= 20$ h. p.

The motor input $= \frac{20}{0.9} = 22.22$ h.p. $= 16576$ watts.

The total current $= \frac{16576}{220} = 75$ amps.

The shunt current $= 2$ amps.

The armature current $= 75 - 2 = 73$ amps.

The torque $= \dfrac{20 \times 33000}{2 \times 3.1416 \times 950} = 110.5$ lb. at 1 ft. radius

The back e. m. f. $= EI_a - R_a = 220 - (73 \times .06)$,
$= 215.6$ volts.

At half speed:

(1) The horse-power output $= \dfrac{\text{torque} \times 2\pi \times \text{r.p-m.}}{33,000}$,

and since the torque remains constant, the output is proportional to the speed, and is therefore equal to 10 h. p.

(2) The torque $= K. \ \phi. \ I_a$, and since the torque is constant, and so also is the excitation, therefore the armature current I_a is the same as at full-speed, and is equal to 73 amps.

(3) The back E. M. F., e, in the armature $=$ constant $\times \phi \times$ r. p. m., and since ϕ is constant, e is proportional to the speed, and is therefore equal to $.5 \times 215.6 = 107.1$ volts.

The voltage applied to the motor $= e + I_a \ R_a = 107.8 + (73 \times .06) = 112.18$ volts.

The voltage drop across the external resistance $= 220 - 112.18 = 107.82$ volts, and the current in this resistance $= 73$ amps.

$\therefore$ the external resistance $= \dfrac{107.82}{73} = 1.477$ ohms.

(4) The loss in the resistance $= 107.82 \times 73$,
$= 7870 .86$ watts.

The total input $= 220 \times (73 + 2) = 16500$ watts.

The motor output $= 10$ h.p. $= \dfrac{10 \times 746}{1000} = 7.46$ Kw.

$\therefore$ The overall efficiency $= \dfrac{7.46}{16.5} = 45$ per cent.

124 For Maximum Efficiency of a Motor, the Copper Loss is Equal to the Stray Loss.— This is proved as follows:

If W=sum of the constant losses, then

$$\text{the efficiency, } \eta = \frac{VI-(W+I^2 R_a)}{VI},$$

$$=1-\frac{W+I^2 R_a}{VI},$$

$$=1-\frac{W}{VI}-\frac{I^2 R_a}{VI},$$

$$\therefore \quad \frac{d\eta}{dI}=\frac{W}{VI^2}-\frac{R_a}{V}=0, \text{ for maximum efficiency.}$$

$$\therefore \quad \frac{W}{VI^2}=\frac{R_a}{V},$$

$$\text{or, } W=I^2 R_a.$$

That is, the constant losses = variable losses (copper losses.)

125. Characteristic Curves of Motors.— These include curves of speed, efficiency, current input and torque, in terms of the horse-power output of the machine.

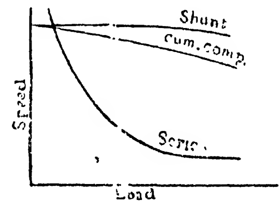
The mechanical characteristic curve connects the speed at constant voltage and torque (Fig. 5.01).

The magnetic characteristic of a machine is the same whether it is running as a generator or motor.

125 A Speed Characteristics :—These are curves between speed and load, the load being the independent variable. For shunt motors the relation between the armature reaction and the IR drop determines whether the motor will speed up or slow down with an increase of load. If the magnetization of the field extends above the knee of the saturation curve the motor will slow down, while below the knee the motor will speed up. A degree of magnetization may be obtained which will result in practically constant speed.

The cumulation compound motor slows down with increase of load, because the effect of the series turn is to increase the field.

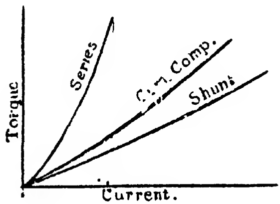
The series motor speed is goverened almost entirely by its field. At light loads the speed becomes high and the operation of the motor is unstable.



Speed Characteristic Curves
Fig. 5.011.

125 B Torque Characteristics:—From the equation, Torque curves may be constructed showing the variation of torque with load current.

The flux ϕ is first to be determined in each case. In the case of Shunt Motor ϕ is nearly constant. Thus torque is nearly proportional to the current. With Series Motor ϕ increases with i and hence torque increases as the square of the current, excepting when the field becoming saturated, the increase of current does not produce much increase of flux.



Torque Characteristics.
Fig. 5.012.

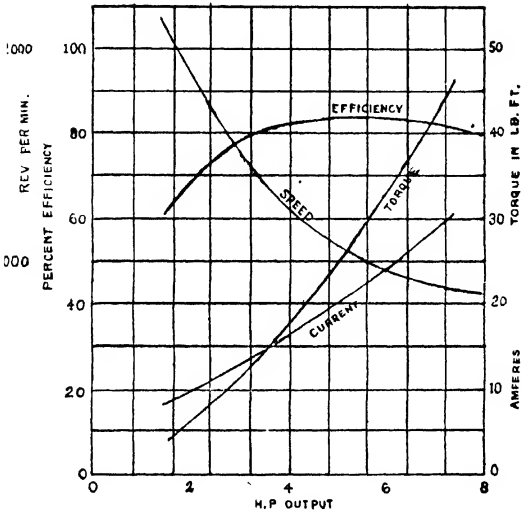


Fig. 5·02
Characteristic Curves of Series Motors.

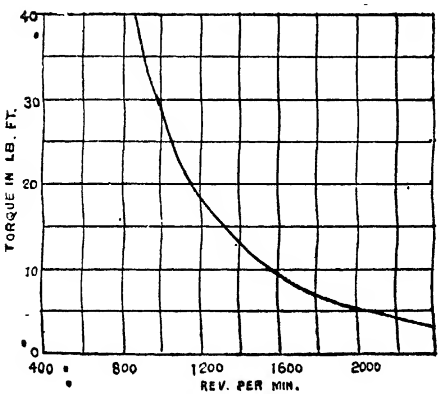


Fig. 5·01
Torque and Speed Curve of a Series Motor.

MOTOR STARTERS.

126. Use of a Starter.—

When a motor is started the back E. M. F. is not established at once, and the current I which flows through the armature is $\frac{E_g}{R_a}$, where E_g is the impressed voltage, and R_a the resistance of the armature. As the counter E.M.F., e, is developed, the current diminishes in value, as I′ becomes equal to $\frac{E_g - e}{R_a}$. Therefore,

$$I \text{ is always } > I'.$$

When the motor is running at full-load, the counter-E. M. F. is very nearly equal to the impressed voltage. The armature conductor, which is intended to carry this rated normal current, when the back E.M.F. is developed, cannot without over-heating, carry the abnormally high current which would flow through the armature conductor when the motor is first started, owing to the absence of the back E. M. F.

Thus, a resistance must always be inserted in the circuit to cause a drop of potential before the current enters the armature, and thus allows $I = \frac{EI - R_s}{R_a}$ to enter the armature circuit, where R_s is the resistance of the starter. As the back E. M. F. is being developed

R., which is to be made in several steps, is gradually entirely cut out of the circuit, so that when the B.E.M.F. is fully established, the normal current from the mains goes straight to the armature. The resistance of the starting rheostat must not be cut too rapidly. If the resistance is cut out more rapidly than the armature can speed up, a sufficient counter-electromotive force will not be generated to properly oppose the flow of the current, and the motor will be overloaded. In starting a direct current motor, close the line switch and move the operating arm of the rheostat step by step over the contacts, waiting a few seconds on each contact for the motor speed to accelerate. If this process is performed too quickly, the motor may be injured by excessive current, if too slowly, the rheostat may be injured.

127. To Start the Motor.—The following directions are to be observed in starting a motor:—

(1) Close the main switch. According to the method of connection this will either establish the current through the field, or will merely bring the line voltage up to the starting box (Fig. 5.03).

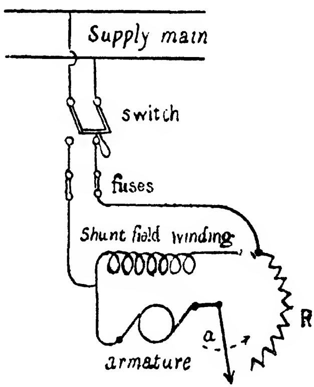


Fig. 5.03.

(2) Move the rheostat lever to the first contact, and hold it there for one or two seconds.

(3) Move the lever to the second stud, and hold it there for about one second, and so on, until all the studs have been passed over and the resistance is short circuited. The lever should then be firmly held by the retaining magnet. The entire operation should not consume more than 15 seconds for motors of 5 H. P. and lesser output, to 30 seconds for greater output.

128. If the Motor does not start when the lever is on the third stud, open the main switch, and look for the trouble. It may be due to one or other of the following causes :—

1. Overload.—The current is very excessive, and the safety fuses blow, or the circuit breaker opens.

   If there is no fuse or circuit breaker, the armature will burn out.

   Remedy.—Open the switch and reduce the load.

2. Very excessive friction due to shaft bearings or other parts being jammed.

3. An open circuit somewhere.

4. A short circuit somewhere.

5. Wrong connections or complete short circuit of field, armature, switch, etc., armature touching 'pole pieces'.

6. Field too weak.— Occurs with adjustable speed motors.

No cause of motor trouble is more common than simple overloading. Always at the time of installation, and occasionally thereafter, an ammeter should be connected into the motor circuit, and its reading compared with the rated current of the motor. Motors are designed to do their work with but little attention. They frequently get none at all: belt, commutator, and bearings are neglected; additional machinery is put into the shop and operated from the original motor; and so on—with the inevitable result of overload.

129. To Stop the Motor.—Open the switch and leave the starter arm to set itself automatically at the starting position. Never force the automatic arm of any automatic starting rheostat back to off position.

130. To Reverse the Motor.—In all cases consider the interpole circuit as a part of the armature circuit.

(1) Series Motor.—(a) Reverse the direction of the field current; or (b) reverse the armature current.

(2) Shunt motor.—(a) Reverse the field current; or (b) reverse the armature current.

(3) Compound Motor.—(a) Reverse both the series and shunt field currents; or (b) reverse the armature current alone.

131. Starting Resistance.—IN A SERIES MOTOR THE RESISTANCE IS CONNECTED IN SERIES with the

armature and the field windings, and, the resistance is gradually cut out as the speed increases.

THE STARTING RHEOSTATS USED WITH SERIES MOTORS are usually of the drum type. Low voltage releases are not essential on series motors, as an attendant is usually near to open the circuit and return the handle to the "off" position in case the current supply fails. Circuit-breakers, which should be connected in the supply leads of each motor, may be depended on to protect the armature from excessive current.

THE SERIES-PARALLEL SYSTEM OF CONTROL is used on street cars, which have two or more motors, start and stop frequently, and are often required to run at low speeds. Its advantage is in the reduction of losses in the starting rheostat, and a correspondingly increased operating efficiency.

IN A SHUNT MOTOR THE RESISTANCE IS IN SERIES with the armature only, but the field winding is connected with the 1st contact stud, and the field circuit is always closed through the armature circuit.

It will be noticed that as the starting resistance is cut out of the armature circuit, it is being added to the field circuit, so that when the entire resistance is cut out from the armature circuit, the field coil current has to pass through the whole risistance of the starter. See Fig. 5·031.

132. Liquid Starting Resistance for shunt motors are more suitable than wire starter. There is a complete absence of trouble with contacts, which frequently

have to be renewed with the ordinary wire starters. The accelerating current is kept more nearly constant, and further they are infinitely more flexible, in that, by adjustment of the strength of the liquid used they can be adapted for a variety of works. The method is very simple, and is always available everywhere. In out of the way places, very often, only common salt or washing soda, an iron drum, and a piece of iron may be used for completing the resistance. It works very well and gives no trouble whatsover. The resistance may be gradually reduced, and finally short-circuted by two contacts on the plates and tank respectively, or by simply touching the drum with the piece of iron.

133. No-volt and Overload Release.—Suppose a motor is suddenly stopped owing to the failure of the supply current. The connections to the motor, however, remain intact, but the resistances are cut out in the starter. Consequently, when the current in the mains is again established, a heavy current flows through the armature which may thus burn out. Hence, a device must be resorted to, so that when the motor stops, the starting rheostat must come back automatically to the starting position. Such a device is called the DEAD-LINE RELEASE or NO-VOLTAGE RELEASE.

Again, the load of the motor may be heavily increased, so that the motor takes excessive current from the mains. Owing to the heavy current, however, the armature may be over-heated and finally burnt out. So the motor, while working, must be protected from damage by a

device called an OVER-LOAD RELEASE, by which the arm of the starting rheostat is automatically thrown to the starting position when such an undesirable heavy current would pass through the armature (Fig. 5·04).

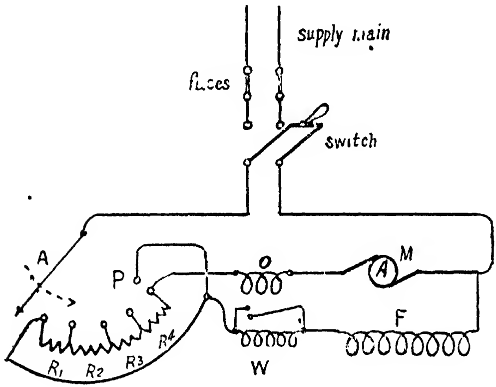


Fig. 5·04.

The Electrical Connections:—The motor armature is represented by M (Fig. 5·04), and the shunt field winding by F. Close the switch to start the motor, and the rheostat arm A is slowly moved in the direction of the dotted arrow until it reaches the point P. As soon as the arm A is moved to the first stud, the shunt field winding is connected directly to the mains, and the armature to the mains through the resistances R_1, R_2, R_3. As the arm is moved the resistances R_1, R_2 are successively cut out of the armature circuit.

In the position shown in the diagram, the resistances are left in series with the field winding as they are cut out of the armature circuit, and the resistances R_1, R_2, etc. are cut off the field circuit when the arm A touches the contact point P, R_4. When in starting the motor, the rheostat arm is pushed to the right, it is held in the running position by an electromagnet, in which the winding W is in series with the field winding F of the motor. When the lines become dead no more current passes through W and F, the electromagnet releases the rheostat arm, and the arm is pulled back to the starting position by the contracting force of a spring, which always tends to bring the arm back to its initial position.

The winding of the electromagnet O, which is instrumental in over-load release, is in series with the motor armature. The armature of this electromagnet is also held back by a spring. The tension of this spring is so adjusted that the lever may be moved by a prescribed value of the current through O. When this value of the current is reached, the movement of the armature of O actuates a small switch which short circuits the winding W. Thus the electromagnet loses its magnetism, and releases the arm A, which comes back to its initial position.

A compound motor is started by a similar starter. The series winding and armature are connected as the armature is connected in the shunt motor.

DESIGN OF MOTOR STARTERS

134. Graphical method for Designing Series Motor Starter.—It is based on the equal fluctuation of current on each step of the starting rheostat.

Let E = the fixed line voltage,

I_m = the maximum allowable current during starting.

Construct the rectangle NI_mQP as in Fig. 5·05, in which (NI_m) is the maximum allowable current during starting, and $(QR_1) = (E \div I_m)$ is the entire resistance of the circuit, when the contact arm of the starter is on the 1st operating contact. Lay off (NI_0) equal to the minimum current during starting. Determine from the magnetization curve of the machine, at some definite speed, the armature voltage E_0 for the current I_0, and the armature voltage E_m for the current I_m at this speed.

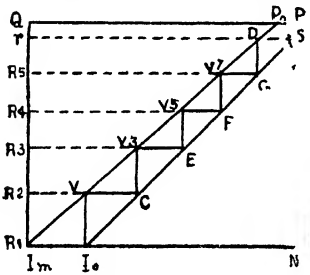


Fig. 5.05

Lay off $(QP_0) = (QP) \times \dfrac{E_0}{E_m}$, and draw the two straight lines ($I_m P_0$) and ($I_0 P$). Lay off (Qr) equal to the resistance of the motor between terminals, and draw (rs) parallel to (QP), and find the point D where (rs) intersects ($I_m P_0$).

Then draw the zig-zag line I_0, V_1, CV_3, FV_5,...... If this line does not meet the point D, then the ratio $I_0 - I_m$ and (QP_0) must be altered until the new zig-zag line meets the new point D. Then extend the horizontal line CV, EV_3, ... until they cut the vertical axis, at R_2, R_3, ... The resistances of the successive steps (resistances between successive contacts) are then equal to the distances (R_1 R_2) (R_2 R_3) ......

For machines working on a straight magnetization curve, the steps for the starter would be equal, as the line (I_0P) would be parallel to the line (I_m P_0).

135. Graphical method for Shunt motor starters:—

The above method may also be applied to the determination of the steps of a shunt motor in which case the point P coincide with P_0.

Design of Shunt Motor Starters.—

136. Analytical Method.—To determine the necessary number of steps, and the resistance in each step of a motor starter to secure equal current increments, and to prevent these exceeding the increase of starting-current above normal-load current, we proceed as follows:

Let n = number of steps in the resistance;

r_1 = resistance in the armature circuit on the first stud, including the armature resistance;

r_2 = resistance in armature circuit on the second stud,

r_3 = resistance in armature circuit on the third stud,

r_n = resistance in the armature circuit on the n^{th} stud,

$r_n + 1$ = resistance in armature circuit on the $(n+1)^{th}$ stud, i. e., the armature resistance $= R_a$.

I'_1 = maximum starting current,

I = normal full-load current,

V = line voltage,

e = back E. M. F.

Then,

$$r_1 = \frac{V}{I_1}.$$

The full-load current I is less than the current I_1 which is necessary to start the motor with its load, for at the start the counter E. M. F. is not developed, and the current $I_1 = \frac{E}{r_1}$; so we do not allow a very heavy current which would flow if there were no suitable resistance inserted at the outset, as it is likely to burn out the armature.

When the switch arm is moved to the first stud, then, when the machine attains the speed corresponding to the voltage across the armature on this stud, a certain back E. M. F. is generated, and the current falls to normal value. The value of the resistance when the switch arm is moved from one stud to the next must be such that the ratio $\frac{I_1}{I}$ is constant during the whole time the motor is accelrating. Then

$$V - e = I \times r_1, \therefore r_1 = \frac{V - e}{I}.$$

On moving the switch arm to the next stud the current should rise to its previous maximum value.

Then,
$$r_2 = \frac{V - e}{I_1} = \frac{I \times r_1}{I_1}.$$

$$\therefore \quad \frac{r_1}{r_2} = \frac{I_1}{I}$$

In like manner the resistances on successive steps satisfy the conditions

$$\frac{r_1}{r_2} = \frac{r_2}{r_3} = \frac{r_3}{r_4} = \frac{r_n}{r_{n+1}} = \frac{I_1}{I},$$

and
$$\therefore \frac{r_1}{r_{n+1}} = \left(\frac{I_1}{I}\right)^n.$$

Thus, $n \log_{10} \frac{I_1}{I} = \log_{10}\left(\frac{r_1}{r_{n+1}}\right) = \log_{10} r_1 - \log_{10} r_{n+1}$

$$\therefore n = \frac{\log_{10} r_1 - \log_{10} r_{n+1}}{\log_{10} I_1 - \log_{10} I}, \text{ where } r_{n+1} = R_a;$$

$$= \frac{\log_{10} r_1 - \log_{10} R_a}{\log_{10} I_1 - \log_{10} I}, \quad \dots\dots\dots\dots\dots\dots(1).$$

$$= \frac{\log_{10}\left(\frac{r_1}{R_a}\right)}{\log_{10}\left(\frac{I_1}{I}\right)} = \frac{\log_{10}\left(\frac{V}{I^1 R_a}\right)}{\log_{10}\left(\frac{I_1}{I}\right)} \quad \dots\dots\dots\dots(2).$$

Thus, if there is no alteration in the current in moving from one stud to the other, i.e., when $I = I_1$, the number of steps into which the starting resistance must be divided $= \infty$, and the greater the number of steps

the smaller will be the momentary increase in current in passing from one stud to another.

I_1 is usually taken as 1.5 I, and,

therefore, $r_1 = \frac{V}{1.5\ I}$.

If we are given the applied voltage and full-load current, we can obtain r_1. If from this, we deduct the armature resistance, we get the total resistance of the starter.

Also, $\frac{r_1}{r_2} = \frac{I_1}{I}$; $\therefore\ r_2 = \frac{r_1\ I}{I_1}$.

From r_1 subtract the resistance r_2, which has been found above, and we have the resistance between the studs 1 and 2. In a similar manner the resistance between the other studs are determined.

In starters for motors taking from 30 to 100 amperes, the starting current is reached in the second stud, and motors taking larger currents should start when the switch comes on to the third stud. The no-load starting current does not generally exceed by more than 50 per cent. of the full-load current.

Example 14. A 20-B. H. P., 220-volt shunt motor takes a current of 76 amperes for running normally at full-load. If the maximum starting current has not to exceed 114 amperes, determine the number of steps required for the starting resistance, and the resistance of each step. The armature resistance equals .18 ohm.

Solution:—

Since the starting current is 114 amperes, it should attain this value in three steps, i. e., the motor should start on the third contact stud.

On the first contact the current allowed to pass through the motor = 38 amperes, so that, the total resistance in the armature circuit

on the first contact $= \frac{220}{38} = 5.8$ ohms.

Similarly, the total resistance in the armature circuit on the second contact $= \frac{220}{76} = 2.9$ ohms,

and, that on the third contact $= \frac{220}{114} = 1.93$ ohm.

But the armature resistance itself $= .18$ ohm

$\therefore$ resistance between contacts 1 and $2 = 5.8 - 2.9$
$= 2.9$ ohms;

and, resistance between contacts 2 and $3 = 2.9 - 1.93$
$= .97$ ohm.

Following the method of Art. 136, we now find the number of steps and the resistance per step, taking the starting current for the first of these steps to be 114 amperes. Thus—

$$r_1 = \frac{V}{I_1} = \frac{220}{114} = 1.93 \text{ ohm.}$$

$$r_{n+1} = R_a = .18 \text{ ohm.}$$

$$\frac{I_1}{I} = \frac{114}{76} = 1.5.$$

$$\therefore \quad n = \frac{\log r_1 - \log r_{n+1}}{\log I_1 - \log I} = \frac{\log 17 - \log \cdot 18}{\log 114 - \log 76},$$

$$= \frac{.27875 - 1.25527}{2.0569 - 1.88081} = \frac{1.0235}{.1761},$$

$$= 5.8 \text{ or } 6 \text{ say.}$$

Hence, the starting resistance will be divided into 6 steps.

$$r_2 = r_1 \times \frac{I}{I_1} = 1.9 \times \frac{1}{1.5} = 1.27 \text{ ohm.}$$

$$r_3 = r_2 \times \frac{I}{I_1} = 1.27 \times \frac{1}{1.5} = .85 \text{ ohm.}$$

$$r_4 = r_3 \times \frac{I}{I_1} = .85 \times \frac{1}{1.5} = .57 \text{ ohm.}$$

$$r_5 = r_4 \times \frac{I}{I_1} = .57 \times \frac{1}{1.5} = .38 \text{ ohm.}$$

$$r_6 = r_5 \times \frac{I}{I_1} = .38 \times \frac{1}{1.5} = .25 \text{ ohm.}$$

$r_7 = R_a$........................$= \cdot 18$ ohm.

$\therefore$ resistance of the first step $= r_1 - r_2 = 1.9 - 1.27$,
$= .63$ ohm.

$\therefore$,, ,, second ,, $= r_2 - r_3 = 1.27 - .85$,
$= .42$ ohm.

and so on.

The values of the resistance of steps 1 to 6 are respectively .63, .42, .28, .19, .13, ,07. ohm.

Hence, the actual number of steps $= n + 2 = 6 + 2 = 8$, so that the number of contact studs $= 8 + 1 = 9$, and the values of 8 resistances are respectively

2.9, .97, .63, .42, .28, .19, .13, .07 ohms.

137. A Graphical Construction—for determining the number of steps and the resistance per step:—

Let $OB = I_1$ and $AO = I$. Draw perpendiculars at A and B, and make $BC = r_1$. Join C to O, intersecting the perpendicular through A in D. Draw DE horizontally. Then $BE = r_2$. Similarly, join EO, intersecting AD in F, and draw FG horizontally. Then $BG = r_3$, and so on. (Fig. 5.06).

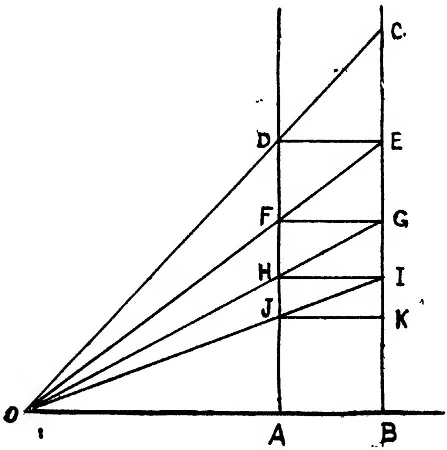


Fig. 5.06

*138. **The Alloys Used for Rheostats** and data relating to them are given in the adjoining table.

Material	Specific Resistance in Microhms per cm. cube at 0° C.	Temperature Coefficient at 0° C.	Specific Gravity.	Specific Heat.
Nickelin ...	33	0.0003	9.0	0.08
Rheostan ...	52	0.00041	8.6	0.097
Platinoid ...	40	0.00031	8.6	0.098
Kruppin ...	84	0.00077	8.7	0.13
Constantan.	43	zero.	8.8	0.1
Eureka ...	47	0.000005	8.8	0.1
Iron (pure).	9	0.00625	7.9	0.104
Resista ...	75	—	—	0.1

*139. **To Determine the Size of Wire Required for a Starter**:–The time taken to start a motor is usually about 30 seconds, and so the loss of heat due to radiation can practically be neglected.

Let I = the average current during the starting period in amperes,

I_1 = the current at the first starting point,

I_2 = the current at the end,

R = the resistance of the starter in ohms,

d = the diameter of the wire in centimetres,

t = the time of starting in seconds,

ρ = the specific resistance of the alloy in ohms per centimetre cube,

θ = the permissible temperature rise in degrees centigrade,

* Modern Electrical Engineering Vol. II.

σ = the specific heat of the alloy,

l = the length of wire in centimetres.

Then, the rate of generation of heat in calories per second.

$$= 0.24\ I^2R = 0.24\ I^2\rho l \times \frac{4}{\pi d^2},$$

$$= l \times \frac{\pi}{4}\ d^2 \times \text{specific gravity} \times \sigma \times \frac{\theta}{t}.$$

$$\therefore\ d = \sqrt[4]{\frac{0.389 \times I_1{}^2 \times \rho \times t}{\text{Specific Gravity} \times \sigma \times \theta}};$$

where $$I_1 = \frac{I_1 + I_2}{2},$$

and θ the permissible temperature rise is about 150° C. For shunt rheostats the permissible temperature rise is about 100°C. Taking the emissivity of the wire at 0.0005 calorie per square centimetre of area of radiating surface per degree Centigrade difference of temperature between the wire and surrounding air, the necessary diameter of wire for the shunt rheostat, is given by $d = \sqrt[3]{2 \times I_m^2 \times \rho}$, where I_m = maximum shunt current.

(P. 204 vol. I)

140. Speed Control of a Series Motor.—To diminish the speed a resistance is inserted in series with the armature, and the field of a series motor. To increase the speed a diverter is used in parallel with the field coil of the motor.

141. Speed control of a Shunt Motor.

THE SPEED OF A SHUNT MOTOR can be varied within certain limits by :—

(1) Armature control : Changing the resistance of the armature circuit by connecting in series a variable resistance of sufficient capacity to allow the maximum current to pass through it.

It is effective, cheap and easily applicable but inefficient, a small change in load causes a large change in speed.

(2) Field control : Changing the resistance of the field circuit. This is done by the field rheostat. If the resistance of the field circuit is increased, the field current is decreased and the speed is increased. If the field resistance is decreased, the field current is increased and the speed is decreased.

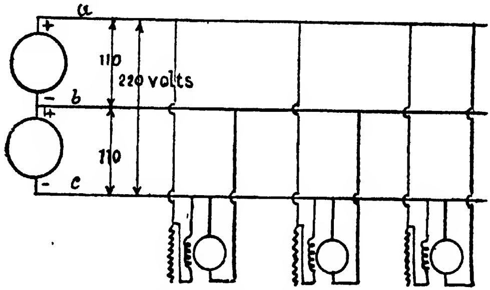
This method is cheap in the first cost and in the operation, but is limited in its operation. Both high and low speeds are unattainable.

(3) By changing the reluctance of the magnetic circuit, which is secured by changing the length of the air-gap between the pole and the armature.

The operation of such motors is satisfactory, but expensive in construction and somewhat complicated in operation.

(4) By changing the electromotive force applied to the armature terminals:—(a) When the change of the voltage is made in steps. (b) When the range of voltage is continuous.

(a) MULTIPLE VOLTAGE SYSTEM: VOLTAGE CHANGED IN STEPS. It requires several supply mains having different voltages. The three-wire system may be used (Fig. 5·07), the field coils are connected to the mains giving the maximum voltage, and thus the maximum flux is obtained. The armature is connected across the mains having the lower voltage. The speed may be increased by inserting resistance in the field coil circuit, and when the speed has been doubled



Multiple Voltage System.

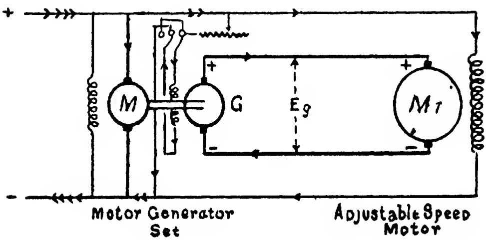
Fig. 5·07

the armature is connected across the mains giving the higher voltage, and all the resistance of the field coil circuit is cut out and the speed may be once more generally increased by reducing the field excitation by means of resistance.

This method is highly efficient.

(b) WARD-LEONARD SYSTEM: VOLTAGE RANGE CONTINUOUS.—Use a separate generator with (i) each adjustable speed motor, and to vary the excitation of

the generator so as to vary the voltage applied to the motor terminals use (ii) a constant speed motor which drives (iii) a generator. The generator supplies current to the armature of the variable speed motor, the field of which is joined to the constant potential circuit. For slow speeds the field excitation of G (Fig. 5.08) is reduced, to increase the speed, the field excitation of G is increased. To reverse the motor



Ward-Leonard System.

Fig 5·08

reverse the excitation of G. The whole control is handled by means of a small field circuit rheostat.

The efficiency of this system is the combined efficiency of the three machines, which is, therefore usually low.

Its initial cost is high and operating efficiency low, but continuous variation of speed is obtained over the widest possible range. So it is used only where such variation of speed is necessary as is usual in printing presses.

142. Speed Control of a Compound Motor.—To reduce the speed, a resistance in series with the armature is used.

To increase the speed, a resistance in series with the shunt turns, or a resistance in parallel with the series turns is to be used.

See that they run almost sparklessly with fixed position at all loads.

***143. Care of Motors.**—All motors should be regularly inspected and the following points noted :—

(1) Bearings filled with proper quantity of oil.

(2) Brushes securely held in proper non-sparking position.

(3) Brushes fit properly.

(4) Commutator smooth and clean.

(5) Commutator not worn in grooves.

(6) Air-gap true.

***144. Inspection and Erection.**—Inspect the following.

(1) Base bolted down.

(2) Bearings clean and filled with oil.

(3) Bearings lined up.

(4) Magnet frame bolted to base.

(5) Field coils secured in place.

(6) Field coils tested for open circuit, wrong connection, and polarity.

(7) Armature in place.

(8) Air gap adjusted.

* H. Pender.

(9) Measure resistance of armature and field.

(10) Measure insulation resistance.

(11) Brushes properly fitted and spaced, and pressure adjusted to about 1.5 to 2 pounds per brush.

(12) Commutator smooth and true.

(13) Substantial connection of field circuit.

(14 Field adjusted for correct direction of rotation.

(15) It must be protected from moisture.

Temperature rise of Motor.—

145. Heating and Cooling of Bodies.—

The energy, lost in a machine, is converted into heat. The rise of temperature will depend upon (1) the capacity of the materials, (2) the facility for radiation or dissipation of heat. The temperature will cease to rise when the rate of generation of heat is equal to the rate of dissipation. The temperature will be constant when the heat generated is equal to the heat lost per sec. (Vol. I. P. 203).

Let Q = heat generated per second,

S = specific heat of substance,

M = mass of the body,

A = radiating surface,

a = coefficient of cooling, or the emissivity of the surface,

θ = temperature of body in degrees Centigrade,

θ_1 = temperature of surrounding medium in degrees Centigrade,

T_e = excess temperature at the given moment after t seconds of loading $=\theta-\theta_1$,

T_l = the limiting temperature rise.

I. **Heating of the Body**.—In a time dt the temperature rises by dT_e degrees, the heat liberated amounts to Qdt, and the body absorbs SM dT_e. The heat dissipated $= A\alpha\, T_e\, dt$ = rise of temperature after t seconds of loading.

$$\therefore\ Qdt = SMd\, T_e + A\alpha\, T_e\, dt.$$

or
$$dt = \frac{SMd\, T_e}{Q - A\alpha\, T_e}$$

Assuming $\theta=\theta_1$ when $t=0$

$$\int_0^t dt = SM \int_{\theta_1}^{\theta} \frac{dT_e}{Q - A\alpha\, T_e}.$$

$$\therefore\ T_e = \frac{Q}{\alpha A}\left(1 - \epsilon^{-\frac{\alpha At}{MS}}\right)\ldots\ldots\ldots\ldots(1)$$

which represents the equation to the heating curve.

When $t=\infty$

$$T_e \cdot t = \infty = \frac{Q}{\alpha A} = T_l\ldots\ldots\ldots\ldots(2)$$

This is the limiting temperature rise of the body.

If the temperature continued to rise, as at the first instant, the limiting temperature rise $T_l = \frac{Q}{\alpha A}$ would be reached in a time $T = \frac{S\,M}{\alpha\,A}$ seconds. The ratio is a constant for the body. It is the ratio of the heat stored to the rate of radiation.

T is called the TIME CONSTANT of the body. It is the time in which the temperature would rise to the final steady value, if no heat were given out to the surrounding atmosphere.

The heating equation can then be written

$$T_e = \frac{Q}{a\,A}\left(1 - \epsilon^{-\frac{t}{T}}\right) = T_f\left(1 - \epsilon^{-\frac{t}{T}}\right) \quad \ldots\ldots\ldots(3)$$

This is very similar to the formula for the rate of growth of current in an inductive circuit, when a P. D. is suddenly applied in which $\frac{L}{R}$ is called the time constant of the circuit.

Example 15.—The final temperature rise of a motor is 50°C. If after 1 hour from the start the temperature rises to 30°C, what is its temperature time constant ?

Solution:—

By eqn. (3), we have—

$$30 = 50\,(1 - e^{-\frac{1}{T}}).$$

$$\therefore \quad e^{-\frac{1}{T}} = 1 - \frac{30}{50} = 1 - 0.6 = 0.4.$$

$$\therefore \quad e^{\frac{1}{T}} = \frac{1}{0.4} = 2.5.$$

$$\therefore \quad \frac{1}{T} = \log_e (2.5) = .916$$

$$\therefore \quad T = 1.09 \text{ hours.}$$

II. Cooling of the body.—In this case no heat is developed. Hence, $Q=0$.

$$\therefore\ 0 = SM\, d\, T_e + A\alpha\, T_e\, dt.$$

If the temperature is ϕ degrees when $t=0$,

$$\int_0^t dt = -SM \int_\phi^\theta \frac{dT_e}{A\alpha\, T_e}.$$

$$\therefore\ T_e = \theta - \theta_1 = (\phi - \theta_1)\epsilon^{-\frac{\alpha A}{SM}t},$$

which is the equation of the cooling curve.

$$\text{If } \phi - \theta_1 = (\theta - \theta_1)t = \infty = \frac{Q}{\alpha A},$$

that is, if the temperature at the beginning of cooling is equal to the limiting temperature at the end of the heating period, the equation of the cooling curve is the same as the variable part of the heating equation, but with the sign changed. Hence, the heating and cooling curves are of the same logarithmic shape, but one is turned upside down with respect to the other.

146. Application to Over-Loads.—Let the maximum permissible temperature be T_f. Then, in the case of continuous rating of the machine it is meant that the load can be applied continuously with the maximum rise T_f, and that if a greater load is applied T_f is reached sooner.

If T_f be the final excess temperature which would be reached if the heat supply Q were maintained long enough we have, $Q = T_f\, A\alpha$

$$T_f\, A\alpha\, dt - T_e\, A\alpha\, dt = SM\; d\, T_e$$

$$\text{or } dt\ \ (T_f - T_e) = \frac{SM\ d\ T_e}{A\ \alpha} = T\ d\ T_e$$

$$\text{or } dt = T.\frac{d\ T_e}{T_f - T_e}$$

and intigrating both sides

$$t = -T \log (T_f - T_e) + \text{a constant}$$

when $t=0$ we know that $T_e=0$, so that the constant must be $T \log T_f$

$$t = -T \log (T_f - T_e) + T \log T_f$$

$$= T \log \frac{T_f}{T_f - T_e}$$

If the rate of heat production is increased m times we have $T_f = m\ T_e$

$$\therefore t\ m = T \log \frac{m\ T_e}{m\ T_e - T_e} = T \log \frac{m}{m-1}$$

If we put $t=T$ we have $\log \frac{T_f}{T_f - T_e} = 1$

or $T = 0.633\ T_f$. Hence the time constant is the time taken to reach 0.633 of the final temperature rise.

Example 16.—The temperature time constant of a motor is 2 hours. Calculate how much the heating rate may be increased if the load is applied for 30 minutes only.

Solution.—

$$.5 = 2 \log_e \frac{m}{m-1} = 4.606 \log_{10} \frac{m}{m-1},$$

$$\therefore \frac{m}{m-1} = \text{antilog } \cdot 1085 = 1.285$$

$$\therefore m = 4.51.$$

i. e., heat may be developed at 4.51 times the normal rate.

147. Intermittent Loads.—When the cooling time constant is known, we can find the rating under intermittent loads. If the machine is run before it has cooled down to the temperature of the air, a high temperature will be reached in the same time, though the temperature rise will be less. This cooling time constant is three to four times the heating time constant and if the machine is stopped, the time constant is further increased, for the ventilation, which always helps to bring down the temperature while the machine is running, is practically stopped.

Example 17.—The final temperature rise of a motor under normal full-load is 50°C. Its heating time constant is 2.5 hours, and cooling time constant 7.5 hours. A load which causes heat to be developed at double the normal rate is applied for 1 hour. The motor is then allowed to cool for 4 hours, and the same load is again applied for one hour. What will be the temperature rise at the end of the last hour ?

Solution:—

Since the heat is developed at twice the normal rate, we have, by eqn. (4)

$$t_1 = 2T_1 \left(1 - \epsilon^{-\frac{t}{T}}\right)$$

$$= 2 \times 50 \times \left(1 - 2.718^{-\frac{1}{2.5}}\right)$$

$$= 33^0 \text{ C.}$$

Hence, by eqn. (5), after 4 hours' cooling

$$t_1 = 33 \times e^{-\frac{4}{7.5}} = 33 \times .886 = 29.2^0 \text{ C.}$$

This temperature rise, reached under double load in a time t from the first start, is given by

$$29.2 = 2 \times 50 \left(1 - 2.718^{-\frac{t}{2.5}}\right) \quad \ldots \quad [\text{Eqn. (4).}]$$

$$\text{or, } 2{\cdot}718^{-\frac{t}{2.5}} = .708$$

$$\therefore t = .86 \text{ hour.}$$

Hence, when the load is again applied for 1 hour the temperature rise will be as if the load remained for 1.86 hours from the start. Thus, the final temperature rise,

$$t_1 = 2 \times 50 \left(1 - 2.718^{-\frac{1.86}{2.5}}\right) \quad \ldots \quad [\text{Eqn. (4).}]$$

$$= 1000 \times .525$$

$$= 52.5^0 \text{ C.}$$

Exercises.

1. Give the diagram of connection and approximate ohmic values of the resistance required for a C. C.

motor starter, 10 h. p., 220 volts. (A. M. I. E. E., 1914. Design E. M. and A., 2nd paper).

2. What is the purpose of a "no-voltage" release attached to a continuous current motor starting switch? Make a sketch of such a device and its connections, and explain briefly how it operates. (Grade II., C. G., 1913).

3. Explain and show by a diagram the relation of speed and torque to the current in series, shunt and compound continuous current motors. (Grade II., C. G. 1912).

4. A continuous-current shunt motor is supplied at constant voltage. What fixes the speed at which it will run, and upon what does the current it will take depend. (Grade II., C. G. 1914).

5. For the frequent stopping and starting, essential to urban and suburban tramway or railway service, what type of continuous-current motor is in general use and why? (A. M. I. E. E., Elec. Trac., 1st Paper, 1914).

6. If a motor of 10 h. p. has an efficiency of 85 per cent., and runs at full load for 12 hours continuously, how many Board of Trade units will it use? (Grade II., C. G., 1916)

7. How would you determine the efficiency of two exactly similar motors of large power, under full-load conditions, if the supply of power available was small? Describe the test with diagram of connections. (Grade II., C. G., 1912).

8. Show how you would test the efficiency of two continuons-current generators of the same rating by circulating the power between them. Make a diagram of the connections, showing all necessary instruments. If two 500-k.w. 500-volt machines are tested near full load and have an efficiency of 92 per cent. how many amperes at 500 volts will be required from an external source to carry out the test? Suppose that no extra current at 500 volts is available, but only engine-driven generator capable of giving 1200 amperes at 100 volts, how would you conduct the test? (Grade II., C. G., 1917).

9. Find an expression for the force on a conductor carrying a steady current in a magnetic field. Hence find the torque on an armature carrying a total of 25,000 ampere conductors, the diameter of the core being 36 inches, the length 12 inches, the polar arc 70 per cent., and the flux-density in the gap 6,000.

(London. Univ., El. Tech., 1912).

10. A six-pole lap-wound motor has poles 20 cm. square and a constant flux-density in the gap of 5,000. The armature is wound with 500 wires having a total length of wire of 24,000 cm. and .07 square cm. area. Find the speed of the motor with 100 volts on the terminals and 120 amperes in the line. (Lond. Univ. El. Tech. 1912).

11. Describe the various methods which have been used in practice for obtaining a wide range of speed

in a continuous current electric motor. Discuss the advantages and disadvantages of each method from the point of view of (a) efficiency and (b) convenience.

(Lond. Univ., El. Tech. 1913).

12. From the following data relating to a 6-pole motor calculate the brake horse-power when the armature carries a current of 300 amps. Of the total power developed by the motor, 2.5 per cent is absorbed in overcoming the friction and iron losses.

Flux entering the armature per pole = 6 megalines.
Total number of armature turns ... = 600.
Number of armature circuits ... = 6.
Speed of armature in R. P. M. ... = 400.

13. A 10-B. H. P. 220-volt motor takes 40 amperes at full load. If the maximum starting current has not to exceed 60 amperes, calculate (1) the number of steps into which the starting resistance must be divided, and (2) the value of the resistance of each step. Resistance of armature brushes and brush loads = .025 of an ohm.

14. A 20-B. H. P. 440-volt shunt-wound motor requires a current of 42 amperes when running normally at full load. If the maximum starting current has not to exceed 60 amperes, determine (1) the number of steps required for the starting resistance, and (2) the resistance of each step. The armature resistance = .025 of an ohm.

15. The armature core of a 4-pole motor has 41 slots, each containing 24 conductors. At full load the

wave-wound armature takes 25 amperes at 450 volts, and a flux of 25,00,000 maxwells enter the armature core per pole. What torque is developed at full load?

16. If the above armature revolve at 800 rev. per min., determine the horse-power developed at full load.

17. From the following name-plate data of a shunt motor determine its regulation and full-load efficiency:

H. P. = 100, Volts. = 220, Amperes = 300,

R. P. M. = 550 at no load, R. P. M. = 500 at full load.

18. A 20-h. p., 220-volt, 1000-r. p. m. shunt motor has an efficiency of 88 per cent. The voltage drop in the armature circuit is 4 per cent. and the exciting current is 1. 4 per cent. of the full-load current.

(a) Find the full-load current in the line, the armature current, the resistance of the armature circuit.

(b) Find the torque developed at the driving pulley at full-load.

(c) Specify the starting resistance to keep the starting current down 1.25 times full-load current, and what will be the starting torque under those conditions?

(d) Would this same starting resistance be suitable for all 120 volt, 30-h. p. shunt motors, no matter of what speed? Give reasons.

(f) What would happen if the 30-h. p. starter were used for the 10-h. p. motor and the 10-h. p. starter for the 30-h. p. motor?

19. A 20-h. p., 220-volt, 1000-r. p. m. series notor has an efficiency of 88 per cent. The voltage lrop in the armature is 4 per cent. and in the exciting oils is 1.5 per cent.

(a) Find the full-load current in the line, the armaure current, the resistance of the armature and that of he field coils.

(b) Find the torque developed at the pulley at ull load.

(c) Specify the starting resistance to keep the tarting current down to 1.25 times full-load current nd what will be approximately the starting torque nder these conditions.

20. If the shunt motor of Exer. 18 is connected vith a starter and is protected by suitable fuses, what vill happen:

(a) If the starting arm is moved over too rapidly?

(b) If the field coil circuit is open and an attempt s made to start the motor?

(c) If the field coil circuit breaks while the motor s running on no load?

(d) If the starter has a no-voltage release and the ield coil circuit then breaks while the motor is running n no load?

(e) If there is an instantaneous overload of 100 per cent.?

(f) If there is an overload of 50 per cent. for some time?

(g) If the torque of the load is increased 50 per cent what is approximately the current taken from the line, also the speed of the motor and the output?

21. If the series motor of Exer. 19 is connected up with a suitable starting resistance, and is protected by fuses and belted to the load, what will happen:

(a) If the field coil circuit is open and an attempt is made to start the motor?

(b) If the field coils are short-circuited and an attempt is made to start the motor?

(c) If the field coil circuit breaks while the motor is running?

(d) If the belt breaks?

(e) If the torque of the load is increased 50 per cent.,

what is approximately the current taken from the line, also the speed of the motor and the output?

(f) If the load on the motor is increased 50 per cent. what is approximately the current taken from the line, the torque developed and the speed in terms of the values at full load?

22. If the shunt motor of Exer. 18 is carrying full load and a resistance of 0.05 ohm is inserted suddenly

in the armature circuit, the torque of the load remaining constant, find:

(a) The back e. m. f. at normal load and speed.

(b) The armature current immediately after the resistance has been inserted.

(c) The torque developed at the same instant.

(d) Explain why the speed drops, also what happens while conditions are becoming steady.

(e) What is the final speed of the motor?

23. A 20-h. p., 220-volt, 1000-r. p. m. shunt motor has an efficiency of 88 per cent. The voltage drop in the armature circuit is 4 per cent., and the exciting current is 1.4 per cent. of the the full-load current.

(a) Find the full-load current in the line, the armature current, the resistance of the armature circuit.

(b) Find the torque developed at the driving pulley at full load.

(c) Specify the starting resistance to keep the starting current down to 1.25 times full-load current; what will be the starting torque under these conditions?

(d) Would this same starting resistance be suitable for a 220-volt,20-h. p. shunt motor?

(e) Specify the starting resistance for a 10-h.p., 220-volt shunt motor to give 1.25 times full-load torque.

(f) What would happen if the 20-h. p. starter were used for the 10-h. p. motor and the 10 h. p. starter for the 20-h. p. motor? The motors are protected by fuses and it is necessary to develop the full-load torque of the motor at starting. Compare results with Exer. 18.

24. A 20-h. p., 220-volt, 1000-r. p. m. series motor has an efficiency of 88 per cent. The voltage drop in the armature is 4 per cent., and in the exciting coils is 1.5 per cent.

(a) Find the full-load current in the line, the armature current, the resistance of the armature and that of the field coils.

(b) Find the torque developed at the pulley at full load.

(c) Specify the starting resistance to keep the starting current down to 1.25 times full-load current and what will be approximately the starting torque under these conditions. Compare the results with those of Exer. 19.

25. A 20-h. p., 220-volt, 900-r. p. m. shunt motor has an efficiency of 87 per cent., and the resistance of the armature circuit is 0.05 ohm.

(a) If the speed is reduced to 500 r. p. m. by means of a resistance in the armature circuit, the torque being constant, what is the value of the resistance, the loss in the resistance, the output of the motor and the overall efficiency ?

(b) Why does the motor run hotter than on normal load ?

(c) If the torque is reduced to half full-load value, the external resistance being unchanged, what is the approximate speed of the motor ?

26. If the armature of the 20-h. p. motor in Exer. 25 is rewound with twice the original number of turns, the wire being of half the original cross-section, what will be the speed on full load, the permissible armature current and the permissible output ?

Explain why this motor with 10-h. p. load will be hotter than the original machine with 20-h. p. load.

27. A shunt dynamo when operated as a generator at 900 r. p. m. delivers 200 amperes (armature) current at a terminal voltage of 220. The resistance of the armature circuit is 0.05 ohm. Calculate the speed at which the armature rotates if operated as a motor, the armature current and the field excitation being the same as in the generator.

28. A 4-pole, wave-wound shunt motor is operated on a 220-volt circuit. Armature slots = 47, conductors per slot = 20, commutator-bars = 141, flux = 1,660,000 maxwells per pole. Neglecting armature resistance, find the speed of the motor.

29. The armature of a 6-pole continuous-current motor is 10 inches long and 24 inches in diameter. The

face of each pole shoe is 10 inches X 11 inches. The armature is lap wound and consists of 600 conductors. The average flux density on the face of the poles is 48,000 lines per square inch. Find the force acting on the armature when the total input to the armature is 300 amperes.

30. A 220-volt shunt motor has a no-load input of 25 amperes, and a full-load input of 700; amperes when operated at 500 r. p. m. Armature resistance $=0.0075$; field resistance$=40$. Find the speed regulation of the motor.

31. You have got a 10 B. H. P. motor of 110 volts at a place where the line voltage is 220, and a 10 B.H.P. motor of 220 volts where the line voltage is 110. State what you will do in each case to run the motor, and show the relation of speed, torque and efficiency in each case.

32. You have a small electric fan running at 1·3 volts but the line voltage is 220. What will you do to run the fan?

CHAPTER VI

ALTERNATING CURRENT

148. Alternating Current.—An alternate current or E. M. F. is a current or E. M. F. which when plotted against time in rectangular co-ordinates consists of half-waves of equal area in successively opposite directions from the zero line (Fig. 6·01).

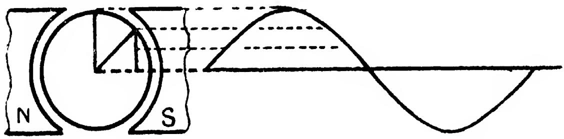


Fig. 6.01.

149. Alternating Current vs. Direct Current.—(1) The alternating current reduces the cost of transmission by the use of high voltages which diminish the amount of copper in the line, when the distance over which the power to be transmitted is large, as the pressure can be altered conveniently, cheaply and efficiently by means of static transformers, which have no moving parts, require little attention and have high efficiency; the continuous current, on the other hand, involves the use of moving-machinery for the purpose.

(2) In the alternating current system, the generators and motors are extremely simple. The commutator, which makes the direct current system more unreliable

than the alternate current system, is eliminated, and in consequence, the machine is made cheaper, the danger of break-down is lessened, the exact position of the brushes is made immaterial, and the risk of the attendant being fatally injured is reduced.

(3) Having stationary armature and revolving field magnets, the portion of an alternator subject to high pressure is relieved from the mechanical stresses incidental to a revolving body, and the liability to break-down is thereby greatly diminished.

(4) As regards transmission of current, it possesses the advantage that if leakage takes place there is lesser danger of electrolytic action ensuing in connection with underground pipes and structures.

Electric welding by certain processes is better done by alternate current.

For lighting, and factory driving each has its advantages and disadvantages.

For heating there is very little difference between the two.

DISADVANTAGES:—THE ALTERNATE CURRENT is not as suitable as direct current for distribution. The high pressure requires more efficient insulation and is dangerous to life. Idle current is absorbed in high pressure circuits. There is some difficulty in running alternators in parallel on the mains, specially in case of a breakdown. For variable speed motors, the direct current system is more suitable. The alternate current cannot be used directly for many purposes such as electroplating, electro-chemical works,

charging storage batteries, etc. It does not admit the use of a battery so readily and conveniently as direct current.

150. Graph of Alternating Current Electromotive Force—

Let an armature with diameter AB (Fig. 6.02) revolve betweent he poles of a magnet N and S, and let P, P′, P″, etc. be the positions of the conductors on the armature. As the armature conductors cut the lines of force an e.m.f. is generated between the ends of the conductor, and when the circuit is complete a current flows through the conductor. The value of the e.m.f. and current may be obtained from the following :—

On AB as diameter draw a circle with

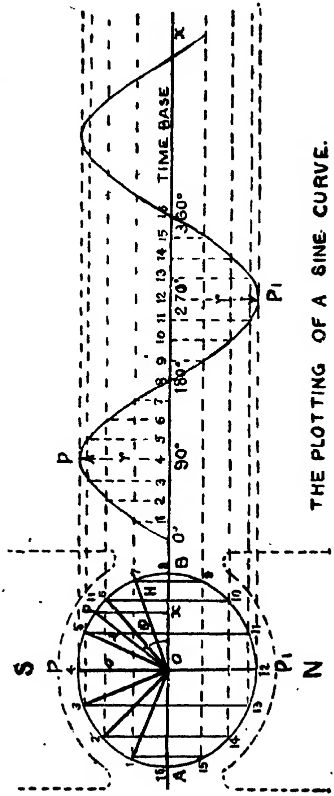


Fig. 6.02

centre O and radius OP equal to r. Draw a horizontal line OX in a line with AB and call it the time base. Distances measured along the time base from O, may represent time or distances moved by P round its circular path from the beginning of measurement. The maximum height is OP, and thus OP and OP′ are the maximum heights, crest or peak points of the sine curve which represents the growth, fall and reversal of the e.m.f. or current. The perpendicular height of P above or below AB depends upon the angle such that PX is always equal to OP sin θ or r sin θ. Complete the figure as in the diagram. Thus the divisions of the straight line may be taken to represent the time or proportional to the angles made by the radius with its first position in its revolution round the centre O.

Hence the e.m.f. or current generated at any position of the point P is directly proportional to sin θ, and as OP is a constant the e.m. f. or current is directly proportional to H.

Thus the various values of the e.m.f. or current are represented in the sine curve by the perpendicular heights on the time base or line O′X.

The ideal pressure or current curve from an alternator is sinusoidal. Commercial alternators do not generate true sinusoidal pressures; there is distorsion, but the curves closely approximate sine curves, and as such, they can be treated with relative simplicity, the

harmonics present are relatively weak compared with the fundamental.

151. Properties of Alternating Current.—

Cycle.—When a current rises from zero to a maximum in one direction and comes down to zero, and again rises to a maximum in the other direction, and finally passes through zero again, it is said to have completed one CYCLE. That is, it completes a cycle when it has returned to the condition from which it first started both as to value and as to direction, and is prepared to repeat the process described, making a second cycle. It takes two alternations to make one cycle. $\sim$ is frequently used to denote a cycle. A cycle is often referred to as 360 ELECTRICAL DEGREES or one electrical revolution. The circumferential distance from the centre of one pole to that of the next pole of the same polarity is referred to as 360 MAGNETIC DEGREES.

The maximum value of the current strength reached during the cycle is called the AMPLITUDE of the current.

Frequency or Periodicity.—It means the number of cycles completed in a unit time, i. e., in one second. It is denoted by f, and ω the angular velocity is equal to $\frac{2\pi}{T} = 2\pi f$. The time of one cycle is called PERIOD and is denoted by T.

To find the frequency of the pressure or the current produced by any alternating-current generator, if R.P.M.

be the number of revolutions per minute, and p the number of pairs of poles, then

$$f = p\frac{\text{R.P.M.}}{60}.$$

Time need not be counted from the instant a current or voltage passes through zero.

Reactance (X). In a simple alternating current circuit, the reactive component of the impedance, (see P. 250) as distinguished from the active component, resistance. Reactance may be divided into two species of mutually opposite signs, namely INDUCTIVE REACTANCE or that species of Reactance, developed in an inductance, $2\pi f L$, and CONDENSIVE REACTANCE, or that species of rectance developed in a condenser, $1/2\pi C$. The unit of reactance in the practical system is the ohm. (Vide chapters VII and VIII.)

Phase.—The curves of the pressure and the current in a circuit can be plotted together, with their respective ordinates and common abscissae. When the zero and the maximum values of the current curve will occur at the same abscissae, as do those values of the pressure curve, the current is said to be in phase with pressure. (Fig. 6.03).

Synchronism means the simultaneous occurrence of any two events. Two alternating currents or pressures are said to be "in synchronism" when they

have the same frequency and are in phase.

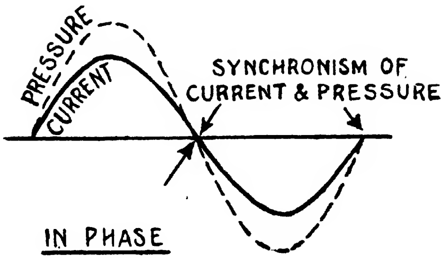


Fig. 6.03.

When the current will reach a maximum or a zero value at a time later than the corresponding values of the pressure, the current is said to be OUT OF PHASE with, and to lag behind the pressure, (Fig. 6.04).

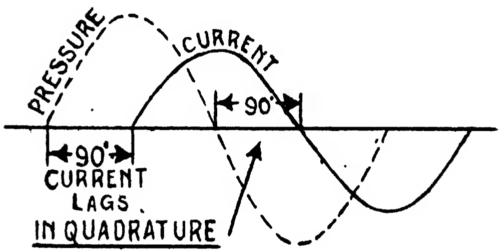


Fig. 6.04.

But the current is said to lead the pressure when it reaches the zero or maximum value sometime before the

corresponding value of the pressure, (Fig. 6.05).

When a curve has its zero ordinate coincident with the maximum ordinate of the other, there is a displacement of a quarter cycle ($\phi = 90^0$), and the curves are said to be AT RIGHT ANGLES or IN QUADRATURE.

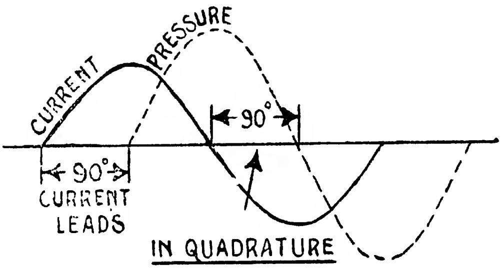


Fig. 6.05.

NOTE (1) that IN PHASE means the simultaneous zero values and the simultaneous maximum values of the current and pressure of the same sign.

(2) that the current is OUT OF PHASE with the pressure when it either lags or leads, and thus the current is not in synchronism with the pressure.

(3) that the current always lags owing to the effect of inductance, and leads due to the effect of capacity; whereas in an ideal circuit containing resistance only, the current is in phase with the pressure, i. e., it does not lag nor lead.

The angular displacement or phase difference measured in degrees is the distance between the ordinate of one sine curve and the corresponding zero ordinate of another. This angle of lag or of lead is usually represented by ϕ (Fig. 6.06). It will be seen hereafter that the power in an alternating current circuit is determined by multiplying the volt amperes by the cosine of this angle.

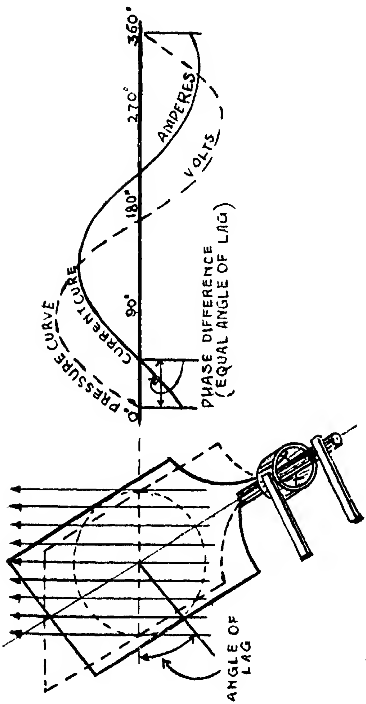


Fig. 60.6.

In Fig. 6.07 the current and the pressure are in opposition. It means that the phase difference between current and pressure is 180°.

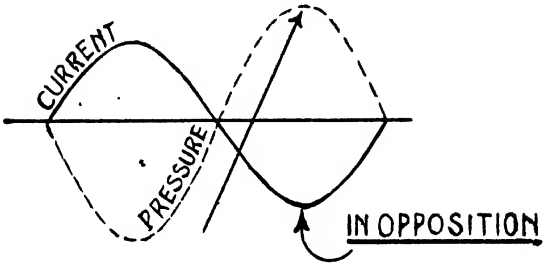


Fig. 6.07.

Distribution Constant:—This is the ratio of the vector sum to the algebraic sum of the pressures generated per pole and per phase.

The Impedance (*z*) of a circuit is the ratio of the difference in effective pressure between the two ends of the conductor to the effective current flowing through the conductor. (For effective pressure and current. Vide P. 254).

152. Synchronous Impedance Curve and Synchronous Impedance.—The synchronous impedance curve shows the relation between the various values of the field current or excitation ampere turns, or armature voltage, and the armature current, when the armature is short circuited, so that the only condition that limits the current for a given voltage generated is called THE SYNCHRONOUS IMPEDANCE of the armature.

The synchronous impedance materially differs from the impedance of the armature when the machine is standing still.

IT IS DETERMINED BY short circuiting the armature

through ammeters and operating at full frequency at various values of the field current. Only fractional values of normal excitation are used. (Fig. 6·08).

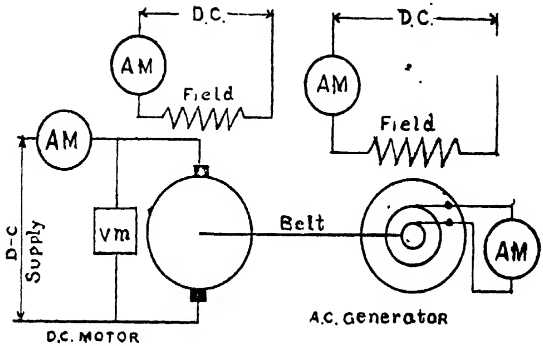


Fig. 6·08.

Synchronous impedance is used as a basis for the predetermination of the regulation of the machine; thereby the trouble and expense of an actual test of regulation under full load is avoided.

The synchronous impedance of an alternator armature is very nearly equal to the Synchronous Reactance of the armature.

153. The synchronous reactance of a generator is the equivalent reactance which would produce the same effect as the armature leakage reactance and armature reaction combined.

The E.M.F. required to overcome Impedance is

$$E=IZ.$$

In the case of direct current $Z = Ir$.

151. The Effective or Alternating Current Resistance of any portion of a circuit, to an alternating current, is the quotient of the average rate P_h at which heat is developed by this current, either directly in the substance through which it passes, or indirectly as a consequence of the hysteresis and eddy current losses produced by its magnetic field, divided by the square of the effective value I of the total current (conductance plus displacement or the charging current) through this portion of the circuit.

$$\text{i. e.,} \quad r = \frac{P_h}{I^2} = \frac{\text{watts absorbed by coil}}{(\text{current})^2}.$$

Calling V the effective value of the potential difference across the given portion of the circuit, the effective conductance G of this portion of the circuit is

$$G = \frac{P_h}{V^2}.$$

In general, both r and G depend upon both the frequency and the wave-shape of the current and voltage. (H. Pender).

It is greater than the ohmic resistance, as it is that value of resistance which represents the total energy loss, and as such, includes in addition to the ohmic resistance, the effect of any other sources of lost energy, such as iron losses in a magnetic circuit dielectric losses, and induced currents in a neighbouring circuit.

This effective resistance produces a potential drop which is in phase with the current. The simplest method of determining it is from the relation, $W = I^2R$, where W = watts measured with a wattmeter, I = current (amperes), and R = alternating-current resistance (ohms).

The current density over the cross section of the conductor is a minimum at the centre increasing to a maximum at the periphery. In a solid conductor of large cross section the current is confined almost entirely to an outer shell or skin. The SKIN EFFECT FACTOR is the number by which the resistance of the circuit to a continuous current must be multiplied to give the effective resistance to an alternating current.

155. To Prove that the Effective Resistance is Greater than the Ohmic Resistance we proceed as follows.—

Let R = resistance of a Conductor,

I = The current flowing through the conductor.

Then, the copper loss in this conductor $= I^2R$. Divide the conductor into two equal parts, the resistance of each part being $2R$.

Now if the density of the current is uniform, the total losses are $\left(\frac{I}{2}\right)^2 \times 2R + \left(\frac{I}{2}\right)^2 \times 2R = I^2R$.

If the current density is not uniform, as happens in the case of effective resistance, the resistance of each part being $2R$, the currents in the two parts are $\left(\frac{I}{2} + \frac{I'}{2}\right)$, and $\left(\frac{I}{2} - \frac{I'}{2}\right)$ although the total current

remains the same.

$$\therefore \text{ Total loss} = \left(\frac{I}{2} + \frac{I'}{2}\right)^2 \times 2R + \left(\frac{I}{2} - \frac{I'}{2}\right)^2 \times 2R.$$

$$= I^2R + I'^2R. \text{ which is greater than } I^2R.$$

156. Equivalent Resistance, Impedance and Reactance.—In alternating current circuits it is sometimes convenient to consider a motor or other load developing a back E. M. F. equivalent to a single resistance or reactance.

Let P = the total power taken by the load,
V = the voltage between its terminals,
I = the current.

The Equivalent Resistance $= R = \dfrac{P}{I^2}$.

The Equivalent Impedance $= Z = \dfrac{V}{I}$.

The Equivalent Reactance $= X = \sqrt{Z^2 - R^2}$.

The difference between the effective resistance and the equivalent resistance, although not always observed, is that the first takes into account only the power dissipated as heat, whereas the latter takes into account the total power of which only part is heat, the rest being converted into some other form, for example, mechanical power.

157. Virtual Volts and Amperes.—As described above, the E. M. F. of an alternator is continually rising, falling, and reversing, and the current in the circuit

must rise, fall, and reverse in sympathy (though not necessarily in step) with the E. M. F.

The maximum points of the pressure or current are only at these maxima for comparatively short periods. An alternating E. M. F. of, say, 100 volts, must at some instants be considerably above or below 100 volts, and at other instants be zero.

The Virtual value is equivalent to that of a direct E. M. F. or current which would produce the same effects, and those effects are taken which are not affected by rapid changes in direction and strength; in the case of E. M. F. or pressure—the reading on an electrostatic voltmeter; and in the case of current—the heating effect.

Thus, A VIRTUAL E. M. F. of 100 volts is one that would produce the same deflection on an electrostatic voltmeter as a direct or continuous E. M. F. of 100 volts: and A VIRTUAL CURRENT of 10 amperes is that current which would produce the same heating effect as a continuous current of 10 amperes—in some kind of electric heater for example. But both pressure and current will be continually varying above and below these values.

The virtual value of an alternating E. M. F. or current having a sine-wave form is .707 of its maximum value.

Thus, a virtual alternating pressure throws more strain on the insulation of a circuit than a continuous pressure of the same value.

158. Method of Deriving Virtual or Effective

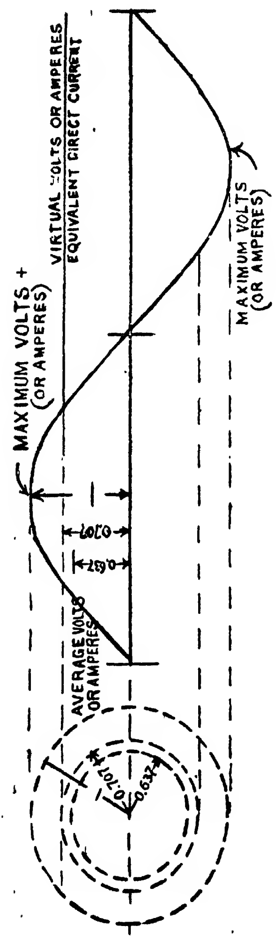


Fig. 6.09.

values of E. M. F. and Current.—In the alternating current system a current of such instantaneous values as to have the same heating effect in a conductor as one ampere of direct current is said to be one ampere.

The heat produced in a conductor carrying current is proportional to the square of the current, and in an alternating current, whose instantaneous current values vary, to the square of the instantaneous current value. Hence, the average heating effect is proportional to the mean of the instantaneous currents.

The average current of a sinusoidal wave of alternating current, whose maximum value is I_m, is equal to the area of one lobe of the curve divided by its base line π.

If I_{av} = average value of the current,

E_{av} = average value of the E. M. F,

I_m = maximum value of the current,

E_m = maximum value of the E. M. F,

I' = Instantaneous value of the current,

E' = instantaneous value of the E.M.F.

I = effective value, virtual or root mean square value of the current.

E = effective value, virtual or root mean square value of the. E M. F.,

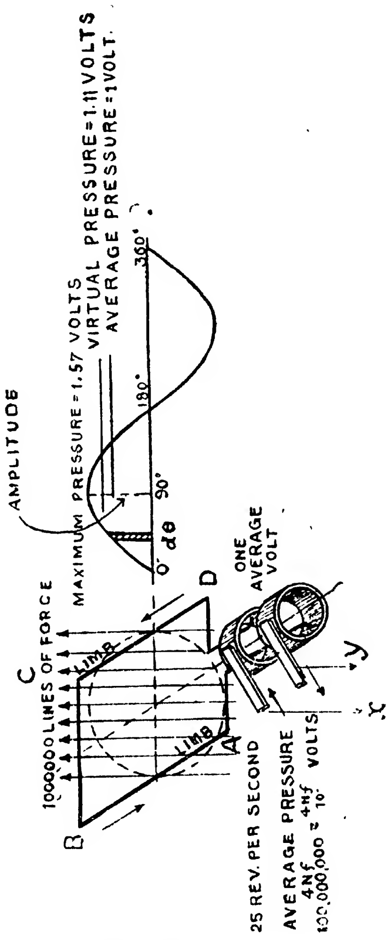


Fig. 6.10.

$$\text{then } I_{av} = \frac{\int_0^\pi I_m \sin\theta\, d\theta}{\pi} = \frac{I_m}{\pi}\Big[-\cos\theta\Big]_0^\pi = \frac{2}{\pi} I_m.$$

The heating value of such a current varies as

$$I^2 = \frac{\int_0^\pi I_m^2 \sin^2\theta\, d\theta}{\pi} = \frac{I_m^2}{\pi}\Big[\frac{\theta}{2} - \tfrac{1}{4}\sin^2\theta\Big]_0^\pi$$

$$= \tfrac{1}{2} I_m^2.$$

$$\therefore\ I = \frac{I_m}{\sqrt{2}} = .707\ I_m,$$

where I has the same heating effect as a direct current I, and THIS EFFECTIVE VALUE IS ALWAY REFERRED TO UNLESS EXPRESSLY STATED OTHERWISE.

$\therefore$ Amplitude factor = .707.

Also, $E = \frac{E_m}{\sqrt{2}} = .707\ E_m$

So, $I_{av} = \frac{2}{\pi} I_m$ and $E_{av.} = \frac{2}{\pi} E_m$.

The maximum value of pressure is frequently referred to in designing alternator armatures, and in calculating dielectric strength of insulation. The instantaneous values are important only in special cases.

In a similar way $\frac{E_m}{\sqrt{2}}$ is the virtual voltage of an alternating current whose maximum voltage is E_m.

In an alternating current system THE ADVANTAGES OF DEFINING AND MEASURING ELECTROMOTIVE FORCE IN VIRTUAL VOLTS, AND CURRENT IN VIRTUAL AMPERES is that when a suitable instrument is calibrated by means of a continuous electromotive force or current, it will then read virtual volts or amperes on an alternating current supply.

(1) The same laws of heating may be applied to Direct Current or Alternating Current.

(2) It makes it possible to measure an alternating current of any wave form.

(3) The electrodynamometer may be used for measuring the Current and Power.

(4) It does not require any factor in the expression for power which varies with the wave form.

The FORM-FACTOR is the ratio $\frac{\text{Effective E. M. F.}}{\text{Average E. M. F.}}$ since its value is determined by the shape of the pressure wave. The form-factor is never less than unity. As a curve becomes more peaked, its form factor increases, due to the superior weight of the squares of the longer ordinates.

In the sinusoid, the values found above give

$$\text{Form-factor} = \frac{\dfrac{1}{\sqrt{2}}E_m}{\dfrac{2}{\pi}E_m} = 1{\cdot}11.$$

$$\text{Crest Factor} = \frac{\text{maximum value.}}{\text{effective value}} = 1.41$$

Example 1. Ten successive instantaneous values of an alternating electromotive force during half a cycle are 0, 30, 60, 80, 90, 95, 90, 80, 60 and 30 volts.

Determine the mean effective values, and its crest and form factors.

Solution.—

The mean value is the sum of the above quantities, 615 volts, divided by the number of values, 10.

∴ The AVERAGE VALUE of this Electromotive force during half a cycle = 61.5 volts.

Squaring each of the above values and adding the squares together, and dividing their sum by the number, here 10, we get the average value of the square of the electromotive force = (900 + 3600 + 6400 + 8100 + 9025 + 8100 + 6400 + 3600 + 900)/10 = 4702.5 volts and the square root of this average square is 68.57 volts which is the EFFECTIVE VALUE or R.M.S. P.D.

$$\therefore \text{Crest Factor} = \frac{95}{68.87} = 1.385,$$

$$\text{and Form Factor} = \frac{68.57}{61.5} = 1.115,$$

which latter is always referred to unless expressly stated otherwise.

Example 2. The P. D. applied to a circuit and the current flowing in it have the following values at corresponding instants $\frac{1}{24}$th period apart. Find (i) the R. M. S. values of P. D. and the current respectively, (ii) the average power, and (iii) the power factor (iv) the form factor, (v) the crest factor.

Phase angle	0°	15°	30°	45°	60°	75°	90°	105°	120°	135°	150°	165°	180°
E instant.	0	80.6	155·6	220	269·5	300·4	311·1	300·4	269·5	220	155·6	80·6	0
I do.	—32·2	—16·7	0	16·7	32·2	45·5	55·7	62·14	64·33	62·14	55·7	45·5	32·13

Solution:—

(i) To find the Root Mean Square value.

Phase angle	0°	15°	30°	45°	60°	75°	90°	105°	120°	135°	150°	165°	180°
E^2 instant	0	6490	24210	48400	72630	90240	96740	90240	72630	48400	24010	6490	0
I^2 do.	1037	278·4	0	278·4	1037	2074	3112	3873	4148	3873	3112	2074	1032
E. I.	0	—1345	0	3674	8678	13668	17350	18700	17350	13668	8678	3674	0

Mean value of $E^2 = \frac{1}{12}\Sigma E^2 \text{ inst} = \frac{1}{12} \times 580680 = 48390$,

and R. M. S. voltage $= \sqrt{48390} = 220{\cdot}1$ volts,

Similarly, mean value of $I^2 = \frac{1}{12}\Sigma I^2 \text{ inst} = \frac{1}{12} \times 24896{\cdot}8 = 2074{\cdot}7$.

and R. M. S. current $= \sqrt{2074{\cdot}7} = 45{\cdot}5$ amps,

(ii) The average power $= \frac{1}{12}\Sigma EI = \frac{1}{12} 104095 = 8674$ watts.

(iii) Power factor $= \dfrac{\text{Watts}}{\text{Volts} \times \text{Amperes}} = \dfrac{8674}{220 \cdot 1 \times 45 \cdot 5}$,

$= \cdot 866$

(iv). Form factor $= \dfrac{\text{Effective value}}{\text{Average value}} = \dfrac{220 \cdot 1}{196 \cdot 9} = 1 \cdot 11.$

(v) Crest factor $= \dfrac{\text{Maximum value}}{\text{Effective value}} = \dfrac{311 \cdot 1}{220 \cdot 1} = 1 \cdot 42.$

159. Fundamental Equation of the Alternator.—The pressure generated in an armature is

$$E_{av} = 2\, p\, \phi\, N \frac{n}{60}\, 10^{-8},$$

where p = number of pairs of poles,

ϕ = maxwells of flux per pole,

N = revolutions per minute,

n = conductors in series between two brushes.

In an alternating circuit $E = kE_{av}$, where k is the form-factor. Hence in an alternator yielding a sine wave E.M.F.,

$$E = 2.22\, p\, \phi\, N \frac{n}{60} 10^{-8} \text{ (effective value).}$$

Inasmuch as $p\frac{N}{60}$ represents the frequency, f,

$$E = 2.22\, \phi\, n\, f\, 10^{-8}.$$

An alternator armature winding may be either concentrated or distributed. In a single phase alternator there is but one slot per pole, and all the inductors

that are intended to be under one pole are laid in one slot, when THE WINDING IS SAID TO BE CONCENTRATED, and if the inductors are all in series the above formula for E is applicable. Nearly all engine-driven alternators have six slots per pole, although twelve slots per pole are used when the output per pole is large and a long armature is undesirable. If now the inductors be not all laid in one slot, but are distributed in n more or less closely adjacent slots, the E.M.F. generated in the inductors of any one slot will be 1/n of that generated in the first case, and the pressure in the different slots will differ slightly in phase from each other, since they come under the centre of a given pole at different times.

If the inductors in four adjacent slots are in series, and if the angle of phase difference between the pressures generated in the successive ones be θ, and E_1, E_2, E_3, and E_4 the respective pressures (represented by A, B. C, D, or the sides A, B′, C,′ D,′ of the polygon in Fig. 6.11) then the total pressure, E, generated in them is equal to the closing side of the polygon. Obviously $E < E_1+E_2+E_3+E_4$.

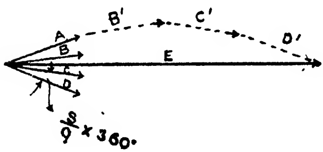


Fig. 6.11

Two wires on the armature, at a distance apart equal to the distance between adjacent north poles, and subtending an angle q, have induced in them

two electromotive forces. These electromotive forces are to be thought of as differing in phase by 360 electrical degrees. Therefore the phase difference of the electromotive forces induced in the wires placed in slot a and those induced in the wires placed in slot b, is

$$\frac{S}{q} \times 360.$$

when S = width of tooth + width of slot.
In other words, this angle is :

$$\frac{\text{width of tooth} + \text{width of slot}}{\text{circumference of armature} \div \text{p.}} \times 360.$$

If the winding had been concentrated, with all the inductors in one slot, the total pressure generated would have been equal to the algebraic sum of the E. M. F. generated in each of the inductors.

The number of slots under the pole vary, and they may be spaced so as to occupy the whole surface of the armature between successive pole centres, or they may be crowded together so as to occupy only one-half, one-fourth, or any other fraction of this space. Both the number of slots and the fractional part of the pole distance which they occupy affect the value of the distribution constant.

Hence the distribution constant k must be considered and we get

$$E = 2\, k\, k_1\, p\, \phi\, n\, \frac{N}{60}\, 10^{-8},$$

or for sine waves,

$$E = 2.22\ k_1\ \phi\ n\ f\ 10^{-8}.$$

In a distributed winding, the turns are not so effective as where they are concentrated.

Example 3. An alternator has 300 turns of wire on its armature and 2000000 lines of magnetic flux from each field pole, and has a frequency of 50. The value of the factor k is 1.11. Assuming a sine wave electromotive force curve and concentrated winding find the effective elecromotive force of the alternator.

Solution:—

$$E = \frac{2 \cdot 22 \times 1 \times 2 \times 10^6 \times 300 \times 2 \times 50}{10^8}$$

$= 666$ volts.

160. E. M. F.s in Series.—If any number of harmonic E. M. F.s of the same frequency, but of different magnitudes and phase displacements, is connected in series, the resulting harmonic E. M. F. will be given in magnitude and phase by the vector sum of the component E. M. F.s. The analytical expressions for E and θ may be derived by inspection of the diagram, and may be laid down thus:—

$$E = \sqrt{\left[\left\{E_1 \sin\theta_1 + E_2 \sin\theta_2 + \ldots\right\}^2 + \left\{E_1 \cos\theta_1 + E_2 \cos\theta_2 + \ldots\right\}\right]^2},$$

and

$$\tan\theta = \frac{E_1 \sin\theta_1 + E_2 \sin\theta_2 + \ldots}{E_1 \cos\theta_1 + E_2 \cos\theta_2 + \ldots}.$$

Example 4. Suppose three alternators (giving sine waves of pressure of values $E_1=60$, $E_2=40$, $E_3=30$ volts respectively) are to be connected in series. Considering the phase of E_1 to be the datum phase, let the phase displacements be $\theta_1=0^0$, $\theta_2=45^0$, and $\theta_3=70^0$ respectively. To find E and θ, complete the parallelograms or complete the force polygon. Then, $E=120$ volts and $\theta=30^0$.

A number of alternating E. M. F.s of DIFFERENT frequencies in series will give, in general, an irregular wave form. It is usual in practice, to have the frequencies of some E. M. F.s as multiples of the frequency of one, called the Fundamental E. M. F., or First Harmonic. The pressure curve which has twice this frequency is called the Second Harmonic; that which has three times this frequency the Third Harmonic, and so on. The resultant E.M.F. at any instant is found by adding the pressure values of all the components at that instant. It is expressed thus:

$$E' = E_1m \sin \omega t + E_{2m} \sin (2\omega t + \phi_1)$$
$$+ E_{3m} \sin (3\omega t + \phi_2) + \ldots$$
$$+ E \sin (n\omega t + \phi_{n-1}),$$

where ϕ_1, $\phi_2, \ldots\ldots \phi_{n-1}$, are the phase differences between E_{1m} and E_{2m}, E_{1m} and $E_{3m}, \ldots E_{(n-1)m}$ and E_{nm} respectively, when $\sin \omega t = 0$.

The presence of both odd and even harmonics will give a curve having unlike lobes. When, however, only odd harmonics are present, as is usual in electrical

machinery, the lobes above and below the horizontal axis will be similar. Fig. 6.12 gives the resultant E. M. F. of three harmonic components for the values, $E_{1m}=100$ volts, $E_{2m}=40$ volts, $E_{3m}=20$ volts, $\phi_1=30°$ and $\phi_2=45°$.

161. Power in Alternating-Current Circuits —The power in an alternating-current circuit is a function of E,I and ϕ where the current lags by the angle ϕ.

Then, $$E'=E_m \sin \alpha,$$

where $$\alpha=2\,\pi ft,$$

and $$I'=I_m \sin (\alpha-\phi).$$

Now, $$E=\frac{E_m}{\sqrt{2}}, \qquad I=\frac{I_m}{\sqrt{3}}.$$

and the instantaneous power

$$P'=E'\,I'=2\,EI \sin \alpha \sin (\alpha-\phi).$$

But $$\sin (\alpha-\phi)=\sin \alpha \cos \phi-\cos \alpha \sin \phi,$$

so $$P'=2\,EI\,(\sin^2 \alpha \cos \phi-\sin \alpha \cos \alpha \sin \phi).$$

Now ϕ is a constant, therefore the average power over 180^0 degrees,

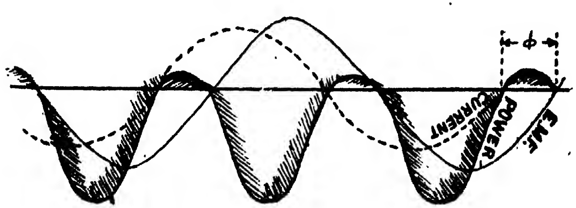


Fig. 6·14

$$P = \frac{2\ EI\cos\phi}{\pi}\int_0^\pi \sin^2 a da - \frac{2\ EI\sin\phi}{\pi}\int_0^\pi \sin a \cos a da.$$

$$= \frac{2\ EI\cos\phi}{\pi}\Big[\tfrac{1}{2}a - \tfrac{1}{4}\sin 2a\Big]_0^\pi$$

$$- \frac{2\ EI\sin\phi}{\pi}\Big[\tfrac{1}{2}\sin^2 a\Big]_0^\pi.$$

Hence $\quad P = EI\cos\phi.$

Should the current lead the pressure by ϕ^0, then the leading equation would be $P' = 2\ EI\sin a\sin(a+\phi)$, which gives the same expression, $P = EI\cos\phi$, the general expression for power in an alternating-current circuit.

(1) Carefully observe that, if there is phase difference, the frequency of the power wave is double that of the voltage or current waves and thus it undergoes a complete cycle of changes in half a period of the current or voltage waves.

(2) Cosine ϕ is called the power factor.

(3) That for series circuits,

$$\text{Cos}\ \phi = \frac{\text{Resistance}}{\text{Impedance.}}\ .$$

For Parallel circuits

$$\text{Cos}\ \phi = \frac{\text{Current through resistance circuit}}{\text{Total current}}.$$

(4) Volt-amperes do not represent true powers but, a knowledge of the voltamperes of a circuit is of very great importance in determining the limit of output of most electrical apparatus.

162. Effect of Phase Difference on A. C. Power. In an alternating current circuit if the current is not in phase with the pressure, the instantaneous value of the power is positive whenever the pressure and current are momentarily in the same direction. On the other hand, the power is momentarily negative whenever the current and pressure are in opposite directions. Now consider only the external potential differences. The power is positive when the apparatus or part of the circuit under consideration is absorbing electrical energy and transforming it into some other form, such as thermal or mechanical energy, and an apparatus has a negative power when it is generating electrical energy at the expense of energy in some other form. In a dynamo the current flows from the negative terminal to the positive, that is, against the terminal pressure, and the power is therefore negative, in other words, electrical energy is being generated. In the motor the current flows from positive to negative and the power is positive, that is, the motor is absorbing electrical energy. Hence a simple alternating current generator in which the current is out of phase with the E. M. F. does not act continuously as a generator, but, twice in each period, takes electrical energy from the circuit and is driven for a moment as a motor.

The same rule can be applied to any apparatus to which electrical energy is supplied. If, at any moment, the current flows in the direction of the terminal

pressure, i. e. from + to —, electrical energy is being absorbed, but if the current is flowing in the other direction, i. e. from − to +, the apparatus is acting momentarily as a generator and returning electric energy to the circuit.

163. Result of Low Power Factor.—Low power factor results in—

(1) Relatively large and costly electrical equipment—including generators, cables, switchgear, and transformers—the dimensions of which are governed by the kilovolt-amperes, rather than by the kilowatt output.

(2) Reduced efficiency for the whole of the electrical equipment owing to the copper losses for a given kilowatt-load being inversely proportional to the square of the power factor.

(3) Poor voltage regulation for the whole system.

(4) The need for a slightly higher charge per kw.-hour, on account of the relatively high initial cost of the electrical plant per kilowatt installed.

164. Methods in Use for Power Factor Correction.—The power factor of the load on a system may be improved by either or both of the methods:—

(1) Installing additional plant of such a nature as to take a leading current.

(2) Employing some means of actually improving the power factor of individual induction motors.

*165. **The Methods actually in use for Improving the Power Factor** are therefore:—

(a) The connection of static condensers in parallel with the load.

(b) The connection of rotary condensers (i.e. over-excited synchronous motors running light), in parallel with suitable sections of the load.

(c) The connection of "phase advancers" of various types in the rotor circuits of individual induction motors.

166. Classification of alternators.—

(1) According to the means employed for causing the conductors to cut the magnetic flux from the field magnet, viz:

(a) Revolving armature type:—Those with fixed field magnet and revolving armature.

Merits:—Revolving armature is in practice limited to about 25 K. V. A. and 600 volts, their inherent regulation is poor, and the alternating voltage is impressed upon the armature and collector rings which are difficult to insulate economically for high voltages.

(b) Revolving Field type:—Those with fixed armature and rotating field magnet.

Merits:—In revolving field machines the revolving parts are not subject to high or live voltage. Revolving field machines in larger capacities are less expensive and in any case have better inherent characteristics.

* Clayton's Power Factor Correction.

(c) Inductor type:—Those in which both the field and armature windings are stationary.

Principle—The direct current excitation is concentrated in one (usually stationary) coil and the variation in the magnetic flux is obtained by revolving a spider with bare projecting poles which alters the reluctance of the magnetic path. Thus the flux threading any particular armature coil is always in the same direction but varies in intensity or quantity.

Merits:—There are no winding or insulation on the moving member of the inductor type which is thus adopted to high speeds. The regulation is poor and is not suitable for parallel operation.

(2) According to the operating characterstics.

(a) Synchronous Generators.—In this type the action of inducing the E. M. F. results from the relative motion of the armature conductors, and a constant magnetic field produced by exciting coils in which a direct current flows. The frequency depends upon the number of field poles and the angular velocity of the revolving parts.

(b) Induction Generators—This is structurally identical with an induction motor but driven above synchronous speed as an alternating current generator.

The mechanical construction is usually that of an induction motor with a short circuited polyphase winding on the revolving number.

The magnetic field is of the rotating polyphase alternating currents flowing in the same windings with the load current. The frequency is determined or set by the characteristics of the external circuit containing synchronous machine either generator or motor. This frequency setter is necessary to supply the exciting current of the induction generator, and by this the power factor of the total load is adjusted to equal the inherent power factor of the generator.

Principle.—Suppose an induction motor is operated at a speed above synchronism by supplying power to the shaft by means of a prime mover. The rotor bars will cut the flux, but in the opposite direction from that when the machine is operating as a motor. The direction of E. M. F., the current and torque will be reversed, and the machine will operate as a Generator.

Use.—Its application is limited. Some electric locomotives are operated by induction motors. When the train descends a grade at a speed slightly above that corresponding to synchronous speed of the motors, the latter automatically become Generators and supply electric power to the line.

These are sometimes used in the development of small water powers. A minority of the units of a station may be of the induction type with advantage, as they will cause less disastrous effects in case of a short circuit in the system.

INDUCTION AND OTHER GENERATORS COMPARED.

(1) The Induction generator will not generate alone and

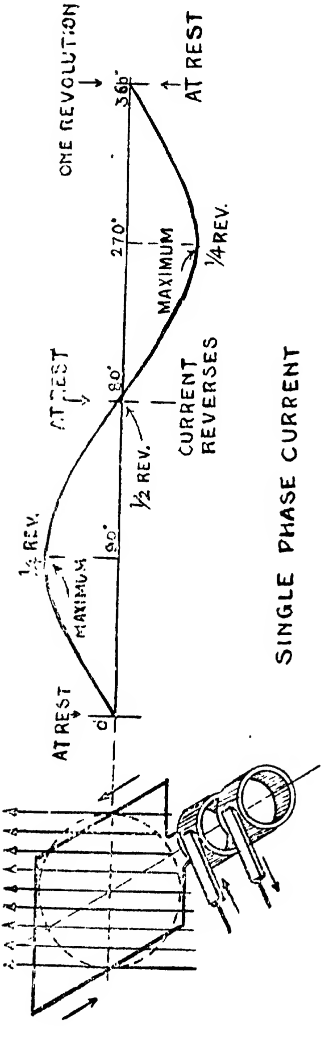


Fig. 6.15

will operate only when a line is already connected to a synchronous machine. (2) The frequency and voltage of the line will be determined by the synchronous machine, and hence the regulation of voltage is effected by means of the field rheostat of the synchronous machine.

(3) As regards the kind of current that may be derived from them, these are usually:

(a) Single phase,
(b) Two phase, } Poly-
(c) Three phase, } phase.

167. Single or Monophase Current.—When an alternator has a single winding on its armature it

generates single-phase current. There are two wires for a lead and return, as in the case of direct current. The alternator has got two slip rings and brushes connected to the lead and return of the external circuit. (Fig. 6.15)

168. Two-phase current.—In most cases two phase current actually consists of two distinct single-phase currents flowing in separate circuits having no electrical connection between them. The currents have equal period and equal amplitude, but differ in phase by one quarter of a period, so that when one is at a maximum the other is at zero. Although both the coils for the two phases are wound on the same armature core at right angles to each other, so that the volts generated in one are at a maximum when those generated in the other are at zero, the alternator may be considered as equivalent to two separate exactly similar alternators of equal capacity; the current of the one is behind that of the other by one right angle, so that when the one sends out the maximum current the other sends out no current. A two-phase alternator thus generally requires four slip rings and four line wires (Fig. 6.16). It is possible although not advisible, to use only three slip rings and only three wires. In this case, however, the common return is a thicker wire.

If only three wires are used, the voltage between two lines will be equal to $\sqrt{2}$ times the voltage in either phase, and the current in the line used as common return provided $\sqrt{2}$ times the current in either line if the two currents in the two phases are equal and

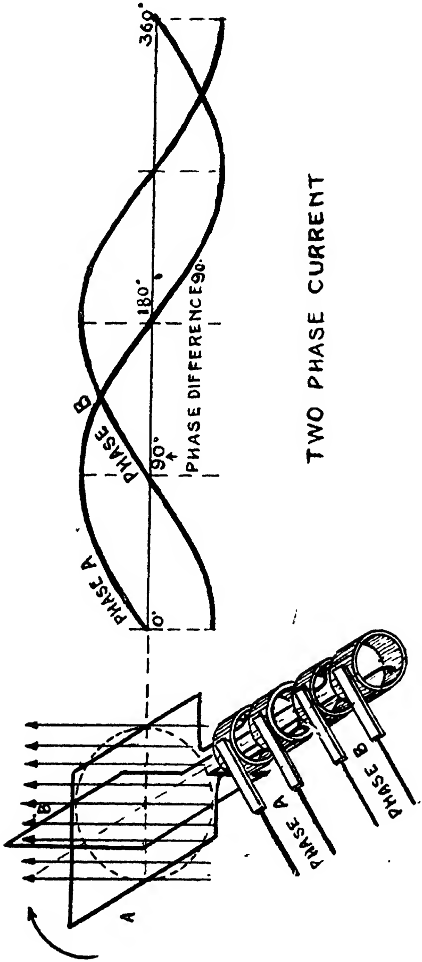

PHASE A
PHASE B
0°
90°
180°
360°
PHASE DIFFERENCE 90°
TWO PHASE CURRENT
A
B
PHASE A
PHASE B

Fig. 6.16

lag behind or lead their respective voltage by equal angles.

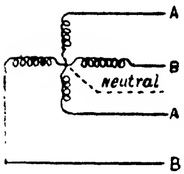
The wire in the armature may be wound :—

(1) Separately.

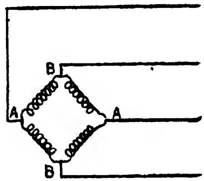
(2) Coupled at a common middle.

(3) Coupled in the armature, requiring only three collector rings.

Two Phase Alternators with four line wires.



(a) Star connection,
Fig. 6.17



(b) Mesh connection.
Fig. 6.18

Two Phase Alternator with three line wires

Example 5. In a two phase interconnected supply the load on the leading phase is non-inductive, and amounts to 10 amperes, while that on the other phase takes a current of 12 amperes, lagging by 30°.

(1) Find the current in the common wire in magnitude and phase.

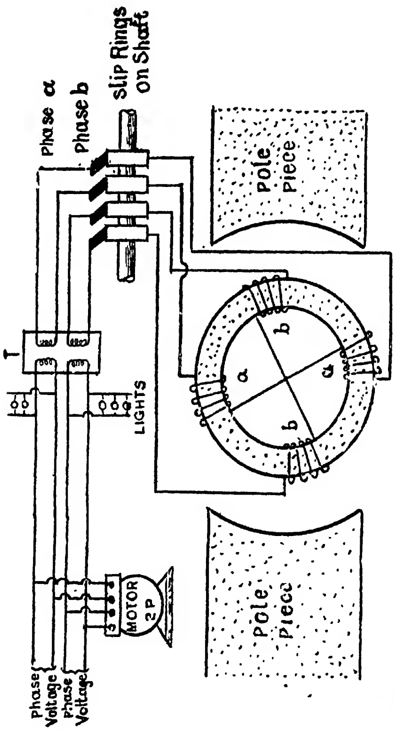
(2) Find the current if the loads are reversed.

Solution :—

(1) The current in the common wire is the vector sum of the currents in the separate phases.

To find its value resolve I_2 (the current of 12 amp) along V_2 and perpendicular to this :—

Component along $V_2 = 12 \times \cos 30° = 10{\cdot}39$ amp.
Component perpendicular to $V_2 = 12 \times \sin 30° = 6$ amp.



Two phase four wire alternator and its receiving circuit.

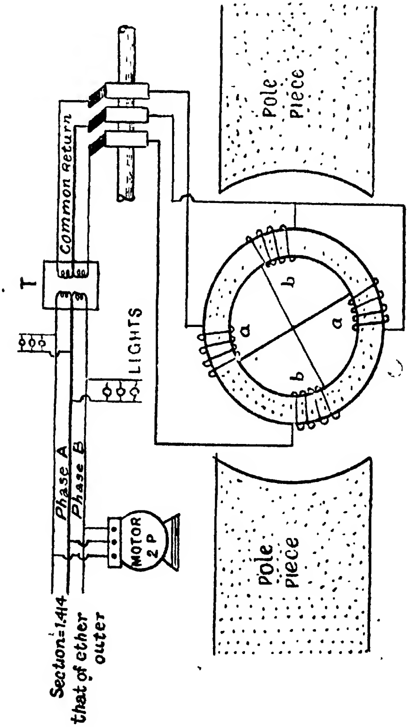
Fig. 6.19

$\therefore$ Resultant component perpendicular to $V_2 = 10 - 6 = 4$ amp.

$\therefore$ Total resultant $= \sqrt{\{(4)^2 + (10\cdot39)^2\}} = 11\cdot13$ amp.

and this lags behind V_1 by an angle $\tan^{-1} \dfrac{10\cdot39}{4}$

$= \tan^{-1} 2\cdot59 = 67^{\circ\prime}\ 40''$



Two phase three wire alternator and its receiving circuit.

Fig. 6.20

(2) In this case resolve I_1 along and perpendicular to V_1, the components being as before 10.39 and 6 amp. respectively.

$\therefore$ Resultant component perpendicular to V_1 $=6+10=16$ amp.

$\therefore$ Total resultant $=\sqrt{\{(16)^2+(10.39)^2\}}=19{\cdot}07$ amp

and this lags behind V_1 by an angle $\tan^{-1}\dfrac{16}{10.39}$

$=\tan^{-1} 1.54=57°$.

Example 6. In an unbalanced two-phase three wire system the load on the leading phase is non-inductive, and takes a current of 10 amperes, while that on the other phase takes a current of 12 amps. lagging by 45°. Find (a) the current in the common wire in magnitude and phase, (b) the current in the return wire if the loads are reversed.

Solution :—

(a) In Fig. 6.21, let E_1 and E_2 represent the two equal separate e. m. f, s. of phases at right angles; I_1 and I_2 the currents in the separate phases, and I the current in the common wire, which is the vector sum of I_1 and I_2.

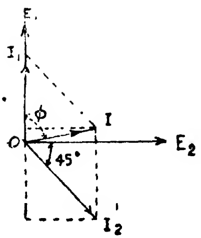


Fig. 6.21

To find I, resolve I_2 in phase and in quadrature with E_2. Here $I_1=10$ amps., $I_2=12$ amps.

$\therefore$ Component in phase $=12\cos 45^0=\dfrac{12}{\sqrt{2}}=8.49$ amp.

Component in quadrature $=12\sin 45^0$

$$=\frac{12}{\sqrt{2}}=8.49 \text{ amp.}$$

$\therefore$ Resultant component in quadrature $=10-8.49$

$$=1.51 \text{ amp.}$$

$\therefore$ Total resultant, $I=\sqrt{(1.51)^2+(8.49)^2}=8.62$ amps.

and this lags behind E_1 by an angle,

$$\phi=\tan^{-1}\frac{8.49}{1.51}=79^0\ 55'$$

(b) With reference to Fig. 6.22, resolve I_1 along and perpendicular to E_1. Here $I_1=12$ amps., $I_2=10$ amps. Therefore, as before, the components are 8.49 amps. and 8.49 amps. respectively.

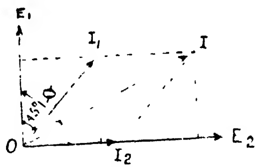


Fig. 6.22

$\therefore$ Resultant component perpendicular to $E_1=8.49+10=18.49$ amps.

$\therefore$ Total resultant, $I=\sqrt{(8.49)^2+(18.49)^2}$,

$$=20.3 \text{ amps.}$$

and this lags behind E_1 by an angle,

$$\phi=\tan^{-1}\frac{18.49}{8.49}=65°\ 21'.$$

169. Three-phase Generator.—This consists of three alternating currents having equal frequency and amplitude but differing in phase by 120 degrees, or by one-third of a period from each other. Three-phase alternators have three equal windings on the armature, so spaced out on it as to be successively one-third and two-thirds of a period apart from each other. There may be six, four or three slip rings and six four or three, line wires. (Figs 6.24—6.29)

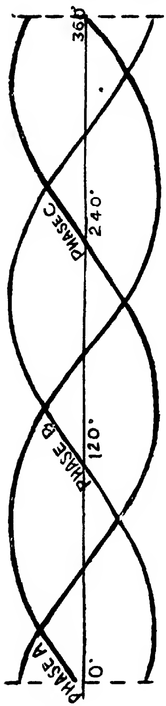


Fig. 6.23

Such an alternator may be viewed as a combination of three distinct single phase alternators, the currents differing from each other in phase by one-third of a period.. Fig 6.23.

If only three wires are used, and the voltage of each one of the phases separately is V the voltage generated between any two of the terminals will be equal to V x $\sqrt{3} = 1.73$ V.

THREE-PHASE CIRCUITS—STAR GROUPING WITH THREE LINE WIRES—The voltage between the line wires is $\sqrt{3}$ times the value of the phase voltage, while the line current is the same as the phase current. That is,

if E_p = phase voltage,
I_p = phase current,
E_L = line voltage,

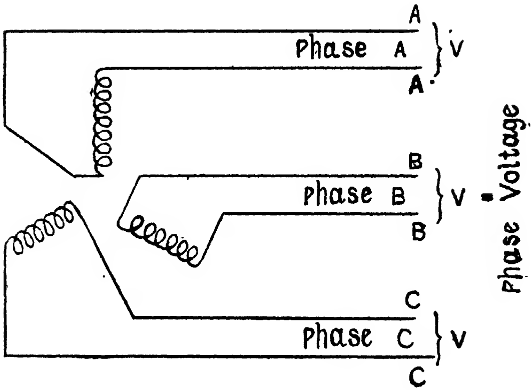


Fig. 6.24

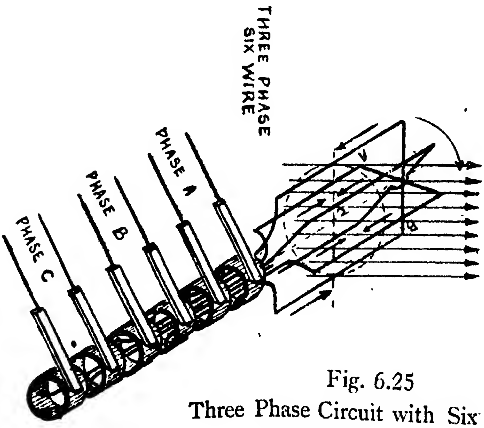


Fig. 6.25
Three Phase Circuit with Six line wires.

I_L = line current,

then, $I_p = I_L$

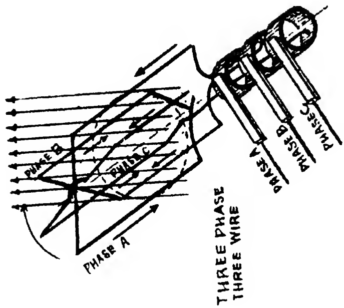


Fig. 6.26

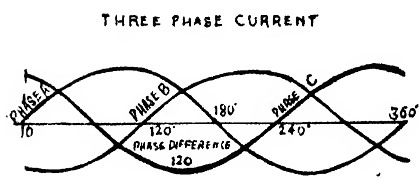
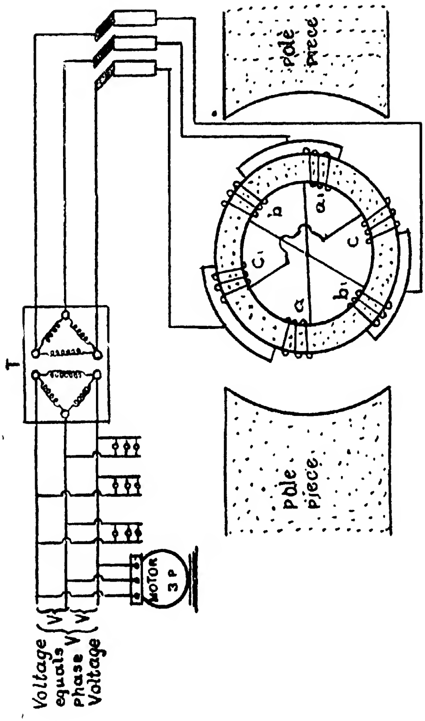


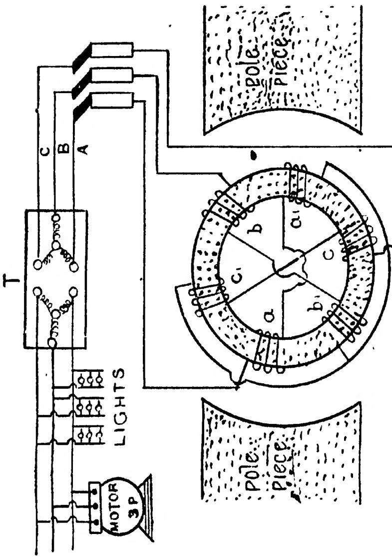
Fig. 6.27

$$E_L = \sqrt{3}\, E_P.$$



Three-phase Δ-connected alternator and its receiving circuit.

Fig. 6.28

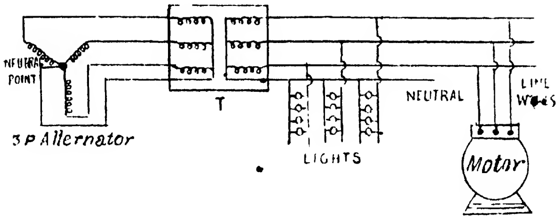


Three-phase star-connected alternator and its receiving circuit.

Fig. 6.29.

THREE-PHASE CIRCUITS.—STAR CONNECTION WITH FOUR LINE WIRES:—The voltage between any line wire and the common return is equal to the phase voltage. The phase currents are not equal, the out-of-balance current flows down the fourth wire. So that when the loads are equal, no fourth wire is necessary. By the use of the fourth wire, lamps may be supplied at 250 volts, and at the same time power may be supplied for motors etc. at $250 \times \sqrt{3}$, i. e. 430 volts.

This causes great saving in copper.



Threephase four wire alternator and its receiving circuit

Fig. 6.30.

170. Three-phase Mesh or Delta Connection. The voltage between any two line wires is equal to the phase voltage. The current flowing in any line wire is the resultant of the current flowing in the two phases whose ends are connected to the line at the common point of junction. That is,

$$E_L = E_p.$$

$$I_L = \sqrt{3} \times I_p.$$

171. Copper Loss in the Armatures of Alternators.— (A. RUCKGABER.) In the armature of any alternating-current, single or polyphase dynamo or motor, the copper loss is always equivalent to $\frac{I^2 R}{2}$ in which I is the total amperes, and R the total resistance between leads of a phase, usually taken as an average of the measurements of the armature resistance of each phase.

Let R_t = average terminal resistance,

r = resistance per phase,

I = total amperes = watts ÷ volts,

I_1 = amperes per lead,

i = amperes per phase, in winding,

then in the case of:

(a) Single-phase:—

$$I = I_1 = i.$$

$$R_t = r\ ;\ I_1^2 \cdot R_t \text{ loss} = I^2 R_t.$$

The copper loss $= I^2 R_t$.

(b) Two-phase independent windings (see Fig. 6·17.):—

$$I = \frac{P}{E} = \frac{\text{watts}}{\text{volts}}.$$

$$I_1 = \frac{I}{2}.$$

The copper loss $= 2. I_1^2 R_t$

$$= 2. \frac{I^2}{4}. R_t = \frac{I^2 R_t}{2}.$$

(c) Two-phase windings connected in series (Fig.6.18.):—

$$I = \frac{P}{E};\ I_1 = \frac{I}{2};\ i = \frac{I_1}{\sqrt{2}} = \frac{I}{2\sqrt{2}}.$$

The copper loss $= 4i^2 r$,

$$= \frac{4\ I^2\ r}{8} = \frac{I^2\ r}{2}.$$

R_t is measured from A to A and B to B, the average of these two being taken for the value of R_t; so that

$$R_t = \frac{(r+r)\ (r+r)}{4r} = r.$$

$$\therefore \text{ The copper loss} = \frac{I^2\ R_t}{2}.$$

(d) Resistance and Reactance of a Three-phase System.—The resistance per phase in a star-or Y-connected system with similar phases is half the resistance between any pair of terminals.

Let I = line current,

R_t = resistance between terminals in ohms.

The total resistance loss $= 3\ I^2\ \frac{R}{2}$ watts.

The total power $= \sqrt{3}\ r\ I\ \cos\phi$; thus $\sqrt{3}\ I$ is called the EQUIVALENT CURRENT.

Adopting this terminology—

$$\text{The Resistance Loss} = (\text{Equivalent Current})^2 \times \frac{\text{Terminal Resistance}}{2}.$$

(e) In a delta-connected system there are two paths in parallel between any pair of terminals, one through a single phase of resistance R, and the other through two phases in series of the total resistance 2 R.

$$\text{Thus, } \frac{1}{R_t} = \frac{1}{R} + \frac{1}{2R} = \frac{2R+R}{R \times 2R}.$$

$$\therefore R_t = \frac{R \times 2\ R}{R+2R} = \frac{2}{3}\ R.$$

The current per phase $= \frac{1}{\sqrt{3}} \times$ line current.

$\therefore$ Total resistance loss $= 3 \times \left(\frac{I}{\sqrt{3}}\right)^2 R$ watts,

$= I^2 R$ watts $= \frac{3}{2} I^2 . R$ watts,

$= (\text{Equivalent Current})^2 \times \frac{\text{Terminal Resistance}}{2}$.

We see that we get the same expression in both cases, and thus it makes no difference if the system is Δ- or Y-connected. Similar reasoning holds good also in the case of Reactances of the three-phase system.

Example 7. If the resistance between the terminals of a balanced three-phase load is 4 ohms, and the current in each line is 150 amperes, find the total resistance loss.

What is the resistance of each phase if the connections are (1) Y and (2) Δ ?

If 150 K. V. A. are supplied what are the values of the impedance and the reactance between the terminals ?

Solution:—

Total resistance loss $= (\sqrt{3}\ I)^2 \times \frac{4}{2}$,

$= 3 \times 150 \times 150 \times \frac{4}{2}$ watts,

$= 135000$ watts $= 135$ Kw.

Resistance of each phase if Y-connected $= \frac{4}{2} = 2$ ohms.

" " " " " Δ " $= \frac{4 \times 3}{2} = 6$ ohms.

Now, Kw $=$ K. V. A. $\cos\phi$; or $135 = 150 \cos\phi$.

$$\therefore \cos\phi = \frac{135}{150} = \frac{R}{Z} = \frac{4}{Z}.$$

$$\therefore Z = 4.44 \text{ ohms.}$$

$$X = \sqrt{(4.44)^2 - (4)^2}$$
$$= 1.93 \text{ ohms.}$$

Example 8. The three windings or phases of a three-phase induction motor are Y-connected to three-phase mains. The voltage between mains is 1000, and each main delivers 40 amps. to the motor. It is required to find the current in each phase of the motor, and the electromotive force acting on each phase of the motor.

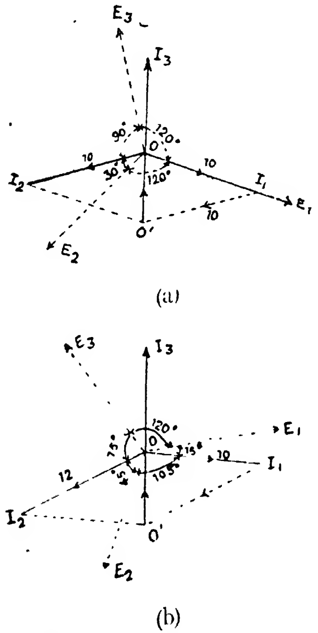
Solution:—

Since the windings are Y-connected, the current in each is the same as the current in each main, namely, 40 amperes; and the electromotive force acting on each phase of the motor winding is $\frac{1000}{\sqrt{3}}$, or 57.4 volts.

The power input is $P = \sqrt{3} \times 1000 \times 40 = 69280$ watts,
$= 69.28$ kw.

Example 9. The currents in two of the phases of a star-connected system are respectively (a) 10 amperes

in phase, and 10 amperes lagging 30°; (b) 10 amperes lagging 15°, and 12 amperes lagging 45°. Find the current in the third phase of the system in each case.



Currents in unbalanced Y-connected system.

Fig. 6.31

Solution:—

In Fig. 6.31, let E_1, E_2, E_3 represent the three equal e. m. fs. of the system, which are 120° out of phase with each other; I_1, I_2 the currents in two of the phases. Then, the current in the third phase, I_3, is the vector sum of I_1 and I_2, and is given by $O'OI_3$, or, OI_3 where $O'OI_3$ is a straight line, and $OI_3 = O'O$.

(a) $\angle I_1 OI_2 = 150°$, $\therefore \angle OI_1 O' = 30°$.

$$\therefore \text{ Current in the third phase, } I_3 = 2 \times 10 \times \sin\frac{30°}{2},$$

$$= 5.18 \text{ amps.}$$

$\angle I_1\, OO' = 75°$, $\quad \therefore \angle I_1\, O\, I_3 = 105°$.

$\therefore I_3$ lags 15° behind its e. m. f. E_3.

(b) $\angle I_1\, OI_2 = 150°$, $\quad \therefore \angle OI_1O' = 30°$.

Component of O′O along OI_1

$$= 10 - 12 \cos 30° = -\cdot 39 \text{ amp.}$$

Component of O′O perpendicular to OI_1

$$= 12 \sin 30° = 6 \text{ amps.}$$

$\therefore$ Current in the third phase,

$$I_3 = \sqrt{6^2 + (.39)^2} = 6.01 \text{ amps.}$$

$$\angle I_1OI_3 = \tan^{-1}\frac{6}{.39} = 86° \; 17'.$$

$$\therefore \angle E_1OI_3 = 71° \; 17'.$$

$\therefore I_3$ lags 48° 43′ behind its e. m. f E_3.

Example 10. A star-connected 12-pole three-phase machine revolving at 50 cycles per minute, requires a magnetic flux 8 mega-lines per pole. The armature contains one slot per pole and phase, and each slot contains 40 conductors. If all these conductors are connected in series, what will be the effective E. M. F. per circuit, and what the effective E. M. F. between the terminals of the machine ?

Solution :—

Number of slots in armature = 12.

$\therefore$,, conductors ,, $= 12 \times 40$.

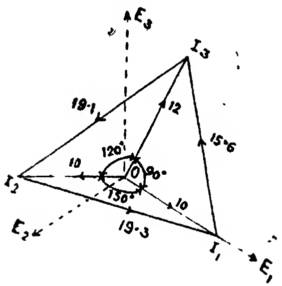
Thus there are only 240 turns in series.

Therefore, the effective E. M. F. is

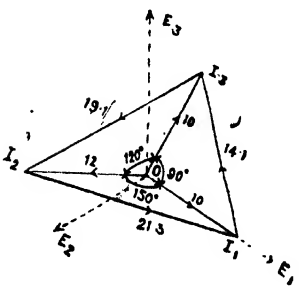
$$E = \sqrt{2}\, \pi f n \phi,$$
$$= 4.44 \times 50 \times 240 \times 8 \times 10^6 \times 10^{-8} \text{ volts},$$
$$= 4.44 \times .5 \times 240 \times 8 \text{ volts},$$
$$= 4262 \text{ volts per circuit.}$$

Now, the E. M. F. between the terminals of a star-connected three-phase machine is the resultant of the E. M. Fs. of the two phases, which differ by 60°, and is thus 2 sin 60°, or $\sqrt{3}$ times that of one phase. Hence, the terminal voltage,

$$E' = \sqrt{3}E = \sqrt{3} \times 4262,$$
$$= 7382 \text{ volts effective.}$$



(a)



(b)

Currents in unbalanced Δ—connected system.

Fig. 6.32.

Example 11. In a three-phase delta-connected system the currents in the phases are respectively 10 amperes in phase with voltage, 10 amperes lagging 30°, and 12 amperes lagging 30°. Find (a) the current in each lead, (b) the current in each lead if the 2nd. and 3rd. loads are interchanged.

Solution :—

(a) In Fig. 6.32. (a), E_1, E_2, E_3 represent the three

equal e. m. fs. of the phases at angles of 120° to one another, and I_1, I_2, I_3 the currents in the three phases.

The current in the lead connecting I_1, I_3

$$=\sqrt{(10)^2+(12)^2}=15.62 \text{ amps.},$$

leading I_3 by an angle $\tan^{-1}\frac{10}{12}=39°\ 49'$, i. e. leading E_3 by $9°\ 49'$.

The current in the lead connecting I_3, I_2 is found by resolving I_3 along and perpendicular to I_2.

Component along $I_2=-12\cos 60°=-6$ amps.

Component perpendicular to $I_2=12\sin 60°$,

$=10.39$ amps.

$\therefore$ Current in the lead$=\sqrt{(10.39)^2+(10+6)^2}$,

$=19{\cdot}1$ amps.

leading I_2 by an angle $\tan^{-1}\frac{10.39}{16}=33°$, i.e. leading E_3 by 3°.

The current in the lead connecting I_2, I_1

$$=\ 2\times 10\times\sin\left(\frac{150}{2}\right)=19.32 \text{ amps},$$

leading I_1 by 15°, i. e. leading E_1 by 15°.

(b) With reference to Fig. 6.32 (b), the current in the lead connecting I_1, $I_3=\sqrt{(10)^2+(10)^2}=14.14$ amps. leading I_3 by 45°, i. e. leading E_3 by 15°.

The current in the lead connecting I_3, I_2 is found by combining its components along and perpendicular to I_2.

Component along $I_2 = 12 + 10 \cos 60° = 17$ amps.

Component perpendicular to $I_2 = 10 \sin 60°$,

$= 8.66$ amps.

$\therefore$ Current in the lead $= \sqrt{(17)^2 + (8.66)^2} = 19.1$ amp.,

leading I_2 by an angle $\tan^{-1} \dfrac{8.66}{17} = 27°$, i. e. lagging E_2 by 3°. [Compare the magnitude and phase with (a)] Similarly, the current in the lead connecting I_2, I_1 is found by combining :—

Component along $I_1 = 10 + 12 \cos 30° = 20.39$ amp.

Component perpendicular to $I_1 = 12 \sin 30° = 6$ amp.

$\therefore$ Current in the lead $= \sqrt{(20.39)^2 + (6)^2}$,

$= 21.25$ amps.,

leading E_1 by an angle $\tan^{-1} \dfrac{6}{20.39} = 16°24'$.

Example 12. The three phases of the induction motor of Ex. 8, are Δ-connected to three-phase mains. The voltage between mains is 577.4, and the current in each main is 69.3 amperes. It is required to find the current in each phase of the motor, and the electromotive force acting on each phase of the motor.

Solution :—

Since the windings are Δ-connected, the electromotive forces acting on each phase is the same as the voltage between the mains, namely, 577.4 volts; and

the current in each phase of the motor is $\frac{69.3}{\sqrt{3}}$, or 40 amperes.

The power input is $P=\sqrt{3}\times 577.4\times 69.3$
$=69280$ watts $=69.28$ Kw.,

the same as in Ex. 8.

172. Summary of Electromotive Force and Current Relations for Δ-and Y-Connections:—

Let V_l be the electromotive force between mains of a three-phase system, and I_l the current in each main; then, for three receiving circuits, the data is as given in the Table below.

Δ-and Y-Connection Data in Receiving Circuits.

Current or Voltage	Δ	Y
Line current	I_l	I_l
Line voltage	V_l	V_l
Phase current	$I_p=\frac{I_l}{\sqrt{3}}$	$I_p=I_l$
Phase voltage	$V_p=V_l$	$V_p=\frac{V_l}{\sqrt{3}}$
Voltage to neutral	$V_n=\frac{V_l}{\sqrt{3}}$	$V_n=V_p$
Total Volt-amperes.	$3\,V_p\,I_p=\sqrt{3}\,V_lI_l$	$3V_pI_p=\sqrt{3}\,V_lI_l$.

The permissible power output or rating of a three-phase alternator is the same whether its armature windings are Y-connected or Δ-connected.

The power output of a three-phase generator is $\sqrt{3} \times$ electromotive force between mains $\times$ current in one main $\times$ power factor of the receiving circuits.

Any Δ-connected motor or generator is considered as equivalent to a Y- connected generator or motor in which—

$$E_y = \frac{E_\Delta}{\sqrt{3}}, \quad R_y = \frac{R_\Delta}{3}, \quad X_y = \frac{X_\Delta}{3}.$$

where E_y, R_y and X_y are the e.m.f, resistance and reactance per phase of the Y- connected machine equivalent to the e. m. f., resistance and reactance of the Δ-connected machine.

Each of the line wires is in series with a corresponding phase of the equivalent Y-connected machine.

The voltages thus calculated are the voltages to neutrals, and the currents are line currents. To find the line voltage multiply the calculated voltage by $\sqrt{3}$; similarly, to find the actual phase current in the Δ-connected generator or load, divide the calculated current by $\sqrt{3}$. (H. Pender).

Example 13. A three phase Y-connected generator delivers power to a balanced Δ-connected load over a transmission line. The resistance of each line wire is 0.3 ohm and the reactance of each line wire is 0.5 ohm.

The potential differences at the terminals of the generator and the load are respectively 500 volts and 400 volts. The power factor of the load is 80 % lagging. Calculate (1) the line current, (2) the power delivered to the load (3) the power output of the generator, (4) the power factor of the generator, and (5) the efficiency of the transmission line at this load. (H. Pender.)

Solution :—

The voltage per phase of the Y-connected generator $= \frac{500}{\sqrt{3}}$. The Δ-connected load can be considered as equivalent to a Y-connected load of which the phase voltage

$$E_Y = \frac{E_\Delta}{\sqrt{3}}.$$

Hence, the voltage per phase of the equivalent Y-connected load $= \frac{400}{\sqrt{3}}$.

Thus, the voltage from line to neutral at the generator $= \frac{500}{\sqrt{3}}$.

The voltage from line to neutral at the generator load

$$= \frac{400}{\sqrt{3}}.$$

(1) Let E denote the line voltage at the load, E_0 that at the generator, and I the current in the line lagging by an angle ϕ. Then,

$$E = \frac{400}{\sqrt{3}},\ E_0 = \frac{500}{\sqrt{3}},\ \cos\phi = .8, \text{ and } \sin\phi = .6.$$

With reference to Fig. 6.33, E_0 is evidently the resultant of E and E_z the impedance drop in the line. Now,

Component of E_R (resistance drop) along $E = .3 \times .8 \times I$.

,, E_X (reactance drop) ,, $= .5 \times .6 \times I$.

,, E_R perpendicular to E $= -.3 \times .6 \times I$.

,, E_X ,, ,, $= .5 \times .8 \times I$.

$\therefore$ Total component along $E = \left(\frac{400}{\sqrt{3}} + .24\ I + .3I\right)$ volts.

,, ,, perp. $E = (-.18I + .4I)$ volts.

$$\therefore \left(\frac{500}{\sqrt{3}}\right)^2 = \left\{\frac{400}{\sqrt{3}} + (.24 + .3)\ I\right\}^2 + \left\{(-.18 + .4)I\right\}^2$$

or, $.34\ I^2 + 249\ .4I - 30{,}000 = 0.$

$$\therefore \quad I = \frac{-249.4 \pm \sqrt{(249.4)^2 + 4 \times 34 \times 300}}{2 \times .34},$$

$$= \frac{-249.4 \pm 320.93}{.63},$$

$$= 105.2 \text{ amps.}$$

(taking the upper sign, since the lower sign gives a negative answer).

(2) The power delivered to the load

$$= \frac{400}{\sqrt{3}} \times 105.2 \times .8 \times 3,$$

$$= 58.3 \text{ kilowatts.}$$

(3) The power lost in the

$$\text{line} = 3R\,I^2 \text{ watts},$$
$$= 3 \times .3 \times (105.2)^2 \text{ watts},$$
$$= 9.96 \text{ kilowatts}.$$

∴ The power output of the

$$\text{generator} = 58.3 + 9.96,$$
$$= 68.3 \text{ kilowatts}.$$

(4) The power factor of the generator

$$= \frac{68.3}{\frac{500}{\sqrt{3}} \times 105.2 \times 3} \times 100 = 75\ \%.$$

(5) The efficiency of the transmission line

$$= \frac{58.3}{68.3} = 85.4\ \%.$$

Example 14. Energy is supplied from a generating station to a substation 6 miles away at a rate of 2000 kilowatts. The system is a balanced three-phase system operating at a frequency of 50 cycles. The transmission line consists of No. 19/.083 strand copper wires spaced 30 inches between centres. Find (1) the voltage between wires at the generating station when the voltage between wires at the substation is 6000 volts, the power factor at the substation being 80 per cent, with the current LAGGING, (2) how much power is lost in the transmission line, and (3) what is the power factor at the generating station. The electrostatic capacity of the line may be neglected.

Solution:—

The current per wire is

$$I = \frac{2000 \times 1000}{\sqrt{3} \times 6000 \times .8} = 241 \text{ amps.}$$

The voltage to neutral at the substation is

$$E_n = \frac{6000}{\sqrt{3}} = 3460 \text{ volts.}$$

The component of this voltage in phase with the line current is $0.8 \times 3460 = 2770$ volts. The component $90°$ ahead of the line current is $0.6 \times 3460 = 2080$ volts (since $\cos\phi = 0.8$, and $\sin\phi = 0.6$).

Now, the resistance per mile of a No. 19/.083 wire $= 0.432$ ohm; and its reactance per mile at 50 cycles $= 0.532$ ohm.

Therefore, the total resistance of each wire $= 2.592$ ohms; and the total reactance of each wire $= 3.192$ ohms.

Hence, the resistance drop in each wire $= 2.592 \times 241 = 625$ volts, and is in phase with the line current; and the reactance drop in each wire $= 3.192 \times 241 = 770$ volts, and is $90°$ ahead of the line current.

Thus, at the generator end the voltage to neutral in phase with the line current $= 2770 + 625 = 3395$ volts; and the voltage to neutral $90°$ ahead of the current $= 2080 + 770 = 2850$ volts. Hence the resultant voltage to neutral at the generator end

$$= \sqrt{(3395)^2 + (2850)^2} = 4432.6 \text{ volts.}$$

Therefore, the line voltage at the generating station is

$$E_l = \sqrt{3} \times 4432.6 = 7677 \text{ volts.}$$

The power lost in the line $= 3\ R\ I^2$, where R is the total resistance of each wire, and I the line current. Hence the power lost in the line $= 3 \times 2.592 \times (241)^2$ watts,

$$= 450 \text{ kilowatts.}$$

The total power delivered to the line and substation is then 2450 kilowatts. Hence, the power factor at

at the generating station $= \dfrac{2450,000}{\sqrt{3} \times 7677 \times 241}$,

$$= \frac{2450000}{3132470} = 78 \text{ per cent.}$$

173. Comparison of Star and Delta Connections :—

1. In a Y-connection $E_p = \dfrac{E_l}{\sqrt{3}}$, but in a Δ-connection $E_p = E_l$. Therefore the Y-connection requires fewer turns per phase than the Δ-type, and so it is cheaper. In a star-connection there is a neutral point to which a ground wire, a meter, or a load may be connected.

2. By the use of a common or neutral main the Y-type can distribute power at a higher voltage, say $\sqrt{3} \times 250 = 430$ volts, while supplying the lights at the lower voltage of 250 ; whereas the Δ-type can use

only 250 volts for such distribution. Thereby a great saving is made in the copper wire used as mains in the Y-connection.

3. The neutral point of the star-wound alternator can be earthed when the P. D. between each line conductor and earth is .58 of the line voltage. The Δ-connection cannot be earthed in this manner, so that if in this case one of the lines becomes earthed through a fault, the voltage between each of the remaining conductors and earth would be equal to the line voltage, and is $\sqrt{3}$ times greater than the voltage to earth in the case of the Y-connection with earthed neutral. The insulating material would be under a greater stress, and there is greater liability to breakdown.

4. Circulating currents cannot flow in the windings of a star-connected system.

5. The electromotive force wave of a star-connected system is more nearly harmonic than that of a Δ-connected winding.

6. The Δ-connection works more satisfactorily in transformers, and is the only connection suitable for such machines as the rotary converters.

174. Comparison of Single-phase and Three-phase Alternators and Motors.—

1. The three-phase machines give about 50 per cent greater output for a given quantity of material, and consequently they are less costly.

2. Three-phase alternators are more satisfactory for the operation of poly-phase motors which have better operating characteristics than single-phase motors.

3. Poly-phase machines are more efficient than single-phase machines.

4. Simplicity in generation, transmission and receiving apparatus are the striking factors of the single-phase system.

Note that an alternator has to be excited with direct current. It cannot convenietly be self-exciting, the exciting current is generally supplied from a small direct current generator called an EXCITER, and is let into the field coils through brushes which bear on slip rings insulated from the shaft of the alternator. The voltage of the exciter is independent of that of the alternator, and is generally chosen to be 110 to 120 volts. The exciting curent may be larger than the full-load current of the alternator.

Example 15. A 1000 Kw. single-phase alternator operates at 20,000 volts at unity power factor. Find the current at full-load. If the exciter voltage is 110 and the excitation loss 2 per cent., find the output of the exciter and also the exciting current.

Solution:—

To find the current:—

$$P = EI \cos \phi,$$

$$10{,}00{,}000 = 20{,}000 \times I,$$

$$\therefore I = 50 \text{ amperes.}$$

To find the exciting current :—

The exciter output $= 2$ percent of 1000 Kw.
$= 20$ Kw.

$$\therefore \text{The exciting current} = \frac{20 \times 1000}{110} = 182 \text{ amp. nearly.}$$

175 Armature Reactance in an Alternator.—An alternator may be taken to be a circuit with a resistance R_g and a reactance X_g, where the value of X_g is generally from four to ten times the value of R_g. Hence when an alternator is operating under normal conditions fully excited, generating voltage and supplying current, a part of this voltage is used up to overcome the reactance and a portion to overcome the resistance. the terminal voltage E_t is obtained by deducting IR_g and IX_g as vectors from E_0, the no-load valtage.

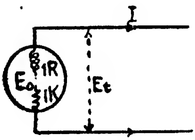


Fig. 6.34
Diagrammatic Representation of Alternator

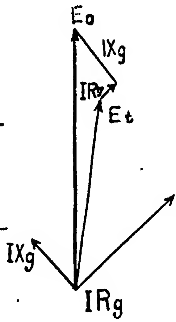


Fig. 6.35
Vector diagram for an Alternator.

176. Vector Diagram at Full-load.—

Let E_0 = the voltage generated by the alternator at no-load,

I = current passing to the external circuit.

Armature resistance drop $= IR_g$, and is in phase with the current.

2. Three-phase alternators are more satisfactory for the operation of poly-phase motors which have better operating characteristics than single-phase motors.

3. Poly-phase machines are more efficient than single-phase machines.

4. Simplicity in generation, transmission and receiving apparatus are the striking factors of the single-phase system.

Note that an alternator has to be excited with direct current. It cannot convenietly be self-exciting, the exciting current is generally supplied from a small direct current generator called an EXCITER, and is let into the field coils through brushes which bear on slip rings insulated from the shaft of the alternator. The voltage of the exciter is independent of that of the alternator, and is generally chosen to be 110 to 120 volts. The exciting current may be larger than the full-load current of the alternator.

Example 15. A 1000 Kw. single-phase alternator operates at 20,000 volts at unity power factor. Find the current at full-load. If the exciter voltage is 110 and the excitation loss 2 per cent., find the output of the exciter and also the exciting current.

Solution:—

To find the current:—

$$P = EI \cos \phi,$$

$$10,00,000 = 20,000 \times I,$$

$$\therefore I = 50 \text{ amperes.}$$

Example 16. A certain alternator gives 1100 volts between its collector rings at full-load current and full-load field excitation. When the current output is decreased to zero by opening the main switch, having the field excitation and speed unchanged, the terminal electromotive force rises to 1170 volts. Determine the per cent regulation of the alternator.

Solution:—

$$\text{Per cent regulation} = \frac{1170-1100}{1100} \times 100,$$

$$= \frac{70}{1100} \times 100 = 6{\cdot}36 \text{ per cent.}$$

178. E. M. F. Method of Calculating Regulation of Alternators.—

If E_a = the electromotive force induced in the armature,

E'_a = the voltage drop in the armature,

E = terminal E. M. F. of the generator,

R_a = the resistance of armature circuit,

X = total reactance of the circuit,

X_a = synchronous reactance due to the combined effect of the inductance of the armature winding, and the flux set up by the current carrying armature conductors.

I_a = the current flowing in the armature,

Cos ϕ = the power factor of the load circuit,

then, $E^a = I_a \sqrt{R^2+X^2}$ (this is assumed.)

$$= \sqrt{(E \cos\phi + R_a I_a)^2 + (E \sin\phi + X_a I_a)^2}.$$

and $E'_a = \sqrt{(R_a I_a)^2 + (X_a I_a)^2}.$

$$\therefore \quad Z_a = \sqrt{R_a^2 + X_a^2}.$$
$$X_a = \sqrt{Z_a^2 - R_a^2}.$$

The open circuit and zero power factor—saturation curves show that the synchronous reactance of an alternator armature is not constant, but decreases as the excitation decreases, and is affected by the power factor of the load circuit.

Example 17. Find the regulation of a single phase alternator having the following data:—

$E = 2200, \quad I_a = 100, \quad R_a = 1, \quad X_a = 10.$

Find the per cent regulation when the power factor of the load circuit is unity and also when the power factor is 80 per cent.

Solution:—

$$E_a = \sqrt{(E_a \cos\phi + R_a I_a)^2 + (E \sin\phi + X_a I_a)^2}$$

Taking $\cos\phi = 1$,

$$\therefore \quad E_a = \sqrt{(2200 + 100)^2 + (1000)^2}$$
$$= 2508 \text{ volts nearly.}$$

$$\text{Regulation} = \frac{(2508 - 2200)}{2200} = 14 \text{ per cent.}$$

At 80 per cent power factor with lagging current,

$$E_a = \sqrt{(E_a \cos\phi + I_a R_a)^2 + (E_a \sin\phi + I_a X_a)^2},$$
$$= \sqrt{(2200 \times 0.8 + 100)^2 + (2200 \times .6 + 1000)^2},$$
$$= 2973 \text{ volts.}$$

$$\therefore \text{Regulation} = \frac{2973 - 2200}{2200} = 35 \text{ per cent.}$$

Example 18 A three-phase Y-connncted alternator has an output of 200 amps, at 2300 volts. With a certain field excitation the no-load voltage between terminals was 2300 volts, and the current in each line on short-circuit was 500 amp. The resistance of each phase is 0.2 ohm. Find (1) the reactance per phase, (2) the regulation of the machine at full-load and 100 per cent power factor, the full-load voltage between terminals being 2300.

Solution :—

E = the E. M. F. of the generator per phase.

E_l = the terminal voltage at no-load $= 2300$ volts.

E_p = the voltage per phase at no-load,

$= 2300/\sqrt{3}$ volts $= 1328$ volts nearly.

$I_l =$ the line current on short-circuit $= 500$ amps.

$I_p =$ the current per phase on short-circuit $= 500$ amp.

$Z =$ the impedance per phase $= 1328/500 = 2.6$ ohms.

$X_g =$ the reactance per phase $= \sqrt{Z^2 - R_g^2}$,

$= \sqrt{2.6^2 - 0.2^2} = 2.59$ ohms.

To find the regulation at 100 per cent power factor we proceed as follows :—

Full-load current in line $= 200$ amps.

Full-load current per phase $= 200$ amps.

The resistance drop $I_p R_g$ per phase $= 200 \times .2$,

$= 40$ volts.

The reactance drop $I_p X_g$ per phase $= 200 \times 2.59$,

$= 518$ volts.

The full-load voltage between the terminals

$= 2300$ volts.

The full-load voltage per phase $=2300/\sqrt{3}$.
$=1328$ volts.

If $E_0=$ the voltage generated per phase by the generator at no-load, then

$$E_0=\sqrt{(1328+40)^2+518^2}=1413 \text{ volts nearly,}$$

Hence the no-load voltage between terminals

$=1413 \times \sqrt{3}$,

$=2447$ volts.

$$\therefore \text{ Regulation}=\frac{2447-2300}{2300}=5.1 \text{ per cent.}$$

Example 19 A three-phase delta-connected alternator has an output of 200 amps. at 2300 volts. With a particular field excitation the no-load voltage between terminals was 2300 volts, and the current in each line on short circuit was 500 amps. The resistance of each phase was 0.6 ohm. Find (1) the reactance per phase, (2) the regulation of the machine at full-load and 100 per cent. power factor, the full-load voltage between terminals being 2300.

Solution:—

$E_l=$ the terminal voltage at no-load$=2300$ volts.

$E_p=$ the voltage per phase at no-load$=2300$ volts.

I_l $=$ the line current on short-circuit$=500$ amps.

$I_p=$ the current per phase on short-circuit,

$$=\frac{500}{\sqrt{3}}, =288.6 \text{ amps.}$$

$$Z = \text{the impedance per phase} = \frac{2300}{288.6},$$

$$= 8 \text{ ohms nearly.}$$

$$X_g = \text{the reactance per phase} = \sqrt{8^2 - 0.6^2},$$

$$= 8 \text{ ohms nearly.}$$

To find the regulation at 100 per cent. power factor we proceed as follows:—

Full-load current in the line = 200 amps.

$$\text{Full-load current per phase} = \frac{200}{\sqrt{3}} = 115 \text{ amps.}$$

$$\text{The resistance drop } I_p R_r \text{ per phase} = 115 \times 0.6,$$

$$= 69 \text{ volts.}$$

$$\text{The reactance drop } I_p X_g \text{ per phase} = 115 \times 8,$$

$$= 920 \text{ volts.}$$

The full-load voltage between terminals = 2300 volts.

The full-load voltage per phase = 2300 volts.

The no-load voltage at the generator,

$$E_1 = \sqrt{(2300 + 69)^2 + 920^2} = 2541 \text{ volts.}$$

$$\therefore \text{Regulation} = \frac{2541 - 2300}{2300} = 10.5 \text{ per cent.}$$

179. Regulation of Alternators by the Magnetomotive Force Method—Assumption:—The voltage induced in the armature windings is due to quadrature fields, one equal to that required to produce the total non-inductive drop, the other equal to that required to produce the total wattless component of the Electromotive Force.

Find the field current I_1 (from the saturation curve) required to induce in the armature winding an E. M. F. equal to the total non-inductive drop ($E \cos \phi + R_a I_a$) of the circuit at rated load; and find $I_{f'}$ to induce in the same winding an E. M. F. equal to the reactive drop. ($E \sin \phi + X_a I_a$) of the circuit at rated load. If I_f = the field current required to produce simultaneously the terminal voltage E and the current I_a in a circuit the power factor of which is $\cos \phi$, then

$$I_a = \sqrt{(I_1)^2 + (I_{f'})^2}.$$

From the saturation curve find the electromotive force E_a induced in the armature winding when the current I_f flows in the field windings. Then

$$\text{Per cent regulation} = \frac{E_a - E}{E} \times 100.$$

180 To Improve the Regulation of an Alternator.—

(1) Decrease the impedance drop by providing a larger flux per pole of the field magnet system.

(2) Keep a stiff field and provide a larger field magnet system. This is expensive.

181. Alternators in Series.—Owing to the synchronizing tendency of alternators, for which it is possible to run them in parallel, they get out of step and become opposed to each other when they are attempted to run in series. They can run in series only when their shafts are rigidly connected, so that they must run exactly in phase, and thus add their waves of E. M. F.

instead of opposing each other. Practically this is seldom taken recourse to.

182. Parallel Operation of Alternators.—Direct current machines may run in parallel only if the voltage is the same in all the machines.

Any alternator can be operated in parallel or synchronized with any other alternator. A single-phase machine can be synchronized with one phase of a polyphase machine. A quarter phase machine can be operated in parallel with a three-phase machine by synchronizing one phase of the former with one phase of the latter.

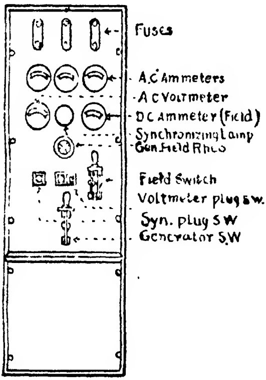


Fig. 6·3

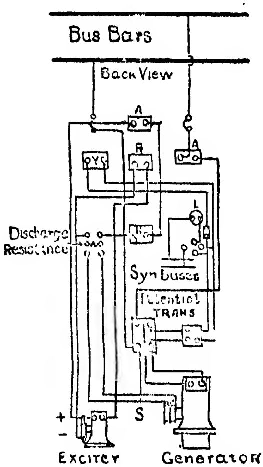


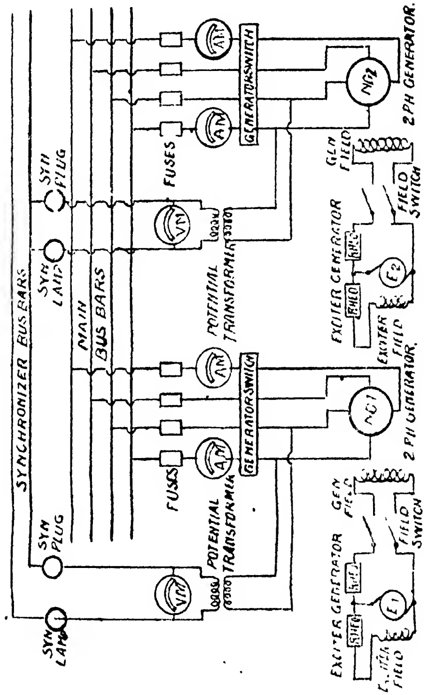
Fig. 6·40

Front View and Complete Wiring Diagram of Switch Board Panel for Single Phase Alternators operating in Parallel

183. To Start an Alternator in Parallel with Others.—

(a) Bring the exciter and generator to speed. Adjust the exciter voltage and close the field switch, the generator field resistance being all in.

(b) Adjust the generator field resistance so that the generator voltage is the same as the bus bar voltage.

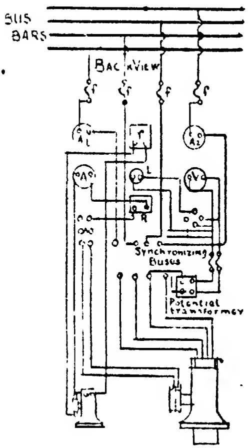


Complete Wiring Diagram for Two Phase Alternators running in Parallel.

Fig. 6·41

(c) Synchronize; close the main switch.

(d) Adjust the field rheostat until cross-currents are a minimum, and adjust the generators of the prime movers so that the load may be properly distributed between the operating units in proportion to their capacities.



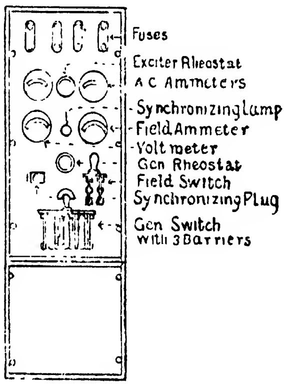


Fig. 6·43

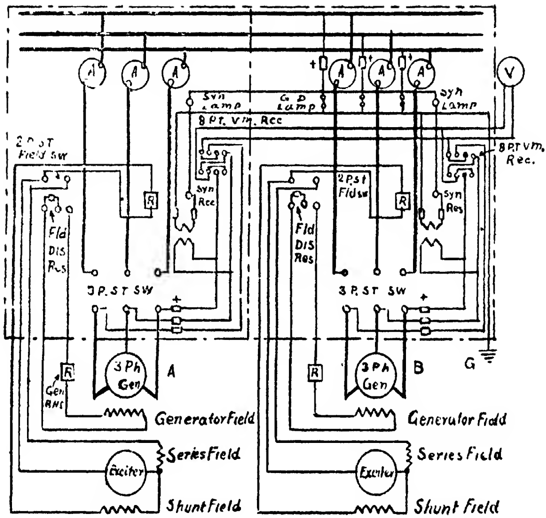
Fig. 6·42

Front View and Complete Wiring Diagram of Switch Board Panel for Two-Phase Alternator operated in Parallel with Other Two-Phase Machines.

184. Alternators to Run in Parallel must have—

(1) Equal terminal voltage secured by adjustment of the field current.

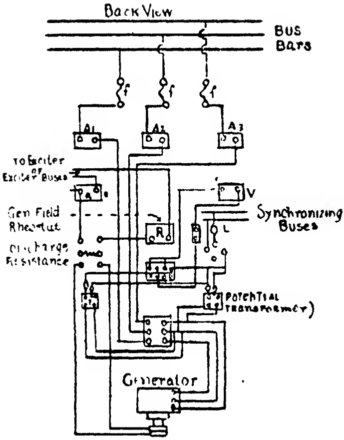
(2) The speed adjusted to have the same frequency, and be constant for an appreciable interval of time.

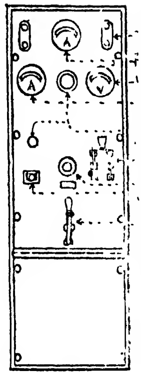


Two Th ee Phase Alternators running in Parallel.

Fig. 6.441

To face page 317.





Fuses. . Field Ammeter. Voltmeter. A.C.Ammeter. Synchroniz-ing Lamp. Exciter Rheostat. Field Switch. Generator Rheostat. Synchronizing Plug. Generator Switch.

Fig. 6·44. Fig. 6.45.

Front View and Complete Wiring Diagram for a Three-Phase Alternator Operated with other Three-Phase Machines.

(3) Their maximum positive values of electromotive force at the same moment, i. e., the same phase.

(4) Similar electromotive-force waves.

If the above conditions are fulfilled the bus-bar P. D. will be exactly counter-balanced by the equal and opposing E. M. F. of the machine, and no rush of current will occur on closing the switch around the local circuit formed by the armatures of the generators.

Further, it is not desirable to connect alternators in parallel if their prime movers have different speed characteristics.

185. Method of Synchronising Alternators.—Suppose A is carrying the load and it is desired to connect B to the system. Start B and regulate its field excitation and the speed of its prime mover until (1) The indication of its voltmeter is equal or slightly greater than the voltage across the bus-bars. Close the switch quickly at a time when the lamps are dark or the pointer is at zero in the Synchroscope. (2) The lamps connected in the synchronising arrangement, and in the case of high voltage system through proper transformer connections, become dark or better still when the pointer of the synchroscope is at zero, as indicated at the top of the instrument. The transformer used for this is as shown in the Fig 6.47

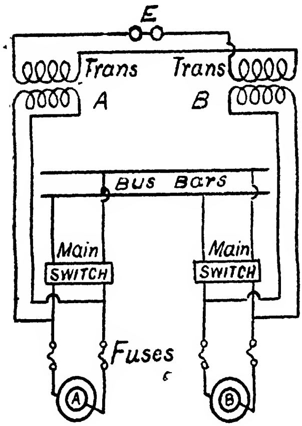


Fig. 6·46.
Wiring Diagram showing Method of Determination of Phase Relation by Synchronizor.

When the pressure of the alternator is high, a lamp cannot be used directly in the circuit. A number of lamps may be used in series for low pressures, but a step down transformer may be used as shown in Fig. 6.47

It is more convenient, if it has three limbs to have the connection so that one winding is connected to the incoming altrenator, the other is joined to the busbars, and the trird with a reduced number of turns carries the lamp.

Thus the core has three limbs; two carry similar windings—one A, A connected to the incoming alternator, the other B B connected to the busbars. The third limb has a winding with a suitably reduced number of turns, connected to the Synchronizing lamp L.

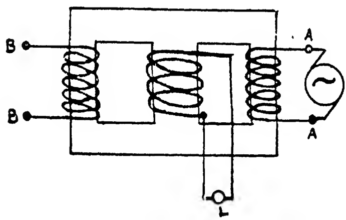


Fig. 6.47

PRINCIPLE :—Whenr the pessures impressed upon A A and B B are in phase, the fluxes in these are increased or decreased at the same time. The flux in the centre limb is the sum of the two, and so the E.M. F. is doubled and the lamp burns very brightly.

When the two pressures in A A and B B are exactly opposite in phase, practically no flux passes through the centre limb, and the lamp will be dark in proportion to the phase defference. This arrangement can be utilized for both bright and dark lamp synchronizing leg by reversing the connection of either A A or B B (but not both). A voltmeter may be joined in parallel to the lamp and the maximum or the zero reading of the

voltmeter will correspond to the instant of the equality of the phase.

186. Parallel Operation of Induction Genarator.—The speed of two induction machines operating in parallel may not have a constant ratio to each other. Such machines need not be synchronized, but simply brought up to approximately synchronous speed and the switch closed.

Knowing the speed and number of poles of the slower machine, say, we can express the frequency in cycles per minute; then counting the pulsations of the lamps we can determine the frequency in cycles per minute of the more quickly running machine.

The ratio of these may be expressed as a percentage, and this value taken off the diameter of the more quickly running machine.

In belt-driven alternators thickness of belt, and tightness of belt, and other conditions, such as surface of belt and pulley, which affect the slip of the belt, are all factors to be noted in speed regulation of such machines.

187. Hunting.—The rotating part of every synchronous machine acts like a pendulum tending to swing ahead and behind its synchronous position. The mass of armature and its flywheel acts like the mass of the pendulum, and the torque of the machine being proportional to the displacement corresponds to a spring or

gravity acting on the pendulum. Such a combination has an "electro-mechanical period" of its own, and if the frequency of this period is the same with any other pulsating force in the system, such as engines, "hunting" or surging may occur.

188. Hunting of Alternators.—In case of engine driven alternators, the angular velocity is not uniform, but is made up of a uniform angular velocity with a superimposed oscillation. The frequency of the generated E. M. F. rises and falls regularly. It describes the oscillatory character or periodic variation of speed of several alternators running in parallel. If it occurs beyond a certain value, the regulation of the machine becomes unstable, it falls out of step, and the huge currents passing through its armature cause its automatics to act and cut it off from the bars.

If one machine lags behind another, due to (1) sudden alteration in load, or (2) field excitation on the electrical side, or (3) irregularity in the turning moment of the engine, or (4) defective steam distribution on the mechanical side, its armature receives current from the other machines running in parallel with it, and driving it as a motor, tending to pull it into phase or step. Its speed increases until it is in correct phase, and attains afterwards a little greater speed than the others. It, in its turn, being accelerated, supplies current to the other machines, resulting in an alternate lagging and leading, retardation and acceleration, of the machines with respect to each other, or we find the phenomena

of hunting which is observed by (1) the variation in the sound given out by the machine, (2) rapid variation in the indications of the ammeter connected to the machine, which do not correspond in any way to the actual variations in the load on the bus-bars, but represent the currents flowing between the various machines themselves.

"Surging" means the rapid current variations whilst the hunting is in progress.

189. Remedies for Hunting.—(1) A dash-pot arrangement is sometimes fitted to the governor of the engine to prevent it responding to the slight variation in speed which may take place in one revolution, and thus commence this action.

(2) Fitting a heavier fly-wheel tends to prevent hunting by producing a more regular turning moment, but this remedy is not very efficient, and may render the hunting more pronounced if it does actually commence.

(3) Damp the oscillations electrically by the use of pole dampers. To produce this, the pole pieces of the alternator are surrounded with copper bands, or copper bars are embedded in holes drilled usually through the pole shoes. The ends of these bars are short-circuited, so as to form a kind of grid or squirrel cage, by copper plates fixed on the outsides of the pole shoes. When the field poles of the alternator are either accelerated or retarded with respect to the synchronous speed, lines of force cut the copper bars

and set up currents in them. The magnetic fields produced by such eddy currents always tend to stop the motion producing them. Hence they tend to damp the oscillations of the alternator and keep an uniform speed.

The eddy currents thus set up explain the fact that alternators with solid pole shoes do not tend to hunt as much as those with laminated shoe pieces.

The high and uniform speeds of steam-and water-driven turbo-generating sets render hunting in this type of machine far rarer than in those driven by reciprocating engines, particularly gas-engines, resulting under such circumstances in far more successful parallel operation of the alternators.

A driving torque is produced as in an inductive motor when the machine is running below synchronous speed, and this tends to speed it up; while a retarding torque is produced when the machine is running above synchronous speed, and this tends to slow it down. When the machine is running in synchronous speed there is no current in the squirrel cage. The torque due to the squirrel cage thus tends at all times to prevent oscillations in speed and therefore to prevent hunting.

A tendency to hunt is damped by solid pole pieces and bridges between poles.

190. Disconnecting Alternators Running in Parallel with Others and from the Bus-bars.—(1) Take off the load by decreasing the supply of steam either by the governor or the main steam valve.

(2) Adjust the resistance in the field current until the armature current is a minimum.

(3) When the load on the alternator is practically zero as indicated by the ammeter, then trip the main switch. It is usually sufficient to simply disconnect the machine from the bus-bars, and throw all the load on the remaining machine without having made any previous adjustment of the load or of the field current.

(4) The main switch should be opened always before the field or exciter switch opens the field circuit. Finally shut down the prime mover.

191. Method of Adjusting the Load after Paralleling.—(1) In DIRECT CURRENT MACHINES operating in parallel, the load distribution is done by the field rheostat regulation, for if the field excitation of one of the machines is increased, the voltage of the machine will be raised and it will take a larger portion of the load. But as the load is increased, the engine and generator slow down, the engine draws the additional amount of steam required for the additional load.

Compound dynamos may sometimes have different characteristics when they are of different makes and sizes; then equalizing bar is not sufficient. Shunt field regulating sometimes suffice to make the proper distribution of load, but usually a diverter has to be placed across the series winding of each machine, and for some positions of the diverter the parallel operation is most successfully obtained.

(2) In ALTERNATE CURRENT MACHINES, however, an increase in the field excitation in one machine increases its voltage, and at the same time makes the current in the machine lag further behind the voltage so as to maintain the load EI cos ϕ constant at the value corresponding to the steam supply. Hence, the load on the alternator must be regulated by the driver by admitting more or less steam to the driving engine or by an electric motor controlling the generator and operated from the switchboard. To share the load properly it is necessary that the governors on the general prime movers should give the same speed load Characteristics.

The field excitation affects the power factor of the load delivered by each machine, and the excitation of the alternators should be so adjusted that it delivers its load at the same power factor as the others.

If the regulation of the prime movers is not the same, the load is not divided proportionately between the alternators. The alternator connected to the prime mover of closer speed regulation takes more than its share of the heavy loads, and less under light loads. Thus too close speed regulation of prime movers is not desirable in parallel operation of alternators.

The division of load between two induction generators operating in parallel is proportional to the ratio of the deviation of the actual speeds of the rotors from the synchronous speed. The load division is adjusted as in the case of the synchronous machines by

manipulating the speed governing apparatus of the prime mover.

In order that two machines may divide the load properly, the engine should have the same per centage drop in speed between no-load and full-load.

192. Efficiency of Alternators.—The losses in an alternator and a direct current generator are the same, viz. armature copper loss, field excitation loss, and stray loss consisting of the hysteresis and eddy current loss in the; magnetic circuit. The friction and windage losses and the load losses are due to eddy current and hysteresis produced by the load current in the armature. The efficiency depends upon the power factor of the load. The efficiency of alternators and synchronous motors is usually so high that a direct determinaton by measuring the mechanical power and the electric power is less reliable than the method of adding the losses and the latter is therefore commonly used.

Example 20. A single-phase alternator has an output of 200 amperes at 2300 volts. The resistance of the armature winding as measured by direct current, is 0·3 ohm. Stray power loss = 20 Kw. Exciting current at full-load = 65 amp, at 100 per cent. power factor, = 80 amp., at 80 per cent. power factor. The excitation voltage is 120.

Find the efficiency of the alternator (a) when the power factor of the load is 100 per cent., and also (b) when 80 per cent; find also the horse power of the driving engine.

Solution :—

(a) At 100 per cent. power factor :—

The output $= 2300 \times 200 = 460$ Kw.

The stray loss $= 20$ Kw.

The excitation loss $= 65 \times 120 = 7{\cdot}8$ Kw.

The armature copper loss $= (200^2 \times 0{\cdot}3) = 12$ Kw.

Total loss $= 39{\cdot}8$ Kw.

$\therefore$ The input $= 499{\cdot}8$ Kw.

The horse power of the driving engine $= 670$ H. P.

The efficiency $= \frac{460}{499{\cdot}8} = 92$ per cent.

(2) At 80 per cent power factor :—

The output $= 2300 \times 200 \times {\cdot}8 = 368$ Kw.

The stray loss $= 20$ Kw.

The excitation loss $= 120 \times 80 = 9{\cdot}6$ Kw.

The armature copper loss $= (200^2 \times 0{\cdot}3) = 12$ Kw.

The total loss $= 41{\cdot}6$ Kw.

The input $= 409{\cdot}6$ Kw.

The horse power of the driving engine

$= 549$ H. P.

The efficiency $= \frac{368}{409{\cdot}6} = 89.8$ per cent.

$$\text{Efficiency} = \frac{368}{409.6} = 89.8 \text{ percent.}$$

Exercises.

1. What must be the speed of a 8-pole alternator to yield an E. M. F. of 50 cycles ?

Find the instantaneous current value in a circuit

in which a 25$\sim$ alternating current of 50.2 amperes flows, 5.0032 seconds after the completion of a cycle.

3. How many amperes flow in a circuit, when the instantaneous value of the current is 10 amperes, 30° after the beginning of a cycle ?

4. What is the frequency of an E. M. F. which assumes its effective value every ·02 second ?

5. What is the phase displacement between E and I, respectively of 220 volts and 10 amperes maximum value, when the power in the circuit is 620 watts ?

6. Three 50$\sim$ alternators, generating respectively 100, 80, and 50 volts, are connected to a circuit. What will be the value of the resulting pressure, and what will be its phase with respect to that of the 100 volts, if the phase difference between successive components is 60 ?

7. If the three E. M. F.s are impressed upon a circuit, what will be the resulting instantaneous voltage 3.012 seconds after the beginning of a cycle ?

8. In what respects does a three-phase alternator differ from a single-phase alternator ? What are the reasons that lead to the use of the former in preference to the latter ? Give diagrams illustrating windings and connections of a multipolar generator of each kind.

(Ord. A. C., 1900).

9. A 1000-kw. three-phase 50-period 6600-volt generator is to be designed to run at 500 r. p. m. Calculate :—

(a) The number of poles.

(b) The currents which will flow in the windings with a power factor of 0.8 with both delta and star connections.

(c) The true efficienency for the 3-phase winding at full load and 0.8 power factor, from the following data :—

Armature resistance between terminals: 0.6 ohm.

Field resistance: 0.5 ohm.

Field current: 150 amperes at full-load and power factor 0.8.

Iron loss: 15 kw.

Friction and windage: 7 kw.

(A. M. I. E. E. Exam., 1914).

10. Describe clearly what is meant by the "hunting" of synchronous electrical machinery, and enumerate the various conditions which may bring it about. Discuss fully the nature of the cross currents which may pass between an engine driven alternator and a distant synchronous motor, and explain their effect upon the losses of the system.

(Honours, 1st Paper, 1911).

11. Deduce a formula for the virtual electromotive force of an alternator, in terms of the number of conductors, flux per pole, and frequency, on the assumption that the electromotive force follows a sine law. Show by a vector diagram the relation between the terminal voltage on load and the electromotive force developed in the armature. (Ord., A. C., 1910).

12, The equation for the electromotive force E, generated in one circuit of an alternator, may be written

$$E = k \times f \times Z \times N \div 10^8;$$

where f is the frequency, Z the number of conductors in series in that circuit, N the flux of any pole (assumed all equal), and k a numerical coefficient, the value of which usually lies between the extreme values of 2 and 2. 4. Discuss the origin of this coefficient k, and point out the features of the design which determine whether its value will be high or low. (Honours, 1st Paper, 1900).

13. Describe, with accompanying sketch. some form of synchronizing gear for paralleling alternators. (A.M.I.E.E. Exam., 1914).

14. Describe the construction of a modern form of synchronizer, and explain in detail the principles upon which it is based. Indicate how such synchronizers can be arranged for automatically switching in the incoming alternator at the right moment; and state in what circumstances it would be profitable to employ snch automatic paralleling arrangements.

(Honours, 2nd Paper, 1909).

15. Make a diagram of connections for a switchboard suitable for two single-phase alternators, which are to be switched to the same bus-bars, and show the arrangements which are necessary to synchronise the machines before switching on to the bars.

(Final, 1st Paper, 1914).

16. Two similar alternators, separately driven by two good engines, are working in parallel on the same bus-bars. It is observed that they are not sharing the load equally. In what way or ways can they be made to take equal loads? In what way or ways can the whole of the load be thrown over from one to the other while they are still connected to the bus bars? What phase changes, if any, occur during the last-mentioned operation ? (Ord., A.C., 1908).

17. What are the effects cf a low power factor on the economical running of a power house ?

(A.M.I.E. E. Exam., 1914).

18. Explain why the terminal voltage of an alternating current geuerator varies with the amount and character of the load. Give numerical values for the voltage drop which may in practice be permitted.

(Gard II,, A.C., 1913).

19. Give some account of the armature reactions in alternators, and explain in what way the cross-magnetizing and the demagnetizing effects depend on the nature, as well as on the amount, of the load.

(Ord., A. C., 1910).

20. What is the object of armouring cables? Why must all the conductors of an alternating-current circuit lie within the same armouring ? (Grade II., A.C., 1913).

21. What are the causes of loss of power in a three-phase. cable? Would you use a three-core steel-armoured cable in preference to three single-core steel-

armoured cables, the core of each cable having the same section ? In which system would you expect to get the greater loss? In what respect does the core of an extra high tension cable differ from that of a low tension cable? (Ord., A.C., 1910).

Note.—In Alterneting Current Systems single conductors should not be run through metal pipe or tube. If the metal be of iron or steel, the inductance of the circuit would be increased and eddy current would be set up, thus resulting in considerable drop of pressure.

If the metal be non-magnetic, there is only eddy current effect and the inductance would be negligible. In both the above cases the conductors are heated and much energy is wasted.

In two cored cables there is only small eddy current and inductance effect, as the field of the current in one conductor is practically neutralized by the other.

In a three cored cable except the concentric ones, such effects are very small, for the algebaric sum of the currents and therefore their magnetic fields at any moment is zero.

A three core concentric cable is never used in alternating current system, as in such cables the inductive effects are not neutralized.

21. How does the presence of self-induction in any circuit affect an alternating current therein (a) when the current is of low frequency; (b) when the current

is of very high frequency; (c) when the circuit is one in which a condenser has been interposed ?

(Ord., A.C., 1908).

22. Describe what is meant by "skin effect."

(A.M.I.E.E. Exam., 1914).

Note—The phenomenon according to which an alternate current tends to have a greater density near the surface than they have along the axis of a conductor is called SKIN EFFECT. This inequality of current density is due to the central or axial filament having greater number of megnetic lines than an element at the surface. This fact produces no effect upon a steady current, after the current has been steady it results in developing greater back E.M. F. of self-induction near the axis than near the outer filaments of the wire, and consequently the current density varies over the section and is less near the axis than it is near the surface of a conductor carrying alternating current. The effect is very perceptible in the case of high frequency or heavy conductor.

23. Show that if for any reason a current does not distribute itself with equal density through the cross-section of a conductor of given material, the energy lost by heat in that conductor will be greater than would be the case if the current density were uniform all over the cross-section. Also show why with high frequencies the resistance offered by a

cylindrical conductor is greater than that offered by the same conductor to an equal current of lower frequency. (Ord., A.C., 1909).

24. If the form of the wave-curve of an alternating electromotive force be given, show how to find, by a graphic construction, the virtual (root-mean-square) value of the electromotive force. (Ord., A.C. 1909).

25. What is the meaning of the term "form-factor" as applied to an alternating quantity? Suppose an alternating current to have the following successive values at successive equal intervals during one period, beginning from zero :—3, 4, 4.5, 5.5, 8, 10, 6, 0, −3, −4, −4.5 −5.5, −8, −10, −5, 0. Find the root-mean-square (or "virtual") value, and find also the form factor. In what conditions is the form-factor equal to 1.11? (Ord., A.C., 1910).

26. An ammeter in a circuit attached to 100-volt mains reads 50, and the power factor is 0. 5. Assume sine curves and plot the volts and amperes to scale showing their proper phase relations. (C. and G., II.)

27. Deduce a mathematical expression for the power in an alternating-current circuit in which a phase difference exists. (Grade II., A. C., 1912).

28. Define the terms, power factor, impedance, periodicity, and state if any of these affect the size and method of running cables. (Wiremen's Final, 1913).

29. Define the term power factor in relation to any

alternating-current apparatus or circuit. Why is the power factor of a distribution system that works through feeders and step-down transformers not constant at all loads? How can such a system be worked so as to improve the power factor? What disadvantage has a low power factor (a) to the station engineer; (b) to the consumer? (Ord., A. C., 1908).

Note on the Supply Tariffs based upon Power Factor :—The normal method of charging is based upon the energy consumption per Kw—hr. Hence, the consumer with a load operating at unity power factor is to a certain extent penalized compared with the one working at a very low power factor; since the latter requires a relatively large proportion of the electrical generating and transmitting plant per Kw and yet charged at the same rate. The scale of charges which would really be equitable from the point of view of the supply companies should therefore be based partly upon the actual energy consumption in Kw-hr. and partly upon the consumption in K. V. A. hr.

An alternative method is to charge a flat rate per annum per K. V. A. installed or per maximum K. V. A. taken, in addition to a charge per Kw-hr. cousumed.

Effect of Low Power Factor on Earning Capacity of the System:—Compared with the case of unity power factor, operation of the lower power factor reduces the earning capacity of the plant etc., and thus it is necessary for profitable operation, to have a slightly higher scale

of charges than would be necessary if the plant could be used to its full capacity at unity power fector.

Note that the value of the power factor only affects the dimensious of the electrical equipment, that is, the switchgear, cables, the rating and performance of the whole of the electrical apparatus concerned in the generation and transmission; it does not appreciably affect the prime movers. (A. E. Clayton's Power Factor Correction.)

30. The power factor on a three-phase public supply system is found to be very low, viz. 0.65: what steps can be taken to improve this?

(A.M.I.E.E. Exam., 1914).

31. A certain motor takes a current of 30 amperes at a pressure of 200 volts and 50 frequency. The power factor is 0.8, lagging. The motor is shunted by a liquid condenser of adjustable capacity. To what value must you adjust the capacity so that the current taken from the supply circuit shall be a minimum?

(Grade II., A. C., 1912).

32. What brake horse power would be required in an engine to drive (a) a D. C. generator having an output of 230 volts 500 amperes at an efficiency of 85 per cent., and (b) an A. C. generator having the same output with an efficiency of 82 per cent. and power factor of 85 per cent. (Wiremen's Final, 1913).

33. What is the relation between the line volts and phase volts, and the between the line current and

phase current, in a three-phase star-connected alternator? If one of the phase windings of a three-phase alternator is connected up wrongly, how would the line voltage be affected? (Ord., A. C., 1911).

34. It is proposed to distribute energy by a three-phase 4-wire system, and the declared pressure at the consumers' terminals is 220 volts. For what pressure should a three-phase motor be wound, and how should the whole installation of motors and lighting be connected so as to prevent any unbalancing of such a distribution system? (A. M. I. E. E. Exam., 1913).

35. Discuss the value of the voltage to earth of the terminals on a three-phase star-connected generator with its star-point (a) earthed, (b) insulated. In case (b) what is the voltage to earth of each of the terminals when one of the terminals is grounded? How is your answer modified if the machine is mesh-connected? (Final, 1st Paper, 1914).

36. The equation of an alternating current is $i = 200 \sin 377\ t$. Find (1) the maxinum value of the current, (2) the rate at which the current is changing when the current is a maximum, (3) the rate at which the current is changing when the current is zero, (4) the effective value, (5) the average value, (6) the frequency, (7) the period, (8) the periodicity.

37. Deduce an expression for the power in a single-phase alternating-current circuit in which a

phase difference exists, and give the power in a three-phase circuit with balanced load in terms of the voltage and current. (Grade II., A.C., 1914).

38. The input to a three-phase star-connected line is 10,000 k. v. a., and the initial pressure is 30,000 volts between phases, the power factor being 0.95. The line is designed for a pressure drop of 10 per cent of the initial pressure.

State:—

(a) The current in each conductor.

(b) The voltage between each conductor and the neutral point at the receiving end.

(c) The total energy received at the end of the line. (A.M.I.E.E. Exam., 1914).

39. Two identical installations, one supplied with three-phase alternating, and one with direct current, each take a steady load of 10 electrical H.P. at 200 volts. What is the line current in each case, and how many Board of Trade units would be consumed per hour, assuming the power factor of the A.C. circuit to be 0.85? (Wiremen's Final, 1914).

40. Determine the relative weights of copper required in the transmission lines to supply a given power over a given distance at the same efficiency of line, when the transmission is made by (a) continuous current, (b) single-phase alternating current, (c) three-

phase alternating current. The maximum voltage to earth of any one wire is to be the same in each case. How is the problem affected by the power factor if alternating current is used ? (Grade II., A C., 1914).

41. A certain factory is fed from a three-phase 50-cycle supply by means of a long aerial line. The kilovolt-amperes measured at the factory are 1040 at a lagging power factor of 0.75, the voltage being 6000. Find the voltage at the generating station if the resistance of each of three-line wires is 1 ohm and the coefficient of self-induction 0.006 henry. If now there is installed a synchronous motor, which takes an input of 900 K. V. A. at 0.6 power-factor leading, in addition to the other load, find the voltage at the generating station. (Final, 2nd Paper, 1913).

42. Explain with diagram of connection the method of measuring power in a three-phase circuit by means of two wattmeters, assuming the loading to be unsymmetrical. (Grade II., A. C., 1914).

CHAPTER VII.

INDUCTANCE.

193. Mechanical Analogy of Resistance, Inductance and Capacity :—Take a piece of steel rod AB (Fig. 7.01) having very little inertia or flexibility, and pivot on the centres PP′ in a frame. If AB is twisted first in one direction and then in the other, it is analogous to applying an alternating E. M. F. to a circuit. As it meets with little resistance in its rotation:, this reprsents an electric circuit containing little resistance and no inductance or capacitance.

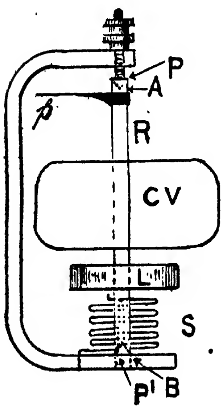


Fig 7.01

Fix a very light but stiff card-board vane C V to the rod AB. The light CARD-BOARD VANE HAVING NO INERTIA REPRSENTS RESISTANCE in the electric circuit. If now the same alternating twist is applied, the rate of rotation (strength of current) will be less than in the first case; but if there is no inertia, the rotation will change directly with the twisting force and will always be at the same rate for a given force.

Take away the vane and next let a small fly-wheel L be attached to the rod AB, and apply the alternating twisting force as before. The maximum force will have to be applied at the start, and less force will be necessary when L gets into motion; and when once L is set in full motion, it will continue to move in the same direction for some time, even when the twisting force is withdrawn. To stop L and reverse its direction of rotation, the twisting force must be applied in the opposite direction. This is ANALOGOUS TO an electric circuit containing INDUCTANCE, and this inductance may be taken to be ELECTRIC INERTIA.

Now, detach the fly-wheel L from AB and introduce a SPRING S REPRESENTING CAPACITY in the circuit, and apply an alternating twist to AB. The elasticity of the spring tends to bring the pointer P to its original position. Thus it helps the pointer P to come back to its orginal position and opposes any force which tends to rotate the pointer P outwards in either direction. This is analogous to the EFFECT OF CAPACITANCE which always helps the current to reverse when an alternating E. M. F. is applied to a circuit.

Thus the whole phenomenon of an alternating current circuit containing resistance, inductance and capacitance may be mechanically represented by the vane, fly-wheel and spring attached to the steel rod as shown in the diagram. It is not necessary that all these factors will be simultaneously present, any combination may be used in the mechanical model

to demonstrate the action of the aforesaid elements in the electric circuit.

194. Difference between Inductance and Ohmic Resistance.—(V. Karapetoff) (1) Suppose a circuit is carrying a current I when the E. M. F. applied to the circuit is E. If we now introduce a resistance or an inductance in this circuit the E. M. F. must be increased to get the same current I. So far, both resistance and inductance affect the circuit exactly in the same way and they are similar.

(2) Ohmic resistance, however, does not depend upon the variable or steady nature of the current; the potential drop due to it depends upon the instantaneous value of the current. But the effect of inductance is apparent only when the current is varying, the induced E. M. F. or the drop in potential due to it is proportional to the rate of change of current, $e = L \frac{di}{dt}$, and not to the absolute value of the current.

(3) As regards the nature of their origin, resistance is due to some molecular friction in the conductor itself, but inductance is caused by the inertia of the magnetic flux surrounding the conductor.

(4) The energy used up in over-coming resistance is converted into heat and is lost electrically. The energy used in overcoming inductance is stored in the form of magnetic energy of the field, and is given back to the electric circuit through the medium of induced E. M. F.

(5) Ohmic resistance does not change with the shape of the conductor, but inductance essentially depends upon the form of the conductor. Thus a conductor whether it is straight or coiled has the same resistance, but the inductance will be altogether different in the two cases.

195. Self Induction.—If a circuit connected to a source of E. M. F. is closed through a switch S, (Fig. 7.02) a current is established in the coil and a magnetic flux is set up. The lines of force produced by a coil grow and die away. They cut not only neighbouring wires but also the turns of the coil itself. Each line of force emerges from the wire of the turn to which it is due, and gradually grows until it finally lies entirely in the iron ring, having cut, on its way, every turn in the coil.

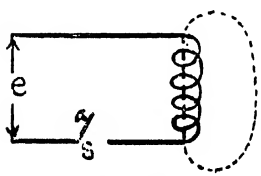


Fig 7.02

Let S be the number of turns in the coil,

A the cross.section of the iron,

and l the length of the magnetic path in the ring.

Assume that the permeabilty μ is constant, which is approximately true for low values of the induction, for which B is roughly proprtional to H. If, now, the current I absolute units increases by an amount dI in the time dt, the increase in the number of lines

of force per sq. cm. will be $dB = \frac{4\pi . S. dI. \mu}{l}$, and the increase in the total flux will be

$$d\phi = \frac{4\pi. S. dI. \mu}{l}. A.$$

But all these lines of force cut S turns in the time dt, and the flux being cut by the conductor produces, a back or counter E. M. F., e in each turn so that $e = -\frac{d\phi}{dt}$ to oppose whatever change of current produced it. Since $d\phi$ represents an increase in the number of lines, we must put a negative sign before the right-hand side of the E. M. F. equation. Substituting this value of $d\phi$ in the equation

$$e = -\frac{d\phi}{dt},$$

or in S turns, $e = -S\frac{d\phi}{dt}$,

$$e = -\frac{4\pi. S.^2 \mu. A}{l} . \frac{dI}{dt} \text{ absolute units.}$$

If the current is expressed in amperes and the electromotive force in volts the right-hand side must be multiplied by 10^{-9}; then,

$$e = -\frac{4\pi. S^2. \mu. A}{l}. 10^{-9}. \frac{di}{dt} \text{ volts.} \quad (1)$$

The expression $\frac{4\pi. S^2. \mu. A}{l}. 10^{-9}$ gives us the

coefficient of self-induction of the coil in practical units, that is, in henries. Denoting this by the letter L, we have

$$e = -L . \frac{d\,i}{d\,t} \text{ volts.}$$

Hence, if $e = 1$ when $\frac{d\,i}{d\,t} = 1$, the self-induction must be 1 henry, i. e., if a coil has a self-induction of 1 henry, an E. M. F. of 1 volt will be induced by a steady change of current at the rate of 1 ampere per second.

The coefficient of self-induction can also be expressed thus :—

$$L = \frac{O.4\,\pi.\,S.\,\mu.\,A}{l} . \quad S.\,10^{-8} \qquad \ldots \qquad (2)$$

Now $\frac{O.4\,\pi.\,S.\,\mu.\,A}{l}$ is the flux produced by a current of one ampere, and multiplying this by S gives the total number of linkages produced by one ampere. Or the INDUCTANCE is the interlinkages of flux and turns per unit current. Thus the coefficient of self-induction in henries is the number of linkages produced by a current of 1 ampere multiplied by 10^8. Thus $L = \frac{S\phi}{I}$; expressed in practical units it is $L = \frac{S\phi}{I \times 10^8}$ henries, where I is the current in amperes.

Further as $e = -L \frac{di}{dt}$, and $e = -\frac{d\phi}{dt}$.

$$L \frac{di}{dt} = \frac{d\phi}{dt}$$

and
$$L = \frac{\phi}{I},$$

$$= \frac{1}{I} \Sigma \text{ Flux} \times \text{interlinkage factor},$$

which is the fraction of the total current enclosed by the flux when ϕ = the total flux linking with the circuit (if the conductor has more than one turn or loop, ϕ is the product of the flux linking with one turn and the number of turns),

I = the current flowing in the circuit, in c. g. s. units,

e = the electromotive force of self-induction, in c. g. s. units,

L = the inductance of the circuit, in c. g. s. units.

The henry is the unit of self-induction and that coil has unit coefficient of self-induction for which the above expression is equal to 1.

From equation $e = -L. \frac{di}{dt}$ volts, it is seen that the electromotive force of self-induction depends not only on the construction of the coil, but also on the rate at which the current is varying. If the coil contains an iron core of crass-section A^1 length. l, and permeability μ, eqn. (2) becames

$$L = \frac{.4\pi S^2}{l}(A - A^1) \times 10^{-8} + \frac{.4\pi S^2 \mu A^1}{l} \times 10^{-8}.$$

$$= \frac{\cdot 4\pi S^2}{l}(A - A^1 + \mu A^1) \text{ henries.}$$

196. Calculation of the E.M.F. of Self-induction in a Coil of known Constants.—The E. M. F. in any coil is proportional to the rate of change of the lines of force threading through the coil, and to the number of turns in the coil. If ϕ be the whole change in the number of lines of force due to any cause in T seconds, and the number of turns is represented by S, the E. M.F. will be

$$E = \frac{\Phi S}{10^8 T}. \qquad \ldots (1)$$

In this equation $\frac{\phi S}{T}$ gives absolute units of E. M. F. or dynes, and 10^8 reduces to practical units of E. M. F. In a long solenoid

$$\phi = \frac{4\pi S' A I}{10}, \qquad \ldots (2)$$

in which S′ is the number of turns per centimetre of length of the solenoid, A the area of the coil, and $\frac{I}{10}$ absolute amperes, I being practical amperes. Substituting this value of ϕ in (1), the self-induced electromotive force

$$E = \frac{4\pi SS' A I}{10^9 T}.$$

The number of lines passing through the coil when the current is 1 C. G. S. unit is $4\pi S'A$. If the coil have iron in it so that its permeability becomes μ instead of 1, (3) will become

$$E = \frac{4\pi SS' A\mu I}{10^9 T}. \qquad \dots (4)$$

Also in (3) $4\pi SS'A = L$, the coefficient of self-induction, and we may write

$$E = \frac{LI}{10^9 T}. \qquad \dots (5)$$

Here L is expressed in absolute units. If L is given in henrys

$$E = \frac{LI}{T}. \qquad \dots (6)$$

When (4) is applicable, $L = 4\pi SS' A\mu$ absolute units; or

$$L = \frac{4\pi SS' A\mu}{10^9} \text{ henrys.}$$

This again gives (6)

$$E = L\frac{I}{T}.$$

Therefore the E. M. F. of self-induction is equal to the coefficient of self-induction in henrys multiplied by the rate of change of the current in amperes per second.

$$\phi = \frac{4\pi S'A\mu I}{10}, \text{ and } E = \frac{4\pi SS' A\mu I}{10^9 T} = \frac{LI}{10^9 T},$$

therefore we may express, for unit current and permeability,

$$L = \frac{S\phi}{10^9}. \qquad \ldots \quad (7)$$

Example 1. How many volts of counter E. M. F. will be developed in a solenoid 40 centimetres long uniformly wound with 200 turns of wire, the area being 4 square centimetres, and in which the current of 4 amperes takes 0.01 second to rise from zero to its maximum value ?

Solution:—

$S=200$ turns, $l=40$ cm.; hence $S'=200\div40=5$ turn' per cm. Also $A=4$ sq. cm., $\mu=1$, and $T=1/100$ second.

Applying (4).

$$E=\frac{4\pi\times200\times5\times4\times1\times5}{10^9\times0.01},$$

$$=0.005023 \text{ volts.}$$

Example 2. Suppose that in the above example a core of iron whose permeability may be taken to be 1000 were put in the solenoid. How many volts of counter E. M. F. will be set up in the coil ?

Solution:—

$$E=\frac{4\pi\times200\times5\times4\times1000}{10^9\times0.01}=5024 \text{ volts.}$$

197. Choking coil.—A coil having a large self-induction is called a Choking Coil. It consists of an iron core bent into a ring with a small air-gap and wound with insulated copper wire. The magnetic flux

density being kept small, the reluctance of the iron may be neglected and only the air-gap is considered in calculating the self-induction.

198. **Design of choking coil.**—Assume the following data:—

Length of magnetic path in air-gap...	l =.75 cm.
Cross-section of path normal to the flux ...	...A=15 sq. cms.
Number of turns in coil ...	...S=250.
Effective value of current ...	... i=10 amps.
Frequency	... f=50.

Then, since the permeability of the air is 1, we have

$$L=\frac{0.4\,\pi.\,S^2.\,\mu.\,A}{l}.10^{-8}=1.25\frac{250^2.\,1.\,15}{.75}.10^{-8}.$$

$$=0.016 \text{ henry}$$

We have also

$$\omega=2\,\pi\,f=314.$$

Hence, for the electromotive force of self-induction, we have

$$E_s=L\omega.\,i=0.016.\,314.\,10=50 \text{ volts.}$$

A voltmeter connected across the terminals of the choking coil will read almost exactly 50 volts, showing that the coil acts like a resistance in causing this drop of pressure.

In order to design a choking coil a suitable cross-section of iron and a convenient number of turns for the coil are to be taken to avoid unsuitable flux density.

Let ϕ=the total flux at the moment of maximum current,

A = the cross section of the iron.

Then, $\phi = B_{max.}\ A.$

For the value of the flux at any moment we have

$$\phi \sin \alpha = \phi \sin \omega t.$$

Hence, the rate of change of the flux will be

$$\phi \frac{d. \sin \alpha}{dt} = \phi\ \omega \cos (\omega t).$$

If the coil have S turns, the momentary value of the E. M. F. will be

$$E = -\phi.\ \omega.\ S. \cos (\omega t)\ 10^{-8} \text{ volts.}$$

The only variable here is the cosine, which has a maxium value of 1.

Hence, $E_{max} = \phi.\ \omega.\ S.\ 10^{-8} = 2\ \pi.\ f.\ \phi.\ S.\ 10^{-8}$ volts.

Now, $E_{max} = \sqrt{2}.\ E.$

$$\therefore E = 4.44\ f.\ \phi.\ S.\ 10^{-8} \text{ volts.} \quad \dots \quad (1)$$

Also, $B_{max} = \mu.\ H_{max} = \dfrac{0.4\ \pi.\ S.\ i_{max}.\ \mu}{l},$

and, $i_{max} = \sqrt{2}.\ i.$

$$\therefore i = \frac{B_{max}.\ l}{1.78\ \mu.\ S.} \text{(amps.)} \quad \dots \quad (2)$$

This equation is very convenient for calculating the magnetising current of a transformer.

Now we chose a suitable value for the magnetic flux-density. If, for example, an E. M. F. of 50 volts is to be induced by a current of 10 amperes at a frequency of 50, we may proceed in the following manner.

Let B = 5,000 lines per sq. cm.; neglect the path in the iron and consider the length l of the air-path.

The cross-section of the iron is assumed to be 15 sq. cms. We have then

$$\phi = B_{max} . A = 5000 \times 15 = 75000.$$

From equation (1) we have

$$S = \frac{E.10^8}{4.44 . \phi . f} = \frac{50 \times 10^8}{4.44 \times 75000 \times 50} = 300.$$

Putting the number of turns S equal to 300 in equation (2) we get

$$l = \frac{1.78 . S . \mu . i}{B_{max}} = \frac{1.78 . 300 . 1 . 10}{5000} = 1 \text{ cm}.$$

This is the length of the air-gap. The cross-section of the path across the air gap has been taken as equal to the cross-section of the iron. The lines of force, however, do not pass straight across the gap, but curve round from pole to pole. The cross-section of the path across the gap is, therefore, considerably greater than the cross-section of the iron. The presence of paper or varnish insulation between the sheet-iron stampings of which the core is built up will still further increase this effect. The above coil will consequently cause a greater drop than 50 volts when carrying 10 amperes.

199. Applications of Choking Coils.—It is used for: (1) Reducing and regulating the pressure of arc lighting circuits without appreciable loss of power, such as would occur if a resistance were used for the same purpose.

(2) Varying or dimming the lights of a number of glow lamps.

(3) Controlling voltages across electric furnaces, and welding and other apparatus.

(4) Protecting electrical machines from lightning in generating or substations in connection with over head lines.

(5) Preventing the flow of an excessive current due to accidental short circuiting when it is used in series with large alternators.

(6) For limiting primary voltage when a large transformer is suddenly switched on to the mains.

Its great advantage is that the power absorbed by it is much less than that absorbed by a resistance.

Let X = reactance of the coil having negligible resistance,

R = resistance of the apparatus with which it is used,

Z = the total impedance.

Then, the power absorbed by the combination

$$= VI \cos\phi = VI \frac{R}{Z} = I^2 R.$$

If instead of a choking coil a resistance R_1 is used, the power absorbed is $I^2 (R+R_1) = VI$, which is greater than that absorbed by the choke coil in the ratio of $\frac{R+R_1}{R}$.

Example 3. A choking coil is required to enable a number of incandescent (non-inductive) lamps to take 5 amps. at 120 volts from one A. C. supply at 225 volts. If the effective resistance is 5 ohms find its reactance, and compare its efficiency with that of a series resistance for the same purpose.

Solution:—

The resistance of the lamps $= 120/5 = 24$ ohms.

$\therefore$ Total resistance in circuit $= 24 + 5 = 29$ ohms.

The total impedance $= 225/5 = 45$ ohms.

$\therefore$ Reactance of coil $= \sqrt{\{(45)^2 - (29)^2\}}$

$= \sqrt{(74 \times 16)} = \sqrt{1184} = 34.4$ ohms.

Efficiency $= 24/29 = 1 - \frac{5}{29} = 1 - .1724 = .8276$ or 82.76 per cent.

With a series resistance:—

Total resistance in circuit $= 225/5 = 45$ ohms.

$\therefore$ Series resistance $= 45 - 24 = 21$ ohms.

$\therefore$ Efficiency $= 24/45 = .533$ or 53.3 per cent.

Alternately:—

Power used by lamps $= 120 \times 5 = 600$ watts.

Power absorbed by coil $= (5)^2 \times 5 = 125$ watts.

$$\therefore \text{ Efficiency of coil } = \frac{600}{600 + 125} = \frac{600}{725} = 1 - \frac{125}{725}.$$

$= 1 - .1724 = .8276$ or 82.76 %.

Power absorbed by series resistance

$= (225 - 120) \times 5 = 525$ watts.

$\therefore$ Efficiency of series resistance $= \dfrac{600}{600 \times 525} = \dfrac{600}{1125}$,

$= .533$ or $53.3\ \%$.

200. Computation of the Self-inductance of a Coil with an Air Core.—The absolute permeability of air, that is μ_a, is always $= 3.192$ perms. per in. cube. Substituting this value in the formula—

$$L = \frac{S^2 \times \mu_a \times A}{100,000,000 \times l},$$

the inductance of an air core coil is:

$$L = \frac{S^2 \times 3.192 \times A}{100,000,000 \times l} \qquad \text{henrys.}$$

Example 4. What is the inductance of a coil of 500 turns wound on an iron core which is ·25 ins. long having a sectional area of 45 sq. ins. ?

Solution :—

Assume $\mu_a = 5000$.

$$L = \frac{S^2 \times \mu_a \times A}{10^8 \times l} = \frac{(500)^2 \times 5000 \times 45}{10^8 \times 25} = 22.5 \text{ henrys.}$$

N. B.—The expression $\dfrac{0.4\ \pi.\ \mu.\ A}{l}$ represents the lines produced by one ampere-turn.

Example 5. A coil of 300 turns is supplied with various amounts of current, and the flux produced, when 1 amp. flows, is 2000 lines. Find the inductance.

$$L = \frac{300 \times 2000}{1 \times 10^8} = 6 \times 10^{-3} \text{ henrys.}$$

Example 6. What is the inductance of the field of a 12-pole alternator, if the 12 field spools are connected in series: each spool contains 500 turns, and a current of 5.65 amps. produces 5.4 megalines per pole ?

Solution:—

Total number of turns in the 12 spools

$$=12 \times 500=6000.$$

Each turn is interlinked with 5.4×10^6 lines.

$\therefore$ Total number of interlinkages at 5.65 amps.

$$=6000 \times 5.4 \times 10^6=32.4 \times 10^9.$$

$\therefore$ Number of interlinkages per unit (absolute) current, or the inductance is $\dfrac{32.4 \times 10^9}{.565}$,

$$=57.345 \times 10^9=57.345 \text{ henrys.}$$

Example 7. What is the indutance of an air-core coil having an internal diameter of 1.6 in. (area $\doteq$ 2.01 sq. in.) 200 turns, and a length of 25 ins. ?

Solution:—

Substitute in the formula above :

$$L=\frac{S^2 \times 3.192 \times A}{100{,}000{,}000 \times l}=\frac{200 \times 200 \times 2.19 \times 2.01}{100{,}000{,}000 \times 25},$$

$$=.000103 \text{ henrys.}$$

$$=0.103 \text{ millihenrys.}$$

Example 8. An electric circuit consists of 200 turns of No. 10 S. W. G. wire (=·013 sq. in.) of 15 inches mean length of turn, wound round an iron core having

a cross-section of 5 sq. inches, and a mean length of 20 inches, the hysteresis coefficient of the iron being $\eta = 2.5 \times 10^{-3}$. By interposing an air gap in the magnetic circuit of a section of 10 sq. inches, (allowing for spread), the reactive coil gives 200 volts e. m. f. of self-inductance at 20 amperes and 50 cycles. (Specific resistance of copper $= 1.8 \times 10^{-6}$).

What should be the length of the air gap, and what is the resistance, the reactance, the effective impedance, and the power-factor of the reactive coil ?

Solution:—

$$R_1 = \rho . \frac{l}{A} = \frac{1.8 \times 10^{-6} \times 200 \times 15}{.013 \times 2.54}.$$

$$= .164 \text{ ohm.}$$

$$E \text{ (effective)} = \sqrt{2}\, \pi f n\, \phi \times 10^{-8},$$

$$= 4.44\, f\, n\, \phi \times 10^{-8} \text{ volts.}$$

$$\therefore 200 = 4.44 \times 50 \times 200 \times \phi \times 10^{-8},$$

$$\text{or, } \phi = 450500 \text{ lines.}$$

$\therefore$ In the air gap, flux density,

$$B = \frac{450500}{10},$$

$$= 45050 \text{ lines per sq. in.}$$

$$= 6980 \text{ lines per sq. cm.}$$

Now, the number of ampere-turns,

$$k = s . I = 200 \times 20 = 4000 \text{ (effective),}$$

$$= 5660 \text{ (maximum).}$$

Thus, 5660 ampere-turns must be provided (neglecting the ampere-turns required by the iron part of the magnetic circuit as relatively very small) by the air gap for producing a flux density, B=6980 lines per sq. cm.

Therefore, substituting numerical values in the formula—

$$B=\frac{4\,\pi\,k}{10\,l}\ ;\ \text{or,}\ l=\frac{4\,\pi\,k}{10\ B},$$

we have,

$$l=\frac{4\,\pi\times 5660}{10\times 6980}=1.02\ \text{cm.}$$

Again, from $\phi=450500$ lines, the density in the iron with a cross-section of 5 sq. in. is

$$B_1=\frac{450500}{5}=90100\ \text{lines per sq. in.}$$

$$=13970\ \text{lines per sq. cm.}$$

∴ Loss of energy per cycle per cubic centimetre,

$$W=\eta\ B_1^{1.6}$$

$$=2.5\times 10^{-3}\times(13970)^{1.6},$$

$$=10725\ \text{ergs.}$$

Hysteresis loss at $f=50$ cycles, and volume $V=20\times 5\times(2.54)^3=1638$ cu. cm., is

$$P=V\,f\,\eta\,B_1^{1.6},$$

$$=1638\times 50\times 10725\ \text{ergs per sec.},$$

$$=87.8\ \text{watts.}$$

Effective hysteretic resistance,

$$R_2=\frac{P}{I^2}=\frac{87.8}{20^2}=.22\ \text{ohm.}$$

$\therefore$ Total effective resistance of the coil,

$$R = R_1 + R_2 = .164 + .22 = .384 \text{ ohm.}$$

Effective reactance, $X = \frac{E}{I} = 10$ ohms.

$\therefore$ Impedance, $Z = 10.01$ ohms.

Power factor, $\cos\phi = \frac{R}{Z} = 3.8$ per cent.

Total apparent power of the coil $= I^2 . Z$

$= 4004$ volt-amperes.

Loss of power $= I^2 R = 153$ watts.

201 Mutual Induction.—In the preceding articles the only reactions which have been considered when a current in a circuit is varied, are those existing or arising in the circuit itself. In general, however, when there are two or more circuits in proximity, any change in the current in one will cause inductive effects in the others. The former is called the PRIMARY circuit, and the latter the SECONDARY. This effect is known as the mutual induction between two or more circuits.

Mutual induction plays an important part in the operation of most alternating-current apparatus, such as the transformer, the induction motor, etc. Without the transfomer and the induction motor, the present development of alternating-current systems of power distribution and utilization would be impossible.

As in self-induction the e. m. f. induced in any circuit 1 as the result of the variation of the

current in any other circuit 2 is called the electromotive force of MUTUAL INDUCTION in circuit 1 due to clrcuit 2. This may be expressed symbolically thus :

$$e_{12} = -M_{12} \frac{d\,i_2}{d\,t},$$

where M_{12} is a constant provided the permeability of the circuits is constant. When the permeability varies with the current in either circuit, M_{12} also varies with the current. M_{12} is called the COEFFICIENT OF MUTUAL INDUCTION or MUTUAL INDUCTANCE of circuit 1 with respect to circuit 2, and may be defined as the ratio of the electromotive force e_{12} induced in any circnit 1, due to a change in the current i_2 in any other circuit 2, to the time rate of change of this current i_2 ; that is,

$$M_{12} = \frac{e_{12}}{\frac{d\,i_2}{d\,t}}.$$

Examp'e 9. Two circuits have a mutual inductance of 2 henrys. Assuming that all the lines of force induced pass through both circuits, what must be the rate of change of current in one in order that the induced e.m.f. in the other may be 10,000 volts ?

(C. & G. II.)

Solution:—

We have

$$e_{12} = -m_{12} \frac{d\,i_2}{d\,t}.$$

$$\therefore \quad 10000 = 2\frac{d\,i_2}{d\,t}.$$

or,
$$\frac{d\,i_2}{d\,t} = 5000 \text{ amps. per sec.}$$

i. e., the current changes at the rate of 5000 amperes per second.

Again, as in the case of self-inductance, mutual inductance may also be expressed in terms of the magnetic linkages produced in circuit 1 by the passage of one C. G. S. unit of current in circuit 2. Thus—

$$e = -\frac{d\,\phi_{12}}{d\,t} = -\frac{d}{d\,t}(M_{12}\,i_2) = -M_{12}\frac{d\,i_2}{d\,t},$$

whence,

$$M_{12} = \frac{d\,\phi_{12}}{d\,i_2},$$

where ϕ_{12} is in maxwells.

The practical unit of mutual inductance is the same as that of self-inductance, viz. the HENRY, and is equivalent to 10^9 or 10^8 magnetic linkages according as the current is in absolute or practical unit.

The number of magnetic linkages threading a coil of S_1 turns, produced by a second coil of S_2 turns carrying a current i_2 will be, for the same coil frames and positions, nearly proportional to $S_1\ S_2\ i_2$.

Moreover, as will be shown later the mutual inductance of one circuit with respect to the other is

the same as the mutual inductance of the second circuit with respect to the first, i. e., $M_{12} = M_{21}$.

202 Relation among the Mutual and Self-inductances of Two Circuits having Constant Magnetic Reluctances.—(1) If no flux leaks between the two circuits, that is, if all the flux is interlinked with both circuits, and if

L_1 = self-inductance of the first circuit,
L_2 = self-inductance of the second circuit,
M = mutual-inductance of the two circuits,

then, from the fundamental relations:

$$L_1 = \frac{S_1 \phi_1}{i_1}, \qquad \phi_1 = \frac{i_1 S_1}{R}.$$

$$L_2 = \frac{S_2 \phi_2}{i_2}. \qquad \phi_2 = \frac{i_2 S_2}{R}.$$

we have
$$L_1 L_2 = \frac{S_1^2 S_2^2}{R^2} = M^2$$

or,
$$M = \sqrt{L_1 \cdot L_2} \quad \ldots \qquad (1)$$

which is the MAXIMUM VALUE of the MUTUAL INDUCTANCE.

(2) If flux leaks between the two circuits, $M^2 < L_1 L_2$. In this case the total flux produced by the first circuit consists of a part interlinked with the circuit itself, and a part interlinked with the second circuit only. Thus, if L_1 and L_2 are the inductances of the two circuits, $\frac{L_1}{S_1}$ and $\frac{L_2}{S_2}$ are the (total) fluxes

produced by unit current in the first and second circuit respectively.

Of the flux $\frac{L_1}{S_1}$ a part $\frac{K_1}{S_1}$ is interlinked with the first only, K_1 being its self-inductance or leakage inductance; and a part $\frac{M}{S_2}$ interlinked with the second circuit also, M being the mutual inductance.

$$\text{Thus,}\quad \frac{L_1}{S_1}=\frac{K_1}{S_1}+\frac{M}{S_2}.$$

Hence, if L_1 and L_2 = inductance.
K_1 and K_2 = self-inductance,
and M = mutual-inductance of two circuits of S_1 and S_2 turns respectively,

$$\frac{L_1}{S_1}=\frac{K_1}{S_1}+\frac{M}{S_2},\qquad \frac{L_2}{S_2}=\frac{K_2}{S_2}+\frac{M}{S_1},$$

$$\text{or,}\quad L_1=K_1+\frac{S_1}{S_2}M.\qquad L_2=K_2+\frac{S_2}{S_1}M.$$

$$\therefore\ M^2=(L_1-K_1)\ (L_2-K_2) \qquad \dots \qquad (2)$$

If K_1 and K_2 are both zero, (2) reduces to (1).

Example 10. Two coils of 9 ohms and 4 ohms reactance, respectively and negligible resistance are connected in series to a. P. D. of 225 volts. Find the current which flows if their mutual inductance is (a) of its maximum value, (b) of half this value, (c) of its maximum negative value.

Solution :—

(a) Maximum value of mutual reactance,

$$\omega M = \sqrt{9 \times 4} = 6 \text{ ohms.}$$

The combined reactance $= L_1 + 2M + L_2$

$$= 9 + 2 \times 6 + 4 = 25 \text{ ohms.}$$

$\therefore$ Current flowing $= \dfrac{225}{25} = 9$ amps.

(b) Mutual reactance $= 3$.

$\therefore$ Combined reactance $= 9 + 2 \times 3 + 4,$

$$= 19 \text{ ohms.}$$

$\therefore$ Current $= \dfrac{225}{19} = 11.85$ amps.

(c) Combined reactance $= 9 - 2 \times 6 + 4 = 1$ ohm.

$\therefore$ Current $= \dfrac{225}{1} = 225$ amps.

203. Coefficient of Electromagnetic Coupling.—The coefficient of electromagnetic coupling betwee two circuits having mutual inductance is the ratio of M t $\sqrt{L_1 L_2}$.

$$\text{Coefficient of coupling} = \frac{M}{\sqrt{L_1 L_2}},$$

$$= \frac{\sqrt{(L_1 - K_1)(L_2 - K_2)}}{\sqrt{L_1 L_2}}$$

The coupling between two circuits is said to be clos when the coefficient of coupling is large. Close couplir is generally required in commercial transformers, sinc

the voltage regulation of a transformer depends very largely on the closeness of coupling between its primary and secondary windings. Good voltage regulation requires small magnetic leakage between primary and secondary windings, and therefore close coupling. Such close coupling as is used in commercial power transformers is, however, undesirable in transformers for radio work. For in radio work, circuits having coupling coefficients greater than .5 are said to be close coupled, whereas in commercial transformers the value may be as high as .98 or .99, when they are close coupled.

204. Calculation of Mutual Inductance.—In determining mutual inductance it is first necessary to ascertain the number of lines of force of one circuit which will cut the other circuit when a current of 1 amp. flows in the first circuit. Then this number representing the cutting flux, is divided by 10^9 to get the result into henrys.

We shall take the simplest case of two solenoids, so associated that there is no magnetic leakage, i. e., all the flux produced by the one, links all the turns of the other.

Let S_1, A_1, l_1 and i_1 represent the total number of turns, the cross-sectional area, the length and the current in the primary circuit; then

$$H_1 = \frac{4\,\pi\, S_1\, i_1}{l_1} \text{ and } \phi_1 = \frac{4\,\pi\, S_1\, i_1\, A_1}{l_1}, \ldots \quad (1)$$

If each turn of the secondary is traversed by this flux, and if there are S_2 turns in the secondary, then the total flux or linkages, ϕ_{21}, through the secondary is given by

$$\phi_{21} = \frac{4 \pi S_1 A_1 i_1}{l_1} \times S_2. \quad \ldots \quad (2)$$

$$\therefore e = -\frac{S_2}{l_1} \cdot \frac{d}{dt} (4 \pi S_1 A_1 i_1),$$

$$= -\frac{4 \pi S_1 S_2 A_1}{l_1} \cdot \frac{di_1}{dt}.$$

from which,
$$M_{21} = \frac{4 \pi S_1 S_2 A_1}{l_1}$$

$$= \frac{4 \pi S_1 S_2 A_1}{l_1 \times 10^8} \text{ henrys} \quad \ldots \quad (3)$$

Note.—We arrive at once at this result, if we make the current i_1 in the primary circuit unity (=1 amp).

If, instead of an air-core we have one of permeability μ, we have to substitute $\mu H_1 = B_1$ for H_1 in eqns. (1), and so,

$$M_{21} = \mu . \frac{4 \pi S_1 S_2 A_1}{l_1 10^8} \text{ henrys.} \quad \ldots \quad (4)$$

Further, if the iron-core has a cross-section different from that of the coil, say A, eqn.. (4) becomes

$$M_{21} = \frac{4 \pi S_1 S_2}{l_1 \times 10^8} (A_1 - A + \mu A) \text{ henrys.} \quad \ldots \quad (5)$$

The similarity between these formulae and those for calculating self-inductance is evident.

The formulae, given below like all practical simple ones for calculating inductance, give approximate results, which, however, are usually sufficiently accurate for most practical purposes :—

$$M = \frac{S_1 \times S_2 \times \mu_a \times A}{100,000,000 \times l} \quad \text{(henrys)},$$

where S_1 = number of turns in one of the concentric coils.

S_2 = number of turns in the other concentric coil.

The other symbols have the usual meanings. Where the coils have an air core, the mutual induction then is :

$$M = \frac{S_1 \times S_2 \times 3.192 \times A}{100,000,000 \times l} \quad \text{(henrys)}.$$

Example 11. Calculate the approximate mutual inductance of two concentric coils wound on the wood (non-magnetic or air) core, 2 ins. in diameter (area =3.14 sq. ins.). The coils are each 20 ins. long, one having 500 turns and the other 300 turns.

Solution:—

Substitute in formula above :

$$M = \frac{S_1 \times S_2 \times 3.192 \times A}{100,000,000 \times l} = \frac{500 \times 300 \times 3.19 \times 3.14}{100,000,000 \times 20}$$

$= 0.00075$ henry $= 0.75$ millihenry.

Example 12. What will be the approximate mutual induction of two concentric coils, each of the

same number of turns as specified in the above example, if they are wound on an iron core 2 ins. in diameter and 20 ins. long? Assume $\mu_i = 6000$.

Solution :—

$$M = \frac{S_1 \times S_2 \times \mu \times A}{100,000,000 \times l} = \frac{500 \times 300 \times 6000 \times 3.14}{100,000,000 \times 20}$$
$$= 1.4 \text{ henry.}$$

Or, instead, 1.4 henry = 1400 millihenry. It is obvious that the inductance in this example is as many times greater than that in the preceeding example as the permeability of this iron is greater than the permeability of air. That is,

$$M : 0.00075 :: 6000 : 3.19$$

Then, $L_m = 1.4$.

205. Magnetic Energy of Two or More Electric Circuits.—We have already seen that when the current in an electric circuit changes, the corresponding change in its magnetic field induces an E. M. F., not only in this circuit, but also in every other neighbouring circuit. If this induced E. M. F. (as it is generally called) establishes a current in such a circuit, there will be a transfer of energy from one circuit to the other. Whether work is done on the current, or is done by the current in a given circuit, depends upon the relative direction of the current and the E.M.F. induced in this circuit.

I. Consider two neighbouring circuits fixed in size, shape and relative position, and let the permeability

of every body in the field be constant. Let L_1 and L_2 be the self-inductances of the two circuits, and M_{12} the mutual inductance of circuit 2 with respect to circuit 1, and M_{21} the mutual inductance of 1 with respect to 2 ; let i_1 and i_2 be the currents in the two circuits at any instant. Then the total linkages of the two circuits are

$$\left.\begin{aligned} \phi_1 &= L_1 i_1 + M_{12} i_2 \\ \phi_2 &= L_2 i_2 + M_{21} i_1 \end{aligned}\right\} \quad \ldots \quad \ldots \quad \text{(a)}$$

where the ϕ, s and i's may be positive or negative. Let di_1 and di_2 be small increments of the two currents in time dt : then the back E. M. Fs. induced in the two circuits are :—

$$e_1 = \frac{d\phi_1}{dt} = L_1 \frac{di_1}{dt} + M_{12} \frac{di_2}{dt},$$

$$e_2 = \frac{d\phi_2}{dt} = L_2 \frac{di_2}{dt} + M_{21} \frac{di_1}{dt}.$$

And the drop of potential in the circuits is

$$v_1 = R_1 i_1 + e_1 = R_1 i_1 + L_1 \frac{di_1}{dt} + M_{12} \frac{di_2}{dt},$$

$$v_2 = R_2 i_2 + e_2 = R_2 i_2 + L_2 \frac{di_2}{dt} + M_{21} \frac{di_1}{dt};$$

so that the total difference of potential if the circuits are connected in series is

$v = v_1 + v_2 =$

$$R_1 i_1 + L_1 \frac{di_1}{dt} + M_{12} \frac{di_2}{dt} + R_2 i_2 + L_2 \frac{di_2}{dt} + M_{21} \frac{dt_1}{dt}$$

$$=(R_1 + R_2)\, i + (L_1 + M_{12} + M_{21} + L_2)\frac{di}{dt}$$

But $M_{12} = M_{21}$ (See P. 372.)

$\therefore$ the combined inductance $= L_1 + 2M + L_2$,
where $M = M_{12} = M_{21}$.

If the coils are joined in parallel,

$$v = R_1 i_1 + L_1 \frac{di_1}{dt} + M_{12}\frac{di_2}{dt} = R_2 i_2 + L_2 \frac{di_2}{dt} + M_{21}\frac{di_1}{dt}$$

and $i = i_1 = i_2$.

If the coils are connected in series so as to oppose magnetically, the combined inductance is zero.

The amounts of energy stored in the magnetic field by the respective currents are:—

$$dw_1 = e_1 i_1\, dt = L_1 i_1\, di_1 + M_{12} i_1\, di_2,$$
$$dw_2 = e_2 i_2\, dt = L_2 i_2\, di_2 + M_{21} i_2\, di_1.$$

Now, let circuit 2 be open, so no current can flow in it, $i_2 = 0$ and $di_2 = 0$. So, the work done by the current in circuit 1 in increasing from zero to I_1 is

$$\int_0^{I_1} L_1 i_1\, di_1 = \tfrac{1}{2} L_1 I_1^2.$$

Next, keep the current in circuit 1 constant, and let the current in 2 increase from 0 to I_2; under these conditions $i_1 = I_1$ and $di_1 = 0$, and therefore the work done by the current in circuit 1 is

$$\int_0^{I_2} M_{12}\, I_1\, di_2 = M_{12}\, I_1\, I_2,$$

and the work done by the current in circuit 2 is

$$\int_0^{I_2} L_2\, i_2\, di_2 = \tfrac{1}{2}\, L_2\, I_2^2.$$

Hence, the total work done by the two currents in establishing the magnetic field corresponding to the final values of the currents I_1 and I_2 is

$$W = \tfrac{1}{2}\, L_1\, I_1^2 + M_{12}\, I_1\, I_2 + \tfrac{1}{2}\, L_2\, I_2^2. \quad \ldots \quad (1)$$

Note that this formula does not contain the coefficient M_{21}. The explanation of this is given below.

(II) THE MUTUAL INDUCTANCE OF ONE CIRCUIT WITH RESPECT TO ANOTHER IS THE SAME AS THE MUTUAL INDUCTANCE OF THE SECOND CIRCUIT WITH RESPECT TO THE FIRST CIRCUIT.—Let the current in 2 be now kept constant and the current in 1 be decreased to zero; under these conditions $i_2 = I_2$ and $di_2 = 0$; and therefore the work done on the current in 1 by the magnetic field is

$$-\int_{I_1}^{0} L_1\, i_1\, di_1 = \tfrac{1}{2}\, L_1\, I_1^2,$$

and the work done on the current in 2 by the magnetic field is

$$-\int_{I_1}^{0} M_{21}\, I_2\, di_1 = M_{21}\, I_1\, I_2.$$

Now open circuit 1 and let the current in 2 fall to zero; under these conditions $i_1 = 0$ and $di_1 = 0$, and therefore

the work done on the current in 2 by the magnetic field when this current falls to zero is

$$-\int_{I_2}^{0} L_2\, I_2\, di_2 = \tfrac{1}{2}\, L_2\, I_2^2.$$

Hence, the total work done by the magnetic field when it disappears is

$$W = \tfrac{1}{2}\, L_1\, I_1^2 + M_{21}\, I_1\, I_2 + \tfrac{1}{2}\, L_2\, i_2^2. \quad \ldots \quad (2)$$

From the Principle of the CONSERVATION OF ENERGY, the energy stored in the field when it is established must be equal to the work done by the field when it disappears. Hence, the expression for the energy stored in the field must be equal to the expression for the energy given back by the field; i. e., (1) and (2) are one and the same expression,

$$\therefore\ M_{21} = M_{12}.$$

(III) THE TOTAL AMOUNT OF ENERGY STORED IN A MAGNETIC FIELD BY TWO CURRENTS IS INDEPENDENT OF THE MANNER IN WHICH THESE TWO CURRENTS ARE ESTABLISHED. This also follows from the Principle of THE CONSERVATION OF ENERGY. Therefore, the expression,

$$W = \tfrac{1}{2}\, L_1\, i_1^2 + M i_1\, i_2 + \tfrac{1}{2}\, L_2\, i_2^2 \quad \ldots \quad (b)$$

is a perfectly general one for the energy of the magnetic field due to the currents i_1 and i_2 in two circuits which have constant self-inductances L_1 and L_2, and a constant mutual inductance M.

Equation (b) may also be written as

$$W = \tfrac{1}{2}\,(L_1\, i_1 + M_{12}\, i_2)\, i_1 + \tfrac{1}{2}\,(L_2\, i_2 + M_{21}\, i_1)\, i_2$$

which, in turn, from equations (a), may be written

$$W=\tfrac{1}{2}\phi_1\, i_1+\tfrac{1}{2}\phi_2\, i_2$$

By exactly similar rasoning it can be shown that the energy of the magnetic field due to the electric currents in any number of circuits is

$$W=\tfrac{1}{2}\,\Sigma\phi\, i,$$

where i is the current in any circuit and, ϕ is the number of linkages between this circuit and the total flux which threads it, and the summation includes all the circuits in the field.

It should be noted that all the formulas in this article are based upon the assumption that every body in the magnetic field has a constant permeability.

Example 13. Calculate the mutual inductance between an alternating transmission line and a telephone wire carried below for 14 miles, and 1.2 metre distant from the one, 1.5 metres distant from the other conductor of the transmission line. What is the e.m.f. generated in the telephone wire if the transmission line carries 100 amperes at 50 cycles ?

Solution :—

The total flux passing between the transmission wires when a current of I amps. flows in the telephone wire is given by

$$\phi=\int_{120}^{150}\frac{.2\, I\, l}{l_r}\, dl_r$$

$l=14$ miles $=2254\times10^3$ cms.

$\therefore\ \phi = 450.8 \times 10^3\ I \log_e \frac{150}{120} = 100.6 \times 10^3\ I$; i. e. $100.6 \times 10^3\ I$ is that part of the magnetic flux produced by I amperes of current in the telephone wire, which passes between the distances of 1.2 and 1.5 metres, or is the number of interlinkages with the alternating circuit produced by I amperes of current in the telephone wire.

Therefore, the number of interlinkages produced by unit (absolute) current in the telephone wire, or the mutual inductance between the telephone wire and the electric circuit is

$M = 100.6 \times 10^4$ absolute units,

(for $I = 10$, or one absolute unit).

$= 100.6 \times 10^{-5}$ henry,

$= 1.006$ millihenry.

Now, 100 amps. effective $= 141.4$ amp . maximum.

Therefore, the maximum flux interlinked with the telephone line when 141.4 amps. or 14.14 absolute units of current flows in the transmission line is

$$= 100.6 \times 10^3 \times 14.14 \times 10,$$
$$= 1.006 \times 14.14 \times 10^6,$$
$$= 14.2 \text{ megalines.}$$

Hence, the e.m.f. generated at 50 cycles is

$$E = \sqrt{2}\pi f \Phi = 4.44 \times .5 \times 14.2 = 31.5 \text{ volts effective.}$$

206. Inductance of a Concentric cable —

To determine the inductance of a concentric cable we have to consider—

(1) The inductance due to the flux in the inner conductor within itself.

(2) The inductance due to the flux in the insulating medium between the conductors.

(3) The current in the inner conductor which interlinks with the entire flux, which is in the outer conductor, but which is caused by the difference in m. m. f. in the inner and outer conductors.

(4) The inductance of the outer conductor which should be subtracted to give the total inductance of the cable, since the current in it is in the opposite direction.

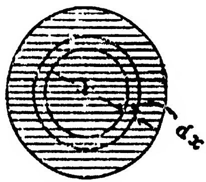


Fig. 7.03

Consider first the flux in the inside conductor due to the assumed uniform distri bution of the current in it.

Let ϕ = the flux, and I = total current.

Then the m. m. f. at a distance x (Fig. 7.03) from the centre $= \frac{\pi x^2}{\pi r^2} I$, where r = the radius of the conductor.

The flux $d\phi_1$ which passes through an element dx wide, and of unit length, is equal to the magnetomotive force divided by reluctance.

$$\therefore d\phi_1 = \frac{4\pi I \frac{\pi x_2}{\pi r^2}}{\frac{2\pi x}{dx}}.$$

$$\text{or,}\quad d\phi_1 = 4\pi\left(\frac{x^2}{r^2}\,I\right)\frac{dx}{2\pi x} = 2\,I\frac{x}{r^2}\,dx.$$

This flux interlinks with $\pi x^2/\pi r^2$ of the total current; thus the interlinkage factor is x^2/r^2 (see P. 346).

$$\text{Now,}\quad L = \frac{1}{I} \times \Sigma \text{ flux} \times \text{interlinkage factor.}$$

Case (1).—The inductance due to the flux in the inner conductor within itself is therefore—

$$L_1 = \frac{1}{I}\int_0^r 2\,I\frac{x^3}{r^4}\,dx = \tfrac{1}{2},\ (\text{assuming } \mu = 1).$$

Case (2).—Between the conductors (Fig. 7.04), the flux interlinks with the whole current. Therefore

$$\therefore\ L_2 = \frac{1}{I}\int_r^R 2\,I\frac{dx_1}{x_1} = 2\log_e\frac{R}{r}.$$

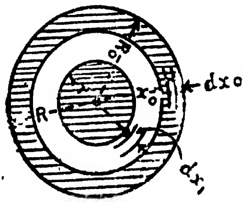


Fig 7.04

Case (3).—The current in the inner conductor interlinks with the entire flux which is in the outer conductor, but which is caused by the difference in m. m. f. in the inner and outer conductors.

At distance x_0 the M. M. F. is thus

$$I - \frac{x_0^2 - R^2}{R_0^2 - R^2}\,I = I\,\frac{R_0^2 - x_0^2}{R_0^2 - R^2}.$$

The interlinkage of this flux with the current in the inner conductor is, of course, unity; thus

$$L_3=\frac{1}{I}\int_R^{R_0}\frac{2\,I}{x_0}\cdot\frac{R_0^2-x_0^2}{R_0^2-R^2}\,dx_0,$$

$$=\frac{2\,R_0^2}{R_0^2-R^2}\log_e\frac{R_0}{R}-1.$$

Case (4).—The inductance of the outer conductor should be subtracted to give the total inductance of the cable, since current in it is in the opposite direction.

The m. m. f. is shown to be $I\,\dfrac{R_0^2-x_0^2}{R_0^2-R^2}$.

$$\therefore\ L_4=\frac{1}{I}\int_R^{R_0}\frac{2\,I}{x_0}\cdot\frac{(R_0^2-x_0^2)}{(R_0^2-R^2)^2}\cdot(x_0^2-R^2)\,dx_0,$$

$$=\frac{1}{2}\frac{R_0^2+R^2}{R_0^2-R^2}+\frac{2\,R_0^2\,R^2}{(R_0^2-R^2)^2}\log_e\frac{R_0}{R}.$$

The total inductance, $L=L_1+L_2+L_3-L_4$, which is already proved to be

$$L=\frac{1}{2}+2\log_e\frac{R}{r}+\frac{2R_0^4}{(R^2-R^2)}\log\frac{R_0}{R}$$

$$-\frac{1}{2}\frac{3R_0^2-R^2}{R_0^2-R^2}\text{ per cm.}$$

This inductance is expressed in the absolute system of units. By dividing by 10^9 the inductance is expressed in henrys.

207. Line Inductance:—The total inductance is that due to the flux outside the conductor plus that within the conductor. To determine the inductance of a single-phase line we proceed as follows :—

Suppose A and B to be two parallel conductors Fig. 70.5 in which equal currents flow in opposite directions. The current in each, if uninfluenced by the other conductor sets up concentric circles of flux round the axis of the wire, and the mutual effect of the currents is to produce flux distribution as shown in the figure.

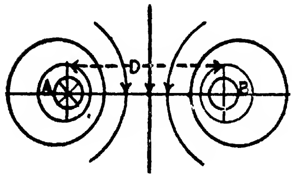
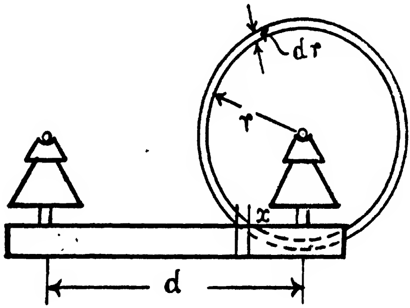


Fig. 7.05.



Fig, 7.06

Let R = radius of the wires in centimetre,

d = distance in centimetres between their centres,

and i = current flowing in the wires.

Let a cross-section of the line be represented as in Fig. 7·06. The flux $d\,\phi_1$, which passes through an element dr wide, and of unit length outside the conductor, is equal to the magnetomotive force divided by the reluctance. Thus—

$$d\phi_1 = \frac{\frac{4\pi i}{2\pi r}}{dr} = \frac{2\,i}{r}dr.$$

Integrating for values of r between $d-R$ and R,

$$\phi_1 = 2\,i \log\left(\frac{d-R}{R}\right),$$

$$= 2\,i \log (d/R) \text{ approximatly.}$$

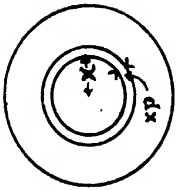


Fig. 7.07

(2) There is some flux which surrounds the axis of the right-hand wire, and which lies inside the metal. This is of appreciable magnitude owing to the greater flux density near the wire. Represent the wire by the circle in Fig. 7·07, and suppose that the current is uniformly distributed over the wire. Then as R is the radius of the wire, the current inside the circle of radius x is $\frac{x^2}{R^2}$ i, and the magnetomotive force which it produces is $4\pi \frac{x^2}{R^2}$ i.

The flux, however, which it produces, (which is $\phi_1 = \frac{\frac{4\pi \frac{x^2}{R^2} i}{2\pi x}}{dx} = \frac{2xidx}{R^2}$), links itself with $\frac{x^2}{R^2}$ ths of the wire. The flux through the element dx, which can be considered as linking the circuit, is therefore,

$$d\,\phi_2 = \frac{2\,\mu\,x^3\ i\,d\,x}{R^4}.$$

Integrating for values of x between o and R,

$$\phi_2 = \frac{2\,\mu\,i}{4}.$$

$$\therefore\ \phi_1 + \phi_2 = 2i\left[\log_e\left(\frac{d}{R}\right) + \frac{\mu}{4}\right].$$

For copper or aluminium wires $\mu = 1$. Hence the total flux linked with the line is

$$\phi_1 + \phi_2 = 2\,i\left[\log_e\left(\frac{d}{R}\right) + \tfrac{1}{4}\right],$$

and the inductance, in absolute units, being the flux per unit current, is

$$l = 2\log_e\left(\frac{d}{R}\right) + \tfrac{1}{2} \qquad \ldots \quad (1)$$

or, more generally,

$$l = \left[4\log_e\left(\frac{2\,d}{d^1}\right) + \mu\right]10^{-9}\ \text{henrys} \qquad \ldots \quad (2)$$

for centimetre length of two parallel conductors, where $d^1 = 2R$ (diameter of conductor).

This gives by reduction the inductance in henrys per wire per mile as—

$$L = \left[80.5 + 741\ \log_{10}\left(\frac{d}{R}\right)\right]10^{-6} \qquad \ldots \quad (3)$$

The drop in volts due to the inductance, per mile of

ne (two conductors) per unit current, is therefore

$$E_L = .00405\, f \left[2.3 \log\left(\frac{d}{R}\right) + \frac{1}{4} \right], \quad \ldots \quad (4)$$

where d and R must be in terms of the same unit.

It will be noted that the inductance depends upon the distance between conductors. This distance should increase with the voltage, but there is no definite relation between them. For each of two parallel iron wires we have the following:—

$$L \text{ per mile} = \left(12070 + 741 \log_{10} \frac{d}{R}\right) 10^{-6}.$$

Example 14. Determine the inductance of an overhead line, $1\frac{1}{2}$ miles long, consisting of two No. 0000 S. W. G. wires, 48 inches apart.

Solution :—

Since it is a metallic circuit with two conductors, the total inductance is due to $2 \times 1\frac{1}{2} = 3$ miles of wire.

$$d = .4'' \therefore r = .2''.$$

The inductance per mile,

$$L = \left(80.5 + 741 \log_{10} \frac{48}{\cdot 2}\right) \times 10^{-6} \text{ henry.}$$

$$= 1245.5 \times 10^{-6} \text{ henry} = 12.455 \text{ millihenries.}$$

$\therefore$ The total inductance

$$= 3 \times 1245.5 = 37.365.$$

Example 15. Determine the self-inductance of 1 mile of a two-wire transmission line of copper or any

non-magnetic wire, diameter of wire $= \cdot 10$ in., the two sides of the circuit being spaced 48 ins. between centres..

Solution :—

Use the formula : $L = 0.741 \times \log\left(2.568 \frac{D}{d}\right)$.

$$L = 0.741 \times \log\left(2.568 \frac{D}{d}\right),$$

$$= 0.741 \times \log\left(2.568 \frac{48}{0.1}\right),$$

$$= 0.741 \times \log\,(2.568 \times 480),$$

$$= .741 \times \log 1232.64,$$

$$= 0.741 \times 3.0906 = 2.29 \text{ millihenries.}$$

This is the inductance of 1 mile of one wire of the circuit. For both wires (1 mile of circuit $=2$ miles of wire) the inductance will be :

$$2 \times 2.29 = 4.58 \text{ millihenries.}$$

Example 16. Calculate the self-inductance per wire of a three-phase transmission line of 20 miles length consisting of three wires of No. 0 S. W. G. (diam. $=.82$ cm.), 48 cms. apart, transmitting the output of a 500 Kw. 6600-volt star-connected three-phase machine.

Solution:—

The effective E. M. F. per circuit $= \dfrac{6600}{\sqrt{3}} = 3810$ volts.

500 Kw. output is $\dfrac{500 \times 1000}{3} = 166666$ watts.

per phase or circuit.

$\therefore$ Current per line $= \dfrac{166666}{3810} = 43.74$ amperes effective,

$= 43.74 \sqrt{2} = 61.85$ amps. maximum.

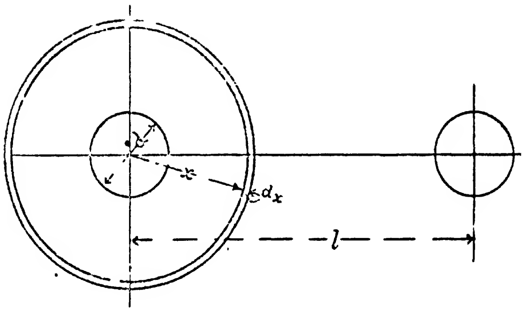


Fig. 7·08.

Now, at the distance x from the centre of one of the conductors (Fig. 7.08), the length of the magnetic circuit surrounding the conductor is $2\pi x$. If I is the current in the line, the m. m. f. is I ampere-turns, and thus the magnetising force, $f = \dfrac{I}{2\pi x}$. Hence the field intensity (in lines of magnetic force per square centimetre) is,

$$H = .4\pi f = \frac{.2\,I}{x},$$

and the flux in the zone dx is $d\phi = \dfrac{.2\,I\,l\,dx}{x}$

l_1 being the length of the conductor.

$$\therefore \phi = \int_{\frac{d}{2}}^{l} \frac{.2\, I\, l_1\, dx}{x} = .2\, I\, l_1 \Big[\log_e x\Big]_{\frac{d}{2}}^{l},$$

$$= .2\, I\, l_1 \log_e \frac{2\,l}{d}.^{*}$$

Now, $I = 61.85$ amps., $l = 20$ units $= 3220 \times 10^3$ cms., $d = .82$ cm., $l_1 = 48$ cms.

Therefore, the flux per wire is

$$\phi = .2 \times 61.85 \times 3220 \times 10^3 \times \log_e \frac{96}{.82},$$

$$= 61.85 \times 644 \times 4.76 \times 10^3,$$

$$= 190 \times 10^6 = 190 \text{ megalines.}$$

Hence, the generated E.M.F. at 50 cycles is

$$E = \sqrt{2}\, \pi f \phi,$$

$$= 4.44 \times 50 \times 190 \times 10^6$$

$$= 4.44 \times 5 \times 19 = 422 \text{ volts effective for line.}$$

The maximum value is

$$E_0 = E \times \sqrt{2} = 607.68 = 608 \text{ volts approximately per line.}$$

* Since the same flux is also produced by the return conductor in the same direction the total flux passing between the two transmission wires is

$$2\,\phi = .4\, I l_1 \log_e \frac{2\,l}{d}.$$

This result is sometimes useful.

Example 17. Find the reactance per wire of a transmission line of length l_1, if d = diameter of the wire, l = spacing of the wires, and f = frequency.

Solution :—

Let I be the current in absolute units, flowing in one wire of the transmission line, then the m.m.f. is I. and the magnetising force in the zone dx (Fig. 7·08) at distance x from the centre of the wire is $f = \frac{I}{2\pi x}$:

Thus, the field intensity in this zone, $H = 4\pi f = 2\frac{I}{x}$,

and the flux in this zone is

$$d\phi = H\, l_1 dx = \frac{2 I l_1 dx}{x}.$$

Therefore, the total flux between the wire and the return wire is

$$L\phi = \int_{\frac{d}{2}}^{l} d\phi = 2 I l_1 \int_{\frac{d}{2}}^{l} \frac{dx}{x} = 2 I l_1 \log_e \frac{2l}{d},$$

neglecting the flux inside the transmission wire.
Hence, the inductance is

$$L = \frac{\phi}{I} = 2 l_1 \log_e \frac{2l}{d} \text{ absolute units,}$$

$$= 2 l_1 \log_e \frac{2l}{d} \cdot 10^{-9} \text{ henrys ;}$$

and, therefore, the reactance

$$X = 2\pi f L = 4\pi f l_1 \log_e \frac{2l}{d} \text{ in absolute units,}$$

$$= 4\pi f l_1 \log_e \frac{2l}{d} 10^{-9} \text{ ohms.}$$

Example 18. The voltage at the receiving end of a 50-cycle three-phase transmission line 14 miles in length is 6600 between the lines. The line consists of three wires, No. 0 S. W. G. (diameter = .82 cm.), 30 inches (76 cms.) apart, and of specific resistance $\rho = 1.8 \times 10^{-6}$. Calculate :—

(a) The resistance, the reactance, and the impedance per line, and the voltage consumed thereby at 50 amperes.

(b) The generator voltage between lines at 50 amperes to (i) a non-inductive load, (ii) a load circuit of 45^0 lag, and (iii) a load circuit of 45^0 lead.

Solution:—

We have $l_1 = 14$ miles $= 14 \times 1.6 \times 10^5$ metres,
$= 2.24 \times 10^6$ cms.

$l = 76$ cms,

$d = 82$ cm.

∴ The cross section, $a = \cdot528$ sq. cm.

(a) **Resistance** per line,

$$R = \rho \frac{l_1}{a} = \frac{1.8 \times 10^{-6} \times 2.24 \times 10^6}{.528},$$

$$= 7.64. \text{ ohms.}$$

Reactance per line, $X = 4\pi f l_1 \log_e \frac{2\,l}{d} \times 10^{-9}$,

$= 4 \times 3.1416 \times 50 \times 2.24 \times 10^6 \times \log_e \; 185 \times 10^{-9}$
$= 7.35$ ohms.

Impedance per line, $z = \sqrt{R^2 + X^2} = 10{\cdot}6$ ohms.

Now, $I = 50$ amperes,

$\therefore$ The e.m.f. consumed by resistance, $E_R = RI = 7.64 \times 50$
$= 382$ volts.

" " reactance, $E_X = XI = 367.5$ volts.

" " impedance, $E_Z = ZI = 10.6 \times 50$,
$= 530$ volts.

(b) (i) 6600 volts between lines in receiving circuit give $\frac{6600}{\sqrt{3}} = 3810$ volts between line and neutral or zero point (Fig. 7.09) per line.

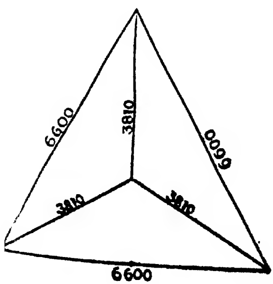


Fig. 7.09

Now, the voltage per line, E at the receiving end with non-inductive load is in phase with the current. Also, the e.m.f. consumed by resistance, E_R, is in phase with the current, and that consumed by resistance, E_X, is 90^0 ahead

of the current. Resolving all e.m.fs. into components in phase and in quadrature with the received voltage E, (Fig. 7.10) we see—

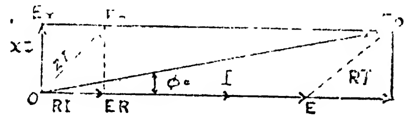


Fig. 7.10.

	Phase component.	Quadrature component.
E.m.f. at receiving end of line, E. gives	E.	0.
E.m.f. consumed by resistance, I_R, ,,	RI.	0.
E.m.f. consumed by reactance, E_X, ,,	0.	XI.

Thus, the total voltage required per line at the generator end of the line,

$$E_0 = \sqrt{(E+RI)^2+(XI)^2},$$

and the phase angle between the current and the e. m. f. per line at the generator end,

$$\phi_0 = \tan^{-1}\frac{XI}{E+RI}.$$

Substituting numerical values, we have—

$$E_0 = \sqrt{(3810+382)^2+(367.5)^2} = \sqrt{(4192)^2+(367.5)^2},$$

$= 4208$ volts per line, or, $4208\sqrt{3} \doteq 7290$ volts (effective) between the lines at the generator.

$$\phi_0 = \tan^{-1}\frac{367.5}{4192} = 5°,$$ the lag of current behind the line voltage at the generator.

(ii). In this case the current lags by an angle $\phi = 45^0$, behind the e.m.f. at the receiving end of the line. As before, resolving all e.m.fs. into components in phase and in quadrature with the received voltage E, (Fig. 7.11).

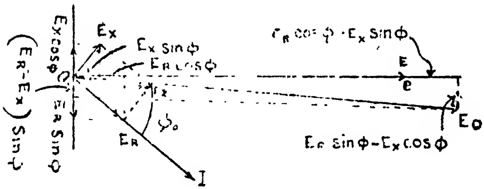


Fig. 7.11

	Phase component.	Quadrature component.
E.m.f. at receiving end of line, E, gives	E.	0.
E.m.f. consumed by resistance, E_R,...RI cos ϕ.		RI sin ϕ.
E.m.f. consumed by reactance, E_X,...XI sin ϕ.		XI cos ϕ.

Hence, the voltage per line at the generator end

$$E_0 = \sqrt{(E + RI\cos\phi + XI\sin\phi)^2 + (XI\cos\phi - RI\sin\phi)^2},$$

and, the angle between the line and the generator voltage,

$$\phi^1 = \frac{XI\cos\phi - RI\sin\phi}{E + RI\cos\phi + XI\sin\phi}.$$

Substituting numerical values, we have :—

$$E_0=\sqrt{\left(3810+\frac{749.5}{\sqrt{2}}\right)^2+\left(\frac{14.5}{\sqrt{2}}\right)^2},$$

$$=\sqrt{\{(4340)^2+(10.27)^2,\}}$$

$=4340$ volts (effective) per line nearly,

or, $4340\sqrt{3}=7518$, the effective voltage between the lines at the generator.

$$\phi^1=\tan^{-1}\left(-\frac{10.27}{4340}\right)=-0^0 8' \text{ (lag)}.$$

$\therefore\ \phi_0=\phi-\phi^1=45^0-8'=44^0\ 52'$, the lag of current behind the line voltage at the generator.

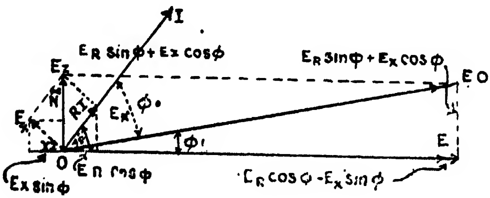


Fig. 7.12

(iii) In this case the current leads the e. m. f. at the receiving end of the line by an angle, $\phi=45°$, and the components of E, E_R and E_X in phase and in quadrature with the e.m.f. at the receiving end, E, are respectively:-

Phase Component.	Quadrature Component.
E	O
RI $\cos\phi$.	RI $\sin\phi$.
$-$XI $\sin\phi$.	XI $\cos\phi$.

Hence, the voltage per line at the generator end

$$E_0=\sqrt{(E+RI\cos\phi-XI\sin\phi)^2+(RI\sin\phi+XI\cos\phi)^2},$$

and the angle between the line and the generator voltage,

$$\phi^1=\frac{RI\sin\phi+XI\cos\phi}{E+RI\cos\phi-XI\sin\phi}.$$

Substituting numerical values,

$$E_0=\sqrt{\left(3810+\frac{14.5}{\sqrt{2}}\right)^2+\left(\frac{749.5}{\sqrt{2}}\right)^2}$$

$$=\sqrt{(3820)^2+(530)^2}$$

$=3856$ volts (effective) per line,

or, $3856\sqrt{3}=6680$,

the effective voltage between the lines at the generator.

$$\phi^1=\tan^{-1}\frac{530}{3820}=7^0\ 54' \text{ (lead).}$$

$\therefore\ \phi_0=\phi-\phi^1=45^0-7^0 54'=37^0\ 6'$, the lead of current in front of the generator voltage.

208. Inductance of Each Conductor in a Three-phase System.—The inductance of a conductor in a three-phase system depends on its position relatively to the other conductors of the system. Consider the following cases.

(a) **When the conductors are placed at the vertices of an equilateral triangle.**—The currents in A, B and C (Fig. 7.13) in a balanced three-phase system are 120 degrees out of phase, the algebraic sum of the currents is zero, and the instantaneous flux set up around any conductor is the algebraic sum of the instantaneous fluxes set up around the other two conductors. Therefore, the inductance of each conductor in a three-phase system when the conductors are placed at the vertices of an equilateral triangle, is equal to one half the inductance of the loop formed by any two of the conductors.

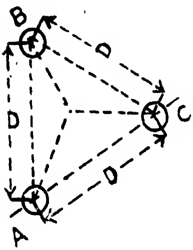


Fig. 7.13

(b) **When the conductors are in a line in the same plane**—From equation 1, the inductance of either A or C (Fig. 7.14) with regard to B is

$$L_1 = \tfrac{1}{2} + 2 \log_e \frac{D-r}{r}, \qquad \ldots \quad (5)$$

The inductance of A with regard to C, or of C with regard to A is

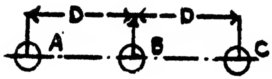


Fig. 7.14

$$L_2 = \tfrac{1}{2} + 2 \log_e \frac{2\,D-r}{r}, \qquad \ldots \quad (6)$$

and the inductance of B with regard to either A or C is

$$L_3 = \tfrac{1}{2} + 2 \log_e \frac{D-r}{r}. \qquad \dots \quad (7)$$

The conductors of a poly-phase alternating-current system are usually transposed, thus the inductances of the lines are sensibly equal. Under such condition the inductance of each conductor in a three-phase system, when the conductors are in the same plane, is

$$L = \tfrac{1}{2} + \frac{4 \log_e \frac{D-r}{r} + 2 \log_e \frac{2D-r}{r}}{3} \qquad \dots \quad (8)$$

$$= \tfrac{1}{2} + 2 \log_e \frac{1.26\,(D-r)}{r} \text{ c. g. s. units per}$$

centimetre length of conductor. ... (9)

209. Inductance of a conductor with earth return—The flux linking with the current in each of two parallel conductors separated D centimetres from each other, passes between the axis of the wire, and a neutral plane distant D/2 centimetres from the axis of the wire. When the return current of a circuit flows through the earth, the surface of the earth is the neutral plane, and the inductance of a conductor placed h centimetres above, and parallel to, the surface of the earth is

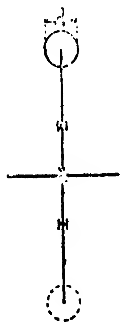


Fig. 7.15

$L = \frac{1}{2} + 2 \log_e \frac{2h - r}{r}$ c. g. s. units per centimetre length of conductor. ...... (10)

Note.—Equation (10) is strictly true only when the resistance of the earth return is zero, but a considerable resistance in the return circuit does not greatly increase the inductance.

210. Means of reducing Self Inductance.—Self inductance is diminished by:—

(1) Reducing the interaxial distance D between two wires forming a metallic circuit, as this distance determines the interlinking of the magnetic lines with the circuit.

(2) Subdividing the conductor and using several smaller wires having the same total sectional area.

(3) Balancing inductance by the effect of capacity.

If C = the capacity in farads, required to neutralize a certain inductance,

L = Inductance in henrys.

f = frequency.

tangent of angle of lag or lead

$$= \frac{2\pi f L - \frac{1}{2\pi f C}}{R}$$

and $$C = \frac{1}{L(2\pi f)^2}.$$

(4) Synchronous motors, having the effect of capacity when their field magnets are over-excited may be used to balance inductance.

211. Means of reducing Mutual Inductance.—

(1) Increasing the distance between the conductors.

(2) Transposing the wires with respect to each other at certain intervals.

212. Energy Stored in a Magnetic Field —

Let L = inductance in any circuit,

e_l = the E. M. F. due to inductance,

p_l = the power delivered in the circuit,

w_l = the energy required to increase or decrease the intensity of the magnet.

Then,

$$e_l = L \frac{di}{dt}. \qquad \ldots \qquad (11)$$

$$p_l = ei, \qquad \ldots \qquad (12)$$

$$= Li \frac{di}{dt}. \qquad \ldots \qquad (13)$$

$$w_L = \int p \, dt, \qquad \ldots \qquad (14)$$

$$= L \int i \, di, \qquad \ldots \qquad (15)$$

$$= L \frac{i^2}{2}, \qquad \ldots \qquad (16)$$

when the current varies between i and zero.

Substituting the value of L found in equation :

$$L = \frac{4 \pi s^2 A \mu}{l} \text{ c. g. s. units,}$$

$$w_1 = \frac{4 \pi s^2 A \mu}{l} \times \frac{i^2}{2}, \qquad \ldots \qquad (17)$$

$$= \frac{16 \pi^2 s^2 i^2}{l^2} \times \frac{A \mu l}{8 \pi}. \qquad (18)$$

But the intensity of the magnetic field is

$$H = \frac{4 \pi s i}{l}. \qquad \ldots \qquad (19)$$

$$\therefore w_1 = H^2 \times \frac{A l \mu}{8 \pi};$$

and the volume of the magnet is

$$V = A\, l \text{ cubic centimetres.} \qquad (20)$$

Substituting the values of H and V in equation (17)

$$w_1 = \frac{\mu H^2}{8\pi} \times V \text{ ergs,} \qquad \ldots \qquad (21)$$

$$= \frac{B H}{8 \pi} \text{ ergs per cubic centimetre,} \qquad (22)$$

$$= \frac{B^2}{8 \pi \mu} \text{ ergs per cubic centimetre.} \qquad (23)$$

213. Force of Magnetic Traction.—Take an iron ring uniformly wound so that there is no magnetic leakage. Divide the ring into two similar halves. Let A be the total cross-section of the two air gaps, the flux density is the same in the iron as in the air gaps.

A small decrease of flux $d\phi = \mathrm{A}d\mathrm{B}$ is caused by separating the two halves of the ring by a small distance dl. The iron loses an amount of stored energy which is given up to the electric circuit, but we have to exert a force of f dynes and do $f\,dl$ ergs of work which must be stored in the magnetic field in the air gaps.

Now, the energy stored in each cubic centimetre in the field in the air

$$= \frac{\mathrm{B}^2}{8\pi} \text{ ergs.} = \mathrm{B}^2/8\pi \text{ ergs.}$$

therefore the energy stored in the air gaps

$$= \frac{\mathrm{B}^2\,\mathrm{A}\,dl}{8\pi} \text{ ergs.}$$

$$\therefore \mathrm{f}\,dl = \frac{\mathrm{B}^2\,\mathrm{A}\,dl}{8\pi} \text{ ergs.}$$

$$\therefore \mathrm{f} = \frac{\mathrm{B}^2\,\mathrm{A}}{8\pi} \text{ dynes.}$$

$$= \frac{\mathrm{AB}^2}{24655} \text{ grams} = \frac{\mathrm{AB}^2}{11183000} \text{ pounds.}$$

If the units involve inches, $f = \dfrac{\mathrm{AB}^2}{72134000}$ pounds.

214. Alternative proof:—

Let each of the two parallel plane surfaces A, B (Fig. 7.15) in contact be oppositely and uniformly charged with m units of magnetism per sq. cm. Then, the strength of the field between them is $4\pi m$; each surface contributing $2\pi m$ lines per sq. cm. For, excepting close to the edges the field is normal to the surfaces, and the strength of the normal field close to a magnetised surface is of value $2\pi m$.

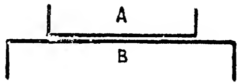


Fig. 7.16

Hence, the force exerted by one surface, say B, on the magnetic charge on each sq. cm. of the other surface A, is

$$2\pi m \times m = 2\pi m^2 \text{ dynes.}$$

If the smaller surface has an area a sq. cms., the total pull between the two surfaces will be

$$P = 2\pi m^2 a \text{ dynes.}$$

Now, if B is the flux density between the surfaces produced by a magnetising field H, we have

$$B - H = 4\pi m.$$

But, H is generally very small in comparison with B, in cases like above, and may therefore be neglected. Hence, the tractive force or pull becomes,

$$P = \frac{B^2}{8\pi} a \text{ dynes.}$$

215. Practical Calculation of Inductance of a Coil.—

It is sometimes of practical importance to calculate the self-inductance of a coil or solenoid from its number of turns and dimensions. This may be done by using the following formulae :—

(a) **Brook and Turner's Formula.**—By means of the formula below the inductance of any closely wound cylindrical coil (Fig. 7.16) may be calculated.

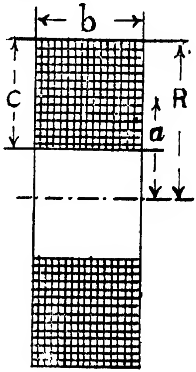


Fig. 7.17

$$L=\frac{4\,\pi^2\,a^2\,N^2\;F'\,F''}{b+c+R}\ \text{milhenries,}$$

when a = the mean radius of the winding, in centimetres,

b = the axial length of the coil, in centimetres,

c = the thickness of the winding, in centimetres,

R = the outer radius of the winding, in centimetres,

N = the total number of turns in the winding,

$$F'=\frac{10\,b+12\,c+2\,R}{10\,b+10\,c+1.4R},$$

$$F''=0.5\log_{10}\left(100+\frac{14\,R}{2\,b+3\,c}\right).$$

The error seldom exceeds 4 per cent for a coil of any dimensions, and becomes less as the relative length of the coil increases.

It is sometimes of practical importance to calculate the self-inductance of a solenoid from its number of turns and dimensions. This may be done by using the following formula :—

(b) **Louis Cohen's Fromula.**—Let r_1 be the mean radius of the first layer of the solenoid, r_2 that of the second, and so on. Also, let r_m be the mean radius of the turns, S the total number of turns, l the length, n the number of layers, and d_1 the distance between consecutive layers of the solenoid. Then, if all the lengths are in centimetres, and

$$A = n\left\{\frac{2\, r_m^4 + r_m^2 l^2}{\sqrt{4\, r_m^2 + l^2}} - \frac{8}{3\pi} r_m^2\right\},$$

$$B = \left[(n-1)\, r_1^2 + (n-2)\, r_2^2 + (n-3)\, r_3^2 + \text{etc.}\right],$$

$$C = \sqrt{r_1^2 + l^2} - \frac{7}{8} r_1,$$

$$D = \left[n\,(n-1)\, r_1^2 + (n-1)\,(n-2)\, r_2^2 + (n-2)\,(n-3)\, r_3^2 + \text{etc.}\right],$$

$$E = \frac{r_1 d_1}{\sqrt{r_1^2 + l^2}} - d_1,$$

$$F = \frac{d_1}{8}\left[n\,(n-1)\, r_1^2 + (n-2)\,(n-3)\, r_2^2 + \text{etc.}\right],$$

Cohen's formula is,

$$L = \frac{4\,\pi^2\, S^2}{n^2\, l^2}\left\{A + 2BC + DE - F\right\} 10^{-9} \text{ henrys.}$$

This value is true to one-half per cent. for values of l as low as twice the diameter d of the solenoid, and

becomes more and more accurate as l/d increases. When this ratio becomes greater than 4, the last two terms D, E and F are comparatively small, and may be neglected unless greater accuracy is desired.

216. Growth of Current in an Inductive Circuit.—If a constant electromotive force is applied to a circuit containing both resistance and inductance, the current takes some time to rise to its final value E/R, because of the counter-electromotive force induced in the circuit by the increasing flux. At the instant of closing the circuit no current flows. Let time be reckoned from this instant. At any subsequent instant, t seconds later, the impressed E.M.F. may be considered as the sum of two parts, E_L and E_r. The first, E_L, is that part which is opposed to, and just neutralizes E_s, the E. M. F. of self-induction, so that $E_L = -E_s$; but

$$E_s = -L\frac{dI}{dt}.$$

$$\therefore E_L = L\frac{dI}{dt}.$$

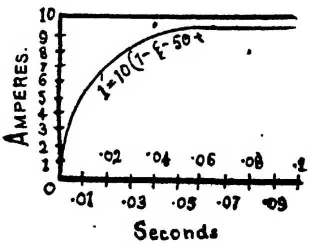


Fig. 7.18

The second part, E_r is that which is necessary to send current through the resistance of the resistance circuit, so $E_r = RI$.

The applied electromotive force, being the sum of E_L and E_r, is therefore

$$E = RI + L\frac{dI}{dt};$$

whence.

$$dt = \frac{L}{E-RI}\,dI = -\frac{L}{R}\cdot\frac{-R\,dI}{E-RI}.$$

Integrating from the initial conditions $t=0$, $I=0$, to any condition $t=t$, $I=I'$,

$$t = -\left[\frac{L}{R}\log_\epsilon(E-RI') - \log_\epsilon E.\right]$$

Therefore, $$-\frac{Rt}{L} = \log_\epsilon\left(\frac{E-RI'}{E}\right),$$

from which the instantaneous current value is

$$I = \frac{E}{R}\left(1-\epsilon^{-\frac{Rt}{L}}\right),$$

where ϵ is the base of the natural system of logarithms, and numerically equal to 2.7183. Thus the rise of current in an inductive circuit follows a logarithmic curve; and when t is of sufficient magnitude to render the second term negligible, the value of the current will be as given by Ohm's law, a condition which agrees with experimental observations.

A curve of the growth of current in a circuit having resistance and inductance is shown in (Fig. 7.18), the values of I being calculated for the conditions noted.

Example 19. A circuit has a resistance of 5 ohms and an inductance of 1 henry. If the applied voltage

is 220 determine the current .1 and 1 second after the application of the E. M. F.

Solution :—

$$i = \frac{220}{5}\left(1 - \epsilon^{-\frac{5t}{1}}\right),$$

$$= 44\left(1 - \epsilon^{-\frac{5t}{1}}\right).$$

$$\left[\text{Since } I = \frac{E}{r}\left(1 - \epsilon^{-\frac{rt}{L}}\right)\right]$$

At the end of 0. 1 second after the E. M. F. has been applied the value of the current becomes

$i = 44 \ (1 - 2.718^{-5})$,

$= 17.6$ amperes.

At the end of 1 second the value of the current becomes

$i = 44(1 - 2.718^{-5})$,

$= 43.7$ amperes.

The ratio L/R is called the TIME CONSTANT, and the larger this ratio is, the greater is the time taken by the current in rising to its final value. Theoretically it takes an infinite time to rise to the value given by the ratio E/R, but as far as practical measurements are concerned the final value is attained very soon. For instance, in the example quoted above, the current has risen to within 0.01 per cent. of its final value at the end of one second.

217. Decay of Current in an Inductive Circuit.—When a circuit containing both resistance and inductance is disconnected from a source of constant electromotive force, and the circuit closed, the current does not immediately fall to zero, but is maintained by the electromotive force induced in the circuit by the decreasing flux.

At any subsequent instant, t seconds, after withdrawing the impressed E. M. F., the current at the instant of interruption of the applied E. M. F. is due solely to the electromotive force of self-induction and may be represented by $\frac{E}{R}$. Therefore,

$$E = RI + L\frac{dI}{dt} = 0,$$

$$\therefore dt = -\frac{L}{R} \cdot \frac{dI}{I}.$$

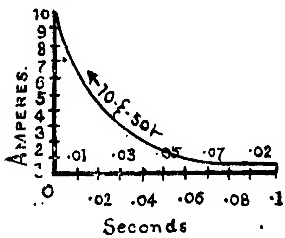


Fig. 7.19

By integrating from the initial conditions $t=0$, $I=\frac{E}{R}$, to any condition $t=t$, $I=I'$, the instantaneous value of the current is found to be

$$I = \frac{E}{R} e^{-\frac{Rt}{L}},$$

which is the term that had to be subtracted in the formula for the growth of current. This shows clearly that, while self-induction prevents the instantaneous attainment of the ultimate value of the current, there

is eventually no loss of energy, since what is subtracted from the growing current is given back to the decaying current.

Fig.7.19 is the curve of decay of current in the same circuit as was considered in Fig. 7.18. The ordinates of the one figure are seen to be complementary to those of the other.

Example 20. An inductance of .17 henry is present in a circuit of 2.9 ohms resistance ; 1110 volts of E. M. F. is maintained on the circuit. Calculate the rate of change (a) at the first application of the E. M. F. ; (b) at the expiration of the time required for the current to grow to 37 amperes ; (c) the same for 350 amperes.

$$\text{(a) } \Sigma = L\frac{di}{dt} \quad \therefore \frac{di}{dt} = \frac{\Sigma}{L} = \frac{1110}{.17}$$

$$= 65.29 \text{ amperes per sec.}$$

(b) When the current has attained the value of 37 amperes, the E. M. F. expended on the inductive coil due to its resistance is its RI drop, or $37 \times 2.9 = 107.3$; $1110 - 107.3 = 1002.7$ volts. This voltage is left free to act, to increase the current and to be in equilibrium with the counter E. M. F. caused by such increase. In other words, the current can only increase at a rate which will produce counter E.M.F. equal to the E.M.F. producing the increase.

$$\therefore \text{ rate of change at the 37 ampere point is } \frac{1002.7}{.17}$$

$$= 5898 \text{ amperes per sec.}$$

(c) Similarly

$$=1110-350\times 2.9$$

$$=95.$$

$$\therefore \frac{di}{dt}=\frac{95}{.17}=559 \text{ amperes per sec.}$$

Example 21. The resistance of an alternator field is 35 ohms, and the inductance 110 henrys. If 250 volts are impressed upon the field, how long will it take for the field to reach (a) $\frac{1}{2}$ strength, (b) $\frac{9}{10}$ strength with full field strength, what is the energy stored as magnetism ?

Solution.—

The final value of current is

$$=\frac{E}{R}=\frac{250}{35}=7.143 \text{ amps.}$$

Hence, the current at time t is

$$i=i_0\left(1-e^{\frac{-Rt}{L}}\right),$$

$$=7.143\left(1-e^{-.3182\,t}\right)$$

(a) $\frac{1}{2}$ strength : $i=\frac{i_0}{2}$,

$$\therefore 0.5=1-e^{-.3182\,t}$$

or $e^{-.3182\,t}=.5.$

or $\log_e(.5)=.3182t$

$$\therefore t=\frac{.6935}{.3182}=2.18 \text{ seconds.}$$

(b) $\frac{9}{10}$ strength : $i=.9\, i_0$,

$$\therefore\ 0.9=1-e^{-.3182\, t}$$

$$\text{or } e^{-.3182\, t}=.1$$

$$\text{or } \log_e (.1)=-.3182\, t$$

$$\therefore\ t=\frac{2.3026}{.3182}=7.23 \text{ seconds.}$$

The energy stored is given by $\frac{i_0^2\, L}{2}=\frac{(7.143)^2\times 110}{2}$

$=2806$ watt-seconds or joules $=2068$ foot-pounds.

Example 22. If 500 volts are impressed upon the field of the alternator in the above example, and a non-inductive resistance inserted in series so as to give the required exciting current of 7.143 amperes, how long will it take for the field to reach (a) $\frac{1}{2}$ strength, (b) $\frac{9}{10}$ strength, and (c) what is the resistance required in the rheostat ?

Solution.—

(a) $i=\frac{i_0}{2}$, after $t=\frac{.6935}{.6364}=1.09$ seconds.

(b) $i=.9i_0$, after $t=\frac{2.3026}{.6364}=3.615$ seconds.

(c) For i_0 to be 7.143 amperes with

$E=500$ volts, a resistance $R=\frac{500}{7.143}=70$ ohms

is required. Thus, the rheostat must have a resistance of $70-35=35$ ohms.

We then have

$$i=i_0(1-e^{-\frac{Rt}{L}})$$
$$=7.143(1-e^{-.6364t}).$$

Example 23. If 500 volts are impressed upon the field of the above alternator without insertion of resistance, how long will it take for the field to reach full strength ? What is then the energy stored as magnetism ?

Solution.—

$$\text{We have } i_0=\frac{E}{r}=\frac{500}{35}=14.286 \text{ amps.}$$

$$\text{and } i=7.143 \text{ amps.}$$

Therefore, at full field strength,

$$i=i_0\ (1-e^{-\frac{Rt}{L}}) \text{ becomes}$$
$$7.143=14.286\ (1-e^{-.3182t}).$$
$$\text{or}\quad e^{-.3182t}=.5$$
$$\text{or}\quad \log_e (.5)=-.3182\ t$$
$$\therefore\quad t=\frac{.6935}{.3189}=2.18 \text{ seconds ;}$$

that is, the same as in Ex. 1., case (a).

Since the field reaches full strength in 2.18 seconds, the average power input, in this case, is (see Ex. 21 p. 406.)

$$\frac{2068}{2.18}=944 \text{ foot-pounds per second}=1.72 \text{ H. P.}$$

In breaking the field circuit of this alternator, 2058 footpounds of energy have to be dissipated in the spark, etc.

Example 24. A circuit has an inductance of 0.1 henry and a non-inductive resistance of 10 ohms in series. If the circuit is connected across a direct current voltage of 120, what will be the voltage drop across the inductance and also that across the resistance:

(a) At the instant the switch is closed?

(b) After the current has reached its maximum value?

What will be the maximum value of the current?

Solution.—

Let i = the Instantaneous Current,

$$i=\frac{E}{R}\left(1-\epsilon^{-\frac{Rt}{L}}\right).$$

(a) At the instant the switch is closed.

$t=0,$

$$\therefore i=\frac{120}{10}\left(1-\epsilon^{-\frac{Rt}{L}}\right)=0 \text{ and } \frac{R}{L}=100.$$

$$\therefore V_R = Ri = 0 \text{ volts.}$$

$$\frac{di}{dt}=12\,\epsilon^{-100t}\times 100$$

$$\text{If } t=0,\ V_L=\frac{di}{dt}=12\epsilon^{-100t}\times 100$$

$$L\frac{di}{dt}=.1\times 12\times 100=120 \text{ volts.}$$

$$\text{(b) } i=\frac{E}{\sqrt{R^2+(2\pi fl)^2}}$$

i is maximum when $2\pi f l=0$ or $l=0$.

$$\therefore\ V_L=L\frac{di}{dt}=0 \text{ and } V_R=Ri=120 \text{ volts.}$$

There being no inductance at the time

$$i=\frac{E}{R}=\frac{120}{10}=12 \text{ amperes.}$$

and $V_R=120$ volts.

Example 25. A coil of resistance .002 ohm and inductance .004 millihenry, carrying a current of 100 amperes, is short-circuited. (a) What is the equation of the current after short-circuit ? (b) In what time has the current decreased to .1 of its initial value ?

Solution :—

(a) When the coil is short-circuited. $R_1=0$, and we get

$$i=i_0\ e^{-\frac{Rt}{L}}$$
$$=100\ e^{-500t}$$

(b)

$$i=.1\ i_0=10,$$
$$\therefore\ 10=100\ e^{-500t} \text{ or } e^{-500t}=.1,$$
$$\therefore\ t=.00461 \text{ second.}$$

Example 26. In the above example, when short-circuiting the coil an e. m. f. of 1 volt is inserted in the circuit in opposite direction to the current.

(a) What is the equation of the current ?

(b) In What time does the current reduce to zero ?

(c) After what time does the current attain its initial value in opposite direction.

(d) What impressed e. m. f. will make the current die out in $\frac{1}{1000}$ second ?

(e) What impressed e. m. f. is required to reverse the current to its initial value in opposite direction in $\frac{1}{500}$ second ?

Solution :—

(a) If e. m. f. $-E$ is inserted, and at time t the current is denoted by i. then

$$e_1 = -L\frac{di}{dt}. \text{ the generated e. m. f.}$$

Hence, $-E + e_1 = -E - L\frac{di}{dt}$, the total e. m. f,

and $i = \frac{-E+e_1}{R} = -\frac{E}{R} - \frac{L}{R}\,\frac{di}{dt}$, the current.

Whence, $-\frac{R}{L}dt = \frac{di}{\frac{E}{R}+i}$.

Integrating, $-\frac{Rt}{L} = \log_e\left(\frac{E}{R}+i\right) - \log_e I$.

Now, at $t=0$, $i=i_0$, and therefore, $I = i_0 + \frac{E}{R}$.

Thus, $$i = \left(i_0 + \frac{E}{R}\right)e^{-\frac{Rt}{L}} - \frac{E}{R}$$

$$= 600\ e^{-500t} - 500.$$

(b) $i=0,\ e^{-500\,t}=\frac{5}{6}=.833$, thus $t=.000364$ second.

(c) $i=-i_0=-100,\ e^{-500\,t}=\frac{2}{3}=.667$, thus $t=.000812$ second.

(d) If $i=0$ at $t=.001$ second,

$$0=(100+500\ E)\ e^{-.5}-500\ E.$$

$$\therefore \quad E=\frac{e^{-.5}}{5\ (1-e^{-.5})}=\frac{.2}{e^{.5}-1}=.31 \text{ volt.}$$

(e) If $i=-i_0=-100$ at $t=.002$,

$$-100=(100+500\ E)\ e^{-1}-500\ E,$$

$$E=\frac{1+e^{-1}}{5\ (1-e^{-1})}=.43 \text{ volt.}$$

218. Quantity of Electricity Traversing a Circuit Due to a Change of Flux Linked with it.—If the circuit have a resistance of r, and in time dt, the flux linked with n turns changes by $d\phi$, then the instantaneous current

$$i=\frac{\frac{n\,d\,\phi}{d\,t}}{r}$$

But the quantity of electricity flowing during the time d t is $dq=idt$, hence

$$dq=\frac{n\,d\,\phi}{r},$$

which is independent of time. So if the flux changes from ϕ_1 to ϕ_2, then

$$q=\frac{\phi_1-\phi_2}{r}\,n.$$

If the resistance of the circuit be expressed in ohms and the flux in maxwells, the quantity of electricity in micro-coulombs will be

$$Q = \frac{n}{100} \cdot \frac{\phi_2 - \phi_1}{R}.$$

219. Voltage and Current Relation in an Inductive Circuit.—The current in an alternating curreut circuit changes from $-I_m$ to $+I_m$ in the time of half a cycle or in $1/2f$ seconds so that E_{av}, the average voltage of self-induction between

b and $c = L\left(\text{average value of } \frac{di}{dt}\right) = L\left(\frac{2I_m}{\frac{1}{2f}}\right) = 4f\ LI_m$

and E_{max}, the maximum voltage of self-induction

$$= \frac{\pi}{2} \times E_{av} = \frac{\pi}{2} \times 4 f L I_m$$

$$= 2 \pi f L I_m$$

$$\therefore E = 2 \pi f L I.$$

In direct current circuits $E = IR$. In inductive circuits of negligible resistance $E = Ix$, where x, called the inductive reactance, is expresssed in ohms and is numerically equal to $2\pi f L$.

Example 27. An alternating E. M. F. of 220 volts sends 2.2 amperes through an inductance coil of negligible resistance at 50 cycles. Find the

reactance at 50 cycles. Find also the coefficient of self-induction and the current at 30 cycles.

Solution.—

$$x \text{ the reactance } = \frac{E}{I} = \frac{220}{2.2} = .100 \text{ ohms.}$$

L the coefficient of self Induction

$$= \frac{X}{2\pi f} = \frac{100}{2\pi 50} = \cdot 318 \text{ henries.}$$

I the current $= \frac{E}{X}$ is inversely proportional to frequency and so has values of 2.2 amperes at 50 cycles and 3.667 amperes at 30 cycles.

220. The Iductance current lags 90° behind the Applied Voltage.

Let i be the current flowing in the inductive circuit.

ω = the angular Velocity

Then $i = I_m \sin \omega t$

The impressed voltage which counteracts the effect of self-induction is

$$e = +L\frac{di}{dt}$$

$$= +L\frac{d\,(I_m \sin \omega\, t)}{dt.}$$

$$= \omega\, L\, I_m \cos \omega\, t.$$

$$= E^m \cos \omega\, t$$

$$= E_m \sin (\omega\, t + 90^0)$$

The sin wave of current therefore lags 90^0 behind the sin e wave of voltage which produces it.

EXERCISES.

1. What is the field winding inductance of a bi-polar generator, having 8000 ampere turns per spool and a total flux of 3·4 mega-maxwells when the exciting current is 3 amperes ?

2. Find the inductance of a cast steel test ring coil of 500 turns when carrying 8 amperes, the test ring being 6″ outside and 5″ inside diameter and 2½″ in axial depth.

3. Determine the inductance of a pole-line 15 miles long and consisting of a pair of No. 1 S. W. G. copper wires separated by a distance. between centres. of 30 inches.

4. Determine the self-inductance of a solenoid consisting of 20 layers of No.16 S. W. G. double-cotton covered wire, 100 turns per layer, wound upon a cylindrical wooden core 2 inches in diameter.

5. Find the value of the current in a circuit having 8 ohms resistance and an inductance of 0·2 henry, ·03 seconds after impressing 110 volts upon that circuit. What would be the current ·02 seconds after suppressing the E. M. F. in the circuit a constant flow having been previously established ?

6. What is the time constant of a circuit in which the current reaches half of its ultimate value ·002 seconds after connection with a source of E. M. F. ?

7. Find the current produced by a 50 ∼ alternating E. M. F. of 220 volts in a circuit having 10 ohms resistance and an inductance of ·05 henry. What is the power factor of the circuit?

8. Find the instantaneous value of a 50 ∼ alternating current, 2·342 seconds after impressing a harmonic E. M. F. of 125 volts maximum upon a circuit which has a resistance of 8 ohms and an inductance of 0·04 henry.

9. What is the "co-efficient of self-induction" of a coil? An iron magnetic circuit has a mean length of path of 50 centimetres and a cross section of 20 square centimetres. It is wound with 200 turns of wire. Find the total flux in the magnetic circuit when the coil carries a current of 1 ampere. Neglecting the resistance, find the voltage which must be applied to the terminals to make the current rise at the rate of 2 amperes per second. Take the permeability of the iron as constant at 2000 c. g. s. units. (C. G. Grade II., A. C., 1914).

10. In an inductive circuit with inductance L and resistance R, determine the relation between current and potential difference when the circuit is carrying an alternating current of 50 cycles per second. Find also the phase difference between current and potential difference for such a circuit. (C. G. Grade II., A. C., 1913).

11. An electromotive force of 100 volts, virtual value, is impressed on a circuit consisting of a resistance of 5 ohms in series with an inductance of 0·01 henry.

State current and power factor. The frequency is 50 cycles per second. (Ord., A. C., 1911).

12. (a) A coil of 300 turns, wound on a wooden ring which has a mean diameter of 10 cm. and a circular cross section of 4 sq. cm., is threaded by a flux of 150 lines of force when the current flowing is 5 amp. Find the inductance of the coil.

(b) When the wooden ring is replaced by a steel ring of the same dimensions, the magnetic flux is 40,000 lines when the current is 1.5 amp. Find the inductance of the coil.

(c) When the steel ring has an air gap of 0.2 cm. length, the magnetic flux produced by a current of 9·5 amp. is 40,000. What is the inductance of this coil ?

13. A coil of negligible resistance has an inductance of 0.2 henry and is connected across a 220-volt line:—

(a) What current will flow at 25,50 and 120 cycles ?

(b) What is the average power taken from the line in each case ?

(c) What is the maximum rate at which energy is given to the circuit during one half cycle and returned by the circuit to the line during the next half cycle ?

(d) Explain without formulæ why the current is inversely proportional to the frequency, the voltage being constant.

14. A coil with a non-inductive resistance of 15 ohms is connected across a 220-volt line:—

(a) What current will flow when the frequency is 25,50 and 120 cycles?

(b) What is the average power taken from the line in each case?

(c) What is the maximum rate at which energy is given to the circuit during each half cycle?

15. An inductance of 0.2 henry and a non-inductive resistance of 15 ohms are connected in series across a 220-volt line:—

(a) What current will flow when the frequency is 25, 50 and 120 cycles?

(b) What is the average power taken from the line in each case?

(c) What is voltage drop across each part of the circuit at 50 cycles?

(d) What is power factor of the circuit in each case?

Note from the above figures that, since the average power taken by an inductance is zero, that taken by a circuit having resistance and inductance in series $= I^2R$ watts.

16. The load on a 220-volt alternator consists of 200 incandescent lamps each of which takes 0.5 amp., also 20 hp. of motors with an average efficiency of 85 per cent. and an average power factor of 80 per cent. Find the current output of the alternator, the kilowatt output of the alternator, the power factor of the total load (the power factor of incandescent lamps is 100 per cent.), and the horse power of the engine,

if the alternator efficiency is 90 per cent.

17. A wooden ring having a mean diameter of 10 cm. and a cross section of 2×5 sq. cm. is wound with 300 turns of wire.

(a) Find the flux per ampere.

(b) Find the inductance of the coil $\left(\frac{N\phi}{I}10^{-8} \text{ henries}\right)$.

(c) Find the reactance of coil at 30 and 50 cycles.

(d) Neglecting the resistance of the coil, find the current when connected across 100 volts with frequencies as in (c).

(e) If a second coil of 600 turns is wound on top of the 200 turns what will be the inductance of this new coil ?

(f) How does the value of L vary with the number of turns of the coil ?

(g) If the two coils (600T and 200T) are connected in series what will be the total inductance ?

(h) If the two coils are connected in series but their magnetic effects are in opposition what will be the total inductance ?

18. A choke coil of negligible resistance takes 5 amp. from 220-volt, 50-cycle mains. What current will it take from 220-volt, 25-cycle mains ? Will the current and voltage be in the same phase relation as before ?

What is the average power taken from the line ?

What is the maximum rate at which energy is given to the circuit during one half cycle and returned by the circuit to the line during the next half cycle for the two frequencies?

19. · Explain exactly what you mean by the inductance of a coil with an iron core (a) when the permeability of the iron is constant, (b) when the iron is saturated and the permeability varies for each new value of the flux density. (C. and G., II).

CHAPTER VIII.

CAPACITANCE.

Two conducting surfaces, insulated from each other, possess electro-static CAPACITY. Such an arrangement embodied as a piece of apparatus is called an ELECTRICAL CONDENSER.

221. Capacity of a Condenser.—The capacity of a condenser is measured by the quantity of electricity which must be given to it to establish unit potential difference between the coatings; if one coating be earthed the capacity of the condenser will be measured by the quantity of electricity necessary to raise the other coating to unit potential, i. e., the capacity of the condenser is numerically the same as the capacity of one plate A when the other plate B is earthed.

222. Charging a Condenser in Series and in parallel.—When the plates of a condenser are connected respectively to the positive and negative terminals of a direct-current generator, the condenser becomes charged.

When several capacities are connected end to end so that the same dielectric flux passes through each of them they are said to be IN SERIES or in CASCADE.

The combination is equivalent to a single condenser, the plates of which have an area equal to the sum of the areas of the separate condensers, provided the dielectrics are of the same material and thickness.

In the case of the series or cascade arrangement (one pole being earthed, i. e., at zero potential) a charge Q resides on the inner surface of each condenser, when a charge of Q units of electricity is given to the end condenser (not earthed), and the potential V is distributed among the individual condensers according to their capacities, we have

$$V=V_1+V_2+V_3$$
$$=\frac{Q}{C_1}+\frac{Q}{C_2}+\frac{Q}{C_3}.$$

If C be the joint capacity of the combination

$$V=\frac{Q}{C}.$$

$$\therefore \frac{Q}{C}=\frac{Q}{C_1}+\frac{Q}{C_2}+\frac{Q}{C_3}+\ldots$$

$$\therefore C=\frac{1}{\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}+\ldots}$$

COMPARE WITH CONDUCTANCES IN SERIES.

The energy acquired by a charged condenser is measured by the amount of work done in charging it to the given difference of potential, and we have

$$W=\tfrac{1}{2}\ QV=\tfrac{1}{2}\ CV^2=\tfrac{1}{2}\frac{Q^2}{C}\text{ ergs.}$$

When several capacities are connected between the same pair of equipotential surfaces so that the same potential drop is established through each they are said to be in PARALLEL. The capacity of a number

of condensers in parallel is equal to the sum of the individual capacities.

If C be the joint capacity of condensers, of capacity C_1, C_2, C_3, etc., connected in parallel,

$$C = C_1 + C_2 + C_3.$$

COMPARE WITH CONDUCTANCES IN PARALLEL

223. The Quantity of Electricity Stored in a Condenser.—called the charge, is equal to the average current flowing into the condenser multiplied by the time during which it flows or is equal to $\int idt$ where i is the charging current at any instant.

In any condenser this charge is found to be directly proportional to the applied voltage or

$$q = C\ e.$$

where q is the charge in coulombs (amperes × seconds).
e is the applied voltage.
C is a constant called the capacity of the condenser and is expressed in farads.

The plates act merely as CARRIERS or DISTRIBUTORS of the charge, while its actual seat, is the surface of the dielectric.

The capacity of any given condenser is determined by the dimensions of its plates, their distance apart, and the nature of the dielectric which separates them.

The charge dq due to current i during interval dt is:

$$dq = idt. \quad \ldots \quad (1)$$

The practical unit of charge or quantity, q, is the coulomb.

Again $q = C\,e.$ ... (2)

Substituting from (1) into (2), since

$$i = \frac{dq}{dt},$$

and $dq = C de,$

$$\therefore\ i = C\frac{de}{dt}. \quad \ldots \quad (3)$$

which is called THE CHARGING CURRENT, OR CAPACITY CURRENT of the condenser.

$$\therefore\ C = \frac{i}{\frac{de}{dt}}\cdot \text{ Compare, } L = \frac{e}{\frac{di}{dt}}$$

The formula for charging current per conductor is

$$i = \frac{2\pi f \times C \times E}{10^6} \times \text{length in miles.}$$

when C (in micro farads mile) $= \dfrac{\cdot 03883}{\log \frac{d}{r}}.$

224. To Transform Capacity Expressed in Electrostatic Units into Electromagnetic Units—the former should be multiplied by $\frac{1}{V^2}$ where V is the velocity of light $= 3 \times 10^{10}$ cm. per sec.

The practical electromagnetic unit of capacity, is farads where C is expressed in electrostatic units,

Capacity in farads $= \dfrac{C}{V^2} \times 10^9$ where C is in e. s. unit

$$\therefore \text{Farads} = \text{Electrostatic unit} \times \frac{1}{9 \times 10^{11}} = \frac{cm}{9 \times 10^{11}}$$

One Microfarad $= 1 \times 10^{-6}$ farad (millionth part of the farad.)

225. Voltage and Current relations in Capacity Circuits.—The charge in the condenser changes from zero to $Qm = CEm$ coulombs in the time of one quarter of a cycle or $\frac{1}{4f}$ seconds so that, since

$$\text{charge} = \text{average current} \times \text{time}$$

$$Qm = CE_m = I_{av} \times \frac{1}{4f}$$

and I_{av} the average current $= 4fCEm$ amp.

Now the maximum charging current I_m

$$= I_{av} \times \frac{\pi}{2} = \frac{\pi}{2} \times 4fCE$$

$$\therefore \; I_{eff.} = 2\pi \times f\,CE_{eff}$$

In direct current circuits $E = IR$.

In capacity circuits $E = IX$,

where X is called the CAPACITY REACTANCE and is expressed in ohms, it is numerically equal to $\frac{1}{2\pi f C}$.

ALTERNATIVE METHOD

$$Q = C\,e = \int i\,dt, \quad \therefore \quad i = C.\frac{de}{dt}$$

If the voltage applied to the capacity circuit be $e = E_m \sin\theta$, where $\theta = 2\pi$ per cycle or $2\pi f$ per second the current i flowing through the circuit $= i = C\frac{de}{dt}$.

$$\text{or } \therefore \; i = C.\frac{de}{dt} = C.\frac{d\,(E_m \sin 2\,\pi\, f\, t)}{d\,t}.$$

$$= 2\,\pi\, f\, CE_m \cos 2\,\pi\, f\, t$$

$$= I_m \cos \theta = I_m \sin (\theta + 90).$$

The sine wave of the current therefore leads the sine wave of voltage by 90 degrees also $E = IX$ where

$$X = \frac{1}{2\,\pi\, f\, C}.$$

Example 1. An alternating E. M. F. of 110 volts sends 2.5 amps. through a capacity circuit at 50 cycles. Find the reactance also the capacity of the condenser.

Solution.—

$$X, \text{ the reactance} = \frac{E}{I} = \frac{110}{2.5} = 44 \text{ ohms.}$$

$$C, \text{ the capacity} = \frac{1}{2\,\pi\, f\, x} = \frac{1}{2\,\pi \times 50 \times 44}$$

$$= 7.24 \times 10^{-5} = 72.4 \text{ microfarads.}$$

If the voltage applied to the above circuit is kept constant at 110, find the current that will flow through the capacity circuit at 25, 50 and 100 cycles.

X, the reactance $= \dfrac{1}{2\,\pi\, f\, C}$ is inversely proportional to the frequency $= 44$ ohms at 50 cycles.

I, the current $= \dfrac{E}{X}$ is therefore proportional to the frequency and so has values of 1.25 amp. at 25 cycles and 5 amps. at 100 cycles.

226. Flux Due to Unit Charge.—The force with which a unit charge acts on another unit charge distant r centimetre in air is

$$f = \frac{I}{r^2} \text{ dynes.} \quad \ldots \quad (1)$$

and the intensity of the dielectric field is

$$F = \frac{I}{r^2} \text{ lines per sq. centimetre.} \ldots \quad (2)$$

But the surface of a sphere, the diameter of which is 2r centimetres, is 4 πr square centimetres. Therefore, the total dielectric flux emanating from unit charge is

$$= \frac{I}{r^2} \times 4 \pi r \quad \ldots \quad (3)$$

$$= 4 \pi \text{ lines.} \ldots \quad \ldots \quad (4)$$

Hence the total dielectric flux emanating from a charge $Q = 4 \pi Q$ lines.

227. Intensity.—The intensity of the electric field (lines per square centimetre in air) is numerically the same as the force which that field exerts on unit charge.

Thus, if F be the intensity of the field at a distance r from a point charge, Q. is

$$F = \frac{4 \pi Q}{\text{area of sphere of radius r}} = \frac{4 \pi Q}{4 \pi r^2} = \frac{Q}{r^2}$$

$$V = -\int F \, dr$$

The minus sign is used because work is done in bringing unit positive charge against the charge Q which is also assumed positive.

To find the potential at N at a distance of d centimetres. from the charge Q at M.

Substitute the value of F just obtained,

$$V_d = -\int_{\infty}^{d} F dr = -\int_{\infty}^{d} \frac{Q}{r^2} dr = \frac{Q}{d}$$

where d is the distance from M to N.

228. Potential Gradient.—The potential gradient, G, or the rate at which the potential changes at a given point is a measure of the electric stress to which the dielectric is subjected. The potential gradient, G, and the electric field intensity, F, are the same numerically. Thus, if the potential of a certain point falls at the rate of 10 units of potential per cm., the actual number of lines per sq. cm. at the point is also 10.

By definition,

$$G = -\frac{dv}{dr}.$$

$$\text{Since, } dV = -F dr,$$
$$G = F.$$

In a dielectric of specific inductive capacity, K, the intensity as well as the potential gradient fo· a given charge is less than in air. It is $1/k$ times the intensity in air. Thus, in the case of a sphere,

$$G = F = \frac{1}{k} \frac{Q}{r^2}.$$

The maximum possible value of G, or F under ordinary conditions in air, is not known exactly, but is

in the neighbourhood of 30,000 volts per cm., or 100 electrostatic units of potential.

229. Capacity of a Sphere—If Q e.s. units be given to a spherical conductor of radius R cm. embedded in a medium of specific inductive capacity K, the potential of the sphere is $\frac{Q}{KR}$ e. s. units. If C denote its capacity in this medium, its potential is also given by the expression Q/C ; hence $C=KR$. With air as medium K is unity, and this reduces to $C=R$ in cm. where V is the potential of the charge Q, (or $C=Q/VK$, but $V=R/Q$, $\therefore C=R/K$, $\therefore K=1$, $C=R$.) Hence the capacity of an isolated spherical conductor in air is, in e.s. units, numerically equal to its radius in centimetres; in a medium other than air, the capacity, in e.s. units, is numerically equal to its radius in centimetres multiplied by K. The value of C in Farads is found by dividing C in centimetres by the constant 9×10^{11}.

Example 2. A charge of 80 C.G.S. units raises the potential of a spherical conductor in air from 15 to 25 units. Find the radius of the conductor.

Solution.—

The capacity of the conductor is from the definition given by

$$C=\frac{80}{25-15}=\frac{80}{10}=8,$$

and since the capacity of a spherical conductor in air is measured by its radius, the radius of the given

conductor is 8 cm.

Note:—To find the capacity of a condenser :—

(1) first find the intensity, then

(2) determine the difference of potential e, then

to find C (3) apply the formula $C = \frac{Q}{e}$.

230. Capacity of Two Parallel Plates so Large that the Effects of their Edges may be Neglected.—The total flux set up by a charge, Q, is $4\pi Q$ (Fig. 8·01.)

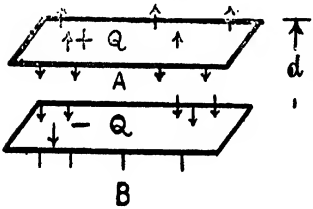


Fig. 8.01

The intensity

$F = \frac{4\pi Q}{A K}$, where K is the dielectric constant and A is the area of one side of the plate.

$e = Fd$. The potential difference is

$$e = -\int_d^0 F dx = -\int_d^0 \frac{4\pi Q}{A K} dx = \frac{4\pi Q d}{A K}.$$

where d is the distance between the plates. or the thickness of the dielectrics.

The capacity

$$C = \frac{Q}{e} = \frac{A K}{4\pi d} = \frac{k A}{4\pi d} \text{ in cm.}$$

C is given in microfarads in the expression

$$C = k \frac{A \text{ (sq. cms.)}}{4 \pi d \text{ (cms.)} \times 9 \times 10^5} \text{ microfarads}$$

when A is given in square centimetres, and the thickness d in centimetres.

The potential gradient, G, is a constant in the dielectric between the plates, since the flux lines are parallel.

Thus,

$$G = -\frac{de}{dx} = \frac{4\pi Q}{A} = \frac{5\pi Ce}{A} = \frac{e\,k}{A}$$

in which e is the difference of potential of the plates and k is a constant $= 4\pi C$.

231. Stack of Plates.—Let $N =$ the number of metal plates in the stack; there will then be $N-1$ effective dielectric sheets or $N-1$ parallel plate condensers in series. $K =$ specific inductive capacity of medium between conductors; medium assumed uniform; for air $K = 1$, $C =$ capacity of the condenser formed by the two conductors. $D =$ thickness of dielectric in inches, $A =$ the surface in square inches, a and d the dimensions in centimetres corresponding to S and D. The capacity of a stack of N metal plates, connected in series, is

$$C = \frac{Ka}{4 \pi d (N-1)} \text{ statfarads}$$

$$= 2.246 \times 10^{-7} \frac{K A}{D (N-I)} \text{ microfarads.}$$

Example 3. A condenser has plates of tinfoil which are 50′ long and 5″ wide and are separated by paraffined paper .003 thick. Find the capacity.

Solution.—

$A = 2 \times 50 \times 12 \times 5 = 6000$ sq. in. since both sides of each plate are active $= 38700$ sq.cm.

$d = .003$ in. $= .0076$ cm.

$K = 2.0$

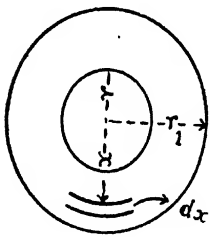
$$\therefore \text{ C in farads} = \frac{1}{4\pi} \times \frac{1}{9 \times 10^{11}} \times 38700 \times 2$$

$$= .9 \times 10^{-6} \text{ farads.}$$

$$= .9 \text{ microfarads.}$$

232. Capacity of a Spherical Concentric Condenser.—In two spherical concentric bodies with charges plus and minus Q.

where the dielectric has a specific inductive capacity K.



$$F = \frac{Q}{K x^2}$$

$$e = -\int F\, dr,$$

Fig. 80.2,

The potential difference is $e = -\int_{x=r_1}^{x=r} F dx;$

where r and r_1 are radii respectively of the inner and outer surfaces of the condenser.

$$\therefore e = -\frac{1}{k}\int_{r_1}^{r} \frac{Q}{x^2} d\,x = Q\frac{r_1 - r}{k\,r\,r_1}.$$

Therefore the capacity of the condenser is

$$C = \frac{Q}{e} = \frac{k\,r\,r_1}{r_1 - r} \text{ in cm.}$$

$$C = k\frac{r_1\,r_2}{9 \times 10^5 \times (r_1 - r_2)} \text{ microfarads.}$$

Potential gradient between concentric spheres.

Since $\quad G = -\frac{de}{d\,r},$

and $\quad de = -F dr,$

$$G = F = \frac{Q}{x^2} = \frac{C\,e}{x^2}.$$

But $\quad C = \frac{r\,r_1}{r_1 - r}. \quad \therefore \; G = \frac{r\,r_1}{r_1 - r}\;\frac{e}{x^2}.$

At the surface of the smaller sphere, $x = r$, whence the gradient is

$$G = \frac{r_1}{r}\;\frac{e}{(r_1 - r)}.$$

Alternative Proof. If the two coatings of a parallel plate condenser be formed into two spheres one within the other, the radii of which are r_1 and r_2 respectively, then the thickness of the dielectric will be

$$d = r_1 - r_2$$

and if d be very small we have

$$C = k.\frac{A}{4\pi\,(r_1 - r_2)} = k\frac{4\pi\,r_1^2}{4\pi\,(r_1 - r_2)}.$$

But $r_1^2 = r_1 r_2$ (approximately), and the capacity of a spherical condenser is

$$C = k \frac{4 \pi r_1 r_2}{4 \pi (r_1 - r_2)} = k \frac{r_1 r_2}{r_1 - r_2} \text{ C.G.S. units}$$ r_1 and r_2 being in centimetres.

233. The Capacity of a Concentric Cylinder. Let the internal radius of the outer cylinder be r_1, and r the radius of the inner cylinder or conductor. Let the charges be $+Q$ per cm. of length of the cylinder (Fig. 8.03). K = the dieletric constant. Then, by Gauss' theorem, the flux emanating from each centimetre of length $= 4 \pi Q$.

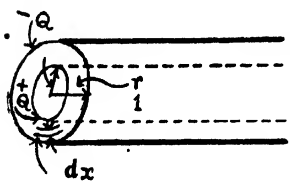


Fig. 8.03

Lines of flux are here assumed to extend radially, At a distance, x, from the centre of the cylinder the intensity at a point is the total number of lines divided by the area, or,

$$F_x = \frac{4 \pi Q}{2 \pi x K} = \frac{2 Q}{x K}.$$

Thus, the potential difference between the two cylinders

$$e = -\int_{r_1}^{r} F dx = -\int_{r_1}^{r} \frac{2 Q}{K x} dx = -\frac{2 Q}{K}\Big[\log r - \log r_1\Big]$$

$$= + \frac{2Q}{K} \log \frac{r_1}{r}.$$

and the capacity is

$$C = \frac{Q}{e} = \frac{K}{2 \log_e \frac{r_1}{r}} \quad \text{in cm.}$$

per cm. length of the concentric cylinder.

Two Concentric Cylindrical, Metallic Surfaces Insulated from each other.—As in submarine and concentric cables form a condenser, and the expression for the capacity for a length l is

$$C = k \frac{l \text{ (cms.)}}{2 \log_e \frac{r_1}{r}} \quad \text{C.G.S. units.}$$

$$= k \frac{l \text{ (cms.)}}{2.3026 \times 2 \log_{10} \frac{r_1}{r}} \quad \text{C.G.S.}$$

$$= k \frac{2.413 \; l \text{ (cms.)}}{10^7 \times \log_{10} \frac{r_1}{r}} \quad \text{microfarads.}$$

Where l is the length of the metallic cylinders in centimetres, r_1 is the internal radius of the outer cylinder, and r the external radius of the inner cylinder.

$$= \frac{7.354 \times 10^{-3} K.}{\log_{10} \frac{r_1}{r} \left(\text{or} \frac{2r_1}{d} \right)} \quad \text{microfarads per 1000 feet.}$$

The gradient at any distance, x, from the centre is

$$G = F = \frac{2Q}{X} = \frac{2Ce}{X} = \frac{2e}{X} \times \frac{1}{2 \log \frac{r_1}{r}}$$

$$= \frac{e}{X \log \dfrac{r_1}{r}}$$

At the surface of the inner conductor, $x = r$.

$$\therefore \; G = \frac{e}{r \log \dfrac{r_1}{r}}$$, and this is the greatest value of the gradient.

In these formulae no account is taken of any effects due to the ends of the concentric cylinder. For the special case of an outer cylinder of radius $r_1 = \infty$, $C = 0$.

234. Capacity of a Transmission Line.—The line is represented in section in Fig. 8.04, with, r, the radius, and D, the distance between centres, of the wires A and B. and K the dielectric constant. Let A be charged $+Q$, and B, $-Q$. The flux lines emanating from A enter B.

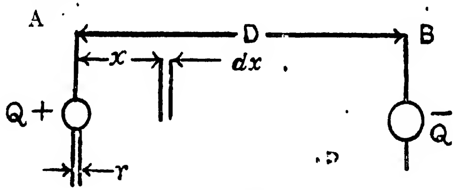


Fig. 8.04

The intensity at a point, P, due to the charge on A, is F_A, that due to the charge on B is F_B.
Then

$$F_A = \frac{4\pi Q}{2\pi x K} = \frac{2\,Q}{xK}.$$

$$F_B = \frac{4\pi Q}{2\pi(D-x)K} = \frac{2Q}{(D-x)K}.$$

The intensity due to the two charges is the sum of F_A and F_B, since the direction of the lines of electrostatic force from A, due to a positive charge, is the same as that due to B, which has a negative charge.

$$\therefore\ F = \frac{2Q}{K}\left[\frac{1}{x} + \frac{1}{D-x}\right]$$

The potential difference is :

$$e = -\int_{D-r}^{r} F\,dx = -\frac{2Q}{K}\int_{D-r}^{r}\left(\frac{1}{x} + \frac{1}{D-x}\right)dx$$

$$= \frac{4Q}{K}\log_e \frac{D-r}{r}. \qquad \ldots \qquad \text{(A)}$$

and the capacity is therefore

$$C = \frac{Q}{e} = \frac{K}{4\log_e \dfrac{D-r}{r}} \text{ electrostatic unit}$$

per cm. length of circuit, not of wire.

When D is greater than 10r the following formula may be used with an error of less than 1 per cent.

$$C = \frac{K}{4\log_e \dfrac{D}{r}} \text{ Stat-farads per centimetre}$$

$$=\frac{3.677\times 10^{-3}K}{\log_{10}\frac{D}{r}}\text{ microfarads per 1000 feet}$$

$$=\frac{k}{4\log\frac{D-r}{r}\times 9\times 10^{11}}\text{ farads.}$$

$$=\frac{k}{\left(4\log_{10}\frac{D-r}{r}\right)\times 9.\times 10^{11}}\text{ farads (electro-magnetic unit).}$$

When connected to a source of alternating E.M.F., the effective value of the charging current is $I_c = 2\pi f C.E.$ when E is the effective value of the line voltage.

The voltage is frequently taken from one side of the line to neutral, that is to the point of zero potential of the system. When this voltage to neutral is used, the capacity to ground or to neutral, is twice as great as the capacity between lines.

This follows, since $I_c = 2\pi f C_n E_n$ where C_n and E_n are capacity and voltage to neutral, and for single phase system, $E_n = \frac{E}{2}$. For three-phase system, $E_n = \frac{E}{\sqrt{3}}$,

$$C_n = \frac{k}{2\times 9\times 10^{11}\times\log\frac{D-r}{r}}$$

farads per cm. of line, since in using the material, the length of the line is the transmission distance.

Reducing values to practical units, note that, $\dfrac{0.0074}{\log_{10}\dfrac{D-r}{r}}$

is the capacity to neutral per 1000 ft. of line in micro-farads and $\dfrac{2\pi f C_n E_n}{10^6}$ is the charging current per 1000 ft. of line in ampere.

$$= \frac{19.41 \times 10^{-3}\ K}{\log_{10}\dfrac{D}{r}\left(\text{or } \dfrac{2D}{d}\right)} \text{ microfarads per mile.}$$

Example 4. What is the capacity of two parallel overhead bare No. 00 S.W.G. wires 18 inches apart and each one mile long ?

$$C \text{ per mile of line} = \frac{2 \times 19.41 \times 10^{-3}}{\log_{10}\dfrac{2\times 18}{.348}} \text{ micro farads.}$$

$$= 0.0193 \text{ microfarads.}$$

235. Capacity of each Conductor in a three-phase Circuit.—The capacity of a conductor in a three-phase system depends on its position relatively to the other conductors in the system.

(a) When the conductors are placed at the vertices of an equilateral triangle. From equation (A) the potential difference between any conductor (Fig 8.05) and the neutral plane halfway between the conductor and either of the other conductors is

$$E = \frac{2Q}{K} \log_e \frac{D-r}{r},$$

and the capacity of each conductor is

$$C = \frac{Q}{\frac{2Q}{K} \log_e \frac{D-r}{r}}$$

$$= \frac{K}{2\log_e \frac{D-r}{r}}$$ electrostatic units per cm. length of conductor

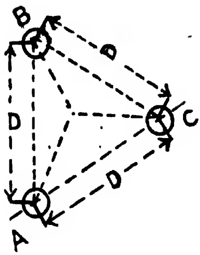


Fig. 8.05.

i.e., the capacity of each conductor in a three-phase system, when the conductors are placed at the vertices of an equilateral triangle is twice the capacity of the loop formed by any two of the conductors.

(b) When the conductors are in the same plane. From equation (A) the potential difference between

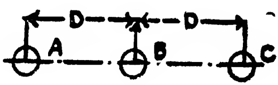


Fig. 8.06.

A or B (Fig. 8.06) and their neutral plane is

$$E_1 = \frac{2Q}{K} \log_e \frac{D-r}{r},$$

the potential difference between A or C and their neutral plane is

$$E_2 = \frac{2Q}{K} \log_e \frac{2D-r}{r},$$

and the potential difference between B or C and their neutral plane is

$$E_3 = \frac{2Q}{K} \log \frac{D-r}{r}.$$

As the conductors of polyphase systems are transposed, the capacities of the conductors are sensibly equal, and are found as follows :—

$$C = \frac{3\,Q}{\frac{4\,Q}{K}\log_e \frac{D-r}{r} + \frac{2\,Q}{K}\log_e \frac{2\,D-r}{r}}$$

$$= \frac{3\,K}{4\log_e \frac{D-r}{r} + 2\log_e \frac{2\,D-r}{r}}$$

$$= \frac{K}{2\log_e \frac{1.26\,(D-r)}{r}}$$ electrostatic units per cm. length of conductor

236. Single Round Wire Parallel to the Ground.—The capacity of the actual condenser formed by the wire and the earth (assumed equivalent to an infinite conducting plane parallel to the wire) is the same as the capacity to neutral of the fictitious condenser formed by the wire and its image, the distance between the two wires of this fictitious condenser being $D = 2H$. Hence using the approximate expressions, since the wire is practically always more than 10 times its diameter above the other, the capacity between the wire and the earth is

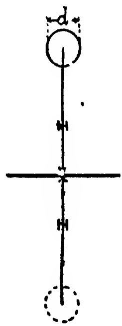


Fig. 8.07.

$$C = \frac{K}{2\log_e \frac{4\,H}{d}}$$ statfarads per centimeter

$$= \frac{7.354 \times 10^{-3}\ K}{\log_{10} \dfrac{4\ H}{d}} \text{ microfarads per 1000 ft.}$$

$$= \frac{38.83 \times 10^{-3} K}{\log_{10} \dfrac{4\ H}{d}} \text{ microfarads per mile.}$$

Example 5. What is the capacity of one mile of single overhead bare no 00 S.W.G wire 20 feet above the ground, with earth return? Take h and r both in inches.

Solution.—

$$C \text{ per mile} = \frac{38.83 \times 10^{-3}}{\log_{10} \dfrac{4 \times 20 \times 12}{.348}} \text{ microfarads}$$

$$= 11.2 \times 10^{-3} = 0.0112 \text{ microfarads.}$$

Capacity of a single conductor cable with ground metallic sheath.

$C = \dfrac{0{\cdot}03883\ K.}{\log_{10} (s/r)}$ m. f. per mile where s = intertnel radius of metallic sheathing and r = radius of conductor.

Examp e 6. What is the capacity of one mile of No. 00 S. W. G. lead covered cable with rubber insulation .15 inch thick? $K = 2.5$; $d = .348$ inch $\therefore r = .174$ and the external diameter of the insulation $D = .348 + .3 = .648$.

Solution.—

$$C \text{ per mile} = \frac{38.83 \times 2.5 \times 10^{-3}}{\log_{10} \dfrac{.648}{.174}} \text{ microfarads}$$

$$= .170 \text{ microfarads.}$$

237. Normal Capacity of a Two-Conductor Cable.—

$$C_{12}=\frac{K}{4\log_e\left(\frac{2a}{d}\cdot\frac{D^2-a^2}{D^2+a^2}\right)}\text{ statfarads per cm.}$$

$$=\frac{3.677\times10^{-3}\ K}{\log_{10}\left(\frac{2a}{d}\cdot\frac{D^2-a^2}{D^2+a^2}\right)}\text{ microfarads per 1000 ft.}$$

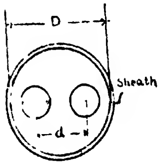


Fig. 8.08

238. Capacity of a Three-Phase Cable.— Capacity to neutral per 1000 ft. of line is given in microfarads.

If R = the radius of the surrounding sheath.

r = radius of one wire.

a = distance from the centre, or neutral point to the centre of one of the wires.

$$\frac{C_n}{1000}=\frac{0.0074}{\log_{10}\frac{\sqrt{3}\,a}{r}\cdot\frac{R^2-a^2}{\sqrt{R^4+a^4+R^2a^2}}}$$

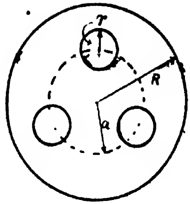


Fig. 8.09

239. Energy stored in the Dielectric Field — When the intensity of a dielectric field increases or decreases, a current i flows to or from the charged body.

Let D = Electrostatic density,

K = Specific inductance capacity,

i = the charging current,

P_c = the power delivered in the circuit.

W_c = the energy required to increase or decrease the intensity of the dielectric field.

$$i = \frac{dq}{dt}.$$

and $$q = Ce.$$

Therefore, $$i = C\frac{de}{dt}.$$

But $$W_c = \int P\,dt$$

$$= \int ei\,dt$$

$$= C\int e\,de$$

$$=\frac{C\,e^2}{2}\text{ ergs.}$$

When the applied voltage varies from e to zero.

Now $\qquad e = Fd$

and $\qquad C = \frac{A\,K}{4\,\pi\,d}$

Substitute the values of e and C.

$$W = \frac{A\,K}{4\,\pi\,d} \times \frac{F^2 d^2}{2}$$

$$= \frac{AKF^2\,d}{8\,\pi}\text{ ergs.}$$

But $\quad F = \frac{D}{K}$

and $\qquad Ad = V$ cubic centimetres.

Therefore, $W_e = \frac{D^2}{8\,\pi\,K} \times V$ ergs,

$$= \frac{D\;F}{8\,\pi}\text{ ergs per cubic centimetres.}$$

$$= \frac{D^2}{8\,\pi\,K}\text{ ergs per cubic centimetres.}$$

240. Charging and discharing Current in a Condenser Circuit.—If an electromotive force of constant value is applied to a circuit containing both resistance and capacitance, a charging current flows in the circuit.

Let E = the applied electromotive force,

R = the resistance of the circuit,

C = the capacity of the circuit,

$\frac{dq}{dt}$ = the rate at which the capacitance charges or discharges.

For a charging current,

$$E = \frac{q}{C} + R\frac{dq}{dt},$$

$$\frac{dq}{q-CE} = -\frac{dt}{RC},$$

$$\int_q^0 \frac{dq}{q-CE} = -\int_t^0 \frac{dt}{RC},$$

$$\log_e\left(\frac{q-CE}{-CE}\right) = -\frac{t}{RC},$$

and $$q = CE\left(1-\epsilon^{-\frac{t}{RC}}\right).$$

But $CE = Q$

and $$i = \frac{dq}{dt}.$$

Substituting.

$$E = \frac{q}{C} - Ri$$

$$\therefore \quad i = \frac{E - \frac{q}{c}}{R} = \frac{EC - q}{RC}$$

$$= \frac{EC - EC\left(1 - \epsilon^{-\frac{t}{RC}}\right)}{RC}$$

Therefore $i = \frac{Q}{RC}\left(\epsilon^{-\frac{t}{CR}}\right)$.

When a circuit containing both resistance and capacitance is disconnected from a source of constant electromotive force, and the circuit closed, a discharging current flows in the circuit.

For a discharging current,

$$\frac{q}{C} + R\frac{dq}{dt} = 0,$$

$$\frac{dq}{q} = -\frac{dt}{RC},$$

$$\int_q^Q \frac{dq}{q} = -\int_q^0 \frac{dt}{RC},$$

$$\log_e \frac{q}{Q} = -\frac{t}{RC},$$

$$q = Q\epsilon^{-\frac{t}{RC}}$$

$$i = \frac{Q}{RC}\epsilon^{-\frac{t}{R}},$$

241. Means of Reducing Capacity.—From the equations it is evident that the capacity of a given length of insulated conductor with metallic covering is decreased by (1) diminishing the dielectric constant of the, insulation, (2) increasing the internal diameter of the metallic covering and (3) reducing the diameter of its conductor.

In most cases the thickness of insulation is determined by its insulating qualities, and strength to withstand breakdown by electrical and mechanical pressures.

To reduce the capacity of overhead wires the distance between them and from the ground should be increased. The advantage gained by the reduction in capacity is small in comparison with additional cost for the increase in distance and is thus not economical. The method of balancing the reactance of capacity and inductance can be used to reduce the effect of capacity in electrical circuits.

242. Dielectric Hysteresis:—Like magnetic hysteresis this is a form of energy loss in dielectrics and is independent of any loss due to pure conduction. It has been experimentally shown that the energy (heat) dissipated in the dielectric of a condenser is greater with alternating current than when a constant difference of potential exists between the condenser terminals. This additional loss is due to dielectric hysteresis. The static Component of dielectric hysteresis probably is proportional to the 1·6th power of the maximum

dielectric flux density. The Viscous component, forming the predominating part, follows the square law. Hence the power loss in dielectric is proportional to the square of the flux density and the square of the frequency which corresponds to the viscous component of hysteresis. Electrostatic hysteresis losses generally amount to no more than a fraction of one per cent of the volt ampere input of the condenser at frequencies from 25 to 125 cycles and are much smaller than magnetic hysteresis when a condenser is connected to an alternating Current system. The changing value of the dielectric flux induces alternating electromotive forces in any conducting particles that may have become embedded in the dielectric. Dielectric hysteresis, therefore, is more nearly analogous to eddy currents than to magnetic hysteresis.

243. Factors Affecting Dielectric Strength.

(1) Internal or external heating,

(2) Chemical change,,

(3) Absorption of moisture,

(4) Nature of surrounding medium.

(5) Size and shape of test electrodes.

(6) Thickness of test specimen.

(7) Time rate of applying the disruptive voltage, whether continuous or alternating test pressure is employed.

(8) Order of magnitude of the test frequency.

244. Corona.—The Phenomenon of luminous discharge into the atmosphere consists of an electrostatic (leakage) discharge between wires of different potential, when this difference exceeds a certain critical value depending on the diameters of the wires and their distance apart. It may result in a considerable loss of power in which case it may be necessary to increase the conductor diameter or the effective size of the conductor but the power losses are negligible for voltages up to 45000.

Example 7. The voltage at the receiving end of a 50 cycle three-phase transmission line 14 miles in length is 6000 between the lines. The line consists of three wires of No. 0 S.W.G. (d=.82 cm.) 30″ apart, and of sepecific resistance $= 1.8 \times 10^{-6}$.

What is the charging current of the line, how many volt-amps. does it represent, and what percentage of the full-load current of 50 amperes is it ?

Solution.—

If l = length of the line per wire,

l_s = distance between wires,

d = diameter of transmission wire,

then, the capacity per wire

$$C = \frac{.24 \times 10^{-6} \times}{\log_{10_e} \dfrac{2\, l_s}{d}} \text{ m}f.$$

Here, $l = 2.23 \times 10^6$ cms., $l_s = 30 \times 2.254$ cms., d = .82cm.

$$\therefore \quad C = \frac{.24 \times 10^{-6} \quad \times 2.23 \times 10^{6}}{\log_{10} \dfrac{2 \times 30 \times 2.54}{.82}} = .236 \text{ m}f.$$

The charging current per line is given by $I = 2\pi f CE$ 10^{-6} amp., E representing the voltage (effective) between the line and the neutral.

$$\text{Thus, } I = 2 \times 3.14 \times 50 \times .236 \times \frac{6600}{\sqrt{3}} \times 10^{-6}.$$

$$= .28 \text{ amp.}$$

or, .56 per cent. of full-load current ;

(hence, its influence on the transmission voltage is negligible). The volt-ampere input of the transmission is

$$\sqrt{3}\ I\ E = \sqrt{3} \times .28 \times 6600$$
$$= 3200$$
$$= 3.2 \text{ Kv. amperes.}$$

Exercises

1. Determine the capacity of a pair of No. 0000 S. W. G. line wires, two feet apart, and three miles long.

2. If a circuit having a resistance of 12 ohms and a capacity of 10 microfarads has a constant E. M. F. of 220 volts impressed upon it, how long will it take for the current to sink to half its initial value ?

3. Determine the energy which can be electrically stored in a cubic inch of mica dielectric when the applied potential is 440 volts per mil thickness.

4. Find the current produced by a 5o ∾ alternating E. M. F. of 220 volts in a circuit having 20 ohms resistance and a capacity of 40 microfarads. What is the power factor of the circuit ?

5. It is desired to construct a condenser of crown glass plates 10×12 inches so that the power factor of its circuit having 10-5 ohms resistance shall be 90 percent for an oscillatory current of 40000 cycles. How many plates will be required if the thickness of each is .12 inch?

6. Determine the instantaneous value of a 50 ∾ alternating current 4.73 seconds after impressing a harmonic E. M. F. of 220 volts (effective) upon a circuit having a resistance of 50 ohms and a capacity of 20 microfarads.

(7) It is desired to connect a single 50-volt 30-watt lamp to 110-volt 50-cycle mains by means of a condenser. Calculate the capacity of the condenser required and the power-factor of the load when the lamp is lighted. (C. and G., II)

(8) Calculate the capacity current of a concentric cable 10 miles long, 0.25 mfd. capacity per mile, when supplied at 6600 volts 50 ∾.

If a non-inductive load of 100 kilowatts comes to the cable, what is the new value of the cable current?

(9) Calculate the capacity current taken by a concentric cable 15 miles long, 0.5 mfd. per mile, when supplied at 6600 volts 50 ∾.

A non-inductive load of 100 kilowatts comes on to the cable. Find the total current now taken.

What difference would it make if the load were inductive ?

(10) A. P. D. of 2000 volts, 50 cycles per sec., is applied to test a cable of 14 mfd. capacity through a resistance of 110 ohms. Find the current which will be taken from the supply, and the phase difference between the current and the impressed P. D.

(C. and G., II).

(11) Prove the relation between the capacity, resistance, and inductance when resonance is established in an oscillating circuit. A circuit has a capacity of 0.003 microfarad and an inductance of 0.011 millihenry. Calculate the frequency at which resonance will take place. (C. and G., II).

(12) What is meant by resonance in an electrical circuit ? A condenser of 1.5 microfarads capacity and a variable choking coil of 15 ohms resistance are connected in series to a 50-cycle 100 volt supply, the wave shape of which has a strong third harmonic. What value of the inductance will give resonance (a) with the third harmonic, (b) with the fundamental frequency ? (C. and G., II).

(13) How many sheets of mica 0.1 mm. thick separating plates of 200 sq. cm. area required to construct a condenser of 1 microfarad capacity (the specific inductive capacity of mica is 6) ?

If plates are 0.1 mm. thick what are the dimensions of the condenser ?

(14) A condenser of 100-microfarad capacity is connected across a 220 volt line :—

(a) What current will flow at 25, 50 and 120 cycles ?

(b) What is the average power taken from the line in each case ?

(c) What is the maximum rate at which energy is given to the condenser during one half cycle and returned by the condenser to the line during the next half cycle ?

(d) What is the maximum charge in the condenser on each half cycle ?

(e) What average current would this maintain in an external circuit for 1/480 sec. ? What effective current if the current changes as a cosine function (compare with a) ?

(f) Explain, without formulae, why the current is directly proportional to the frequency, the voltage being constant ; the reverse is the case for an inductive circuit.

(15) A condenser of 100-microfarad capacity and a non-inductive resistance of 10 ohms are connected in series across a 220-volt line :—

(a) What current will flow when the frequency is 25, 50 and 120 cycle ?

(b) What is the average power taken from the line in each case ? Explain this is equal to $I^2 R$.

(c) What is the voltage drop across each part of the circuit at 60 cycles ?

(d) What is the power factor of the circuit in each case ?

16. Explain the effect of applying to the terminals of a condenser, the resistance of which is many megohms, an alternating electromotive force. What electromotive force will be required to drive 10 virtual amperes through a circuit containing a condenser of which the resistance is 1200 megohms, and its capacity 22 microfarads, the frequency of supply being 80 periods per second. (Ord., A. C., 1908).

17. Give the physical (non-mathematical) reasons why a condenser produces a lead, and why a self-induction produces a lag, in an alternating current. At what frequency will a capacity of 1 microfarad, and a self-induction of 1 henry exactly annul one another's effects ? (Ord A. C. 1910).

Page 97

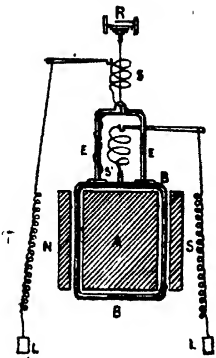


Fig. 3.07

Page 133

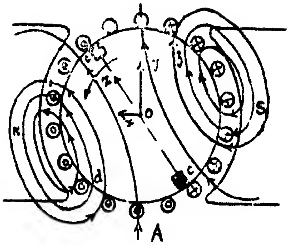


Fig. 4.211

Page 206

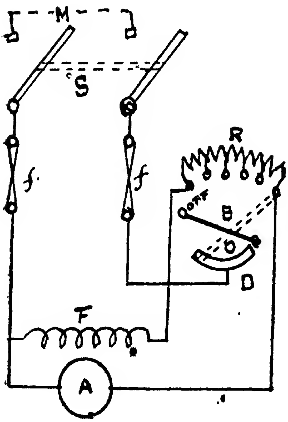


Fig. 5.031

Page 265

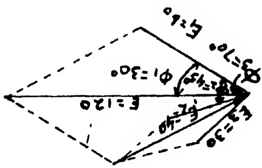


Fig. 6.12

Page 266

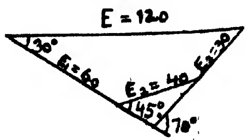


Fig. 6.13

Page 300

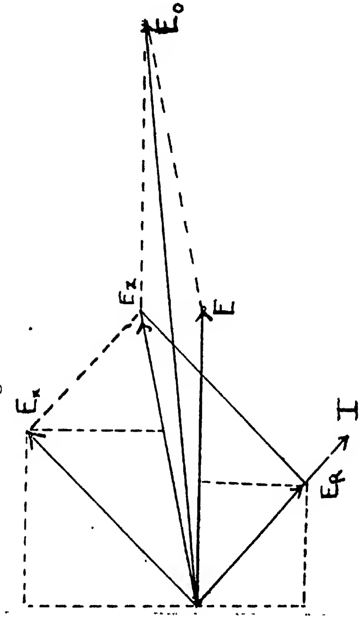


Fig. 6.33

Page 110

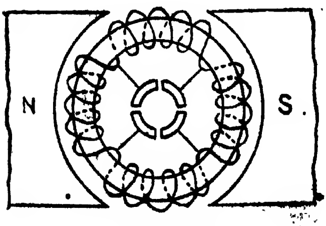


Fig 4·051

Page 276

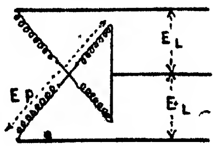


Fig 6.16

INDEX

(The References are to Pages)

Magnetic circuit.	Dielectric circuit.	Electric circuit.
Magnetic flux (magnetic current): ϕ = lines of magnetic force.	Dielectric flux (dielectric current): ψ = lines of dielectric force.	Electric current: i = Electric current
Magnetomotive Force: $F = ni$ ampere-turns.	Electromotive Force: e = volts.	Voltage: e = volts.
Permeance: $B = \frac{\phi}{4\pi F}$	Condensance, capacitance permittance or capacity: $C = \frac{\psi}{e}$ farads	Conductance: $g = \frac{i}{e}$ mhos
Inductance: $L = \frac{n^2\phi}{F} 10^{-8}$ $= \frac{n\phi}{i} 10^{-8}$ henry.	Elastance: $S = \frac{1}{C} = \frac{e}{\psi}$	Resistance: $r = \frac{e}{i}$ ohms
Reluctance: $R = \frac{F}{\Phi}$		
Magnetic Energy: $w = \frac{Li^2}{2} = \frac{F\Phi}{2}$ 10^{-8} joules	Dielectric Energy: $w = \frac{Ce^2}{2} = \frac{e\psi}{2}$ joules	Electric Power: $P = ri^2 = ge^2 = ei$ watts
Magnetic density: $B = \frac{\Phi}{A} = \mu K$ lines per $\overline{cm}^2$.	Dielectric density: $D = \frac{\psi}{A} = kK$ lines per $\overline{cm}^2$.	Electric-current density: $I = \frac{i}{A} = \gamma G$ amp. per $\overline{cm}^2$.
Magnetizing Force: $f = \frac{F}{l}$ amp.-turns per cm	Dielectric gradient: $G = \frac{e}{l}$ volts per cm.	Electric gradient: $G = \frac{e}{l}$ volts per cm.
Magnetic-field intensity: $H = 0.4\pi f$.	Dielectric field intensity: $K = \frac{G}{4\pi v^2}$	
Permeability: $\mu = \frac{B}{H}$	Condensivity, permittivity or specific capacity. $k = \frac{D}{K}$	Conductivity: $\gamma = \frac{I}{G}$ mho-cm.
Reluctivity: $\rho = \frac{f}{B}$	Elastivity: $\frac{1}{k} = \frac{K}{D}$	Resistivity: $\rho = \frac{1}{\gamma} = \frac{G}{I}$ ohm-cm.
Specific magnetic energy: $w_0 = \frac{0.4\pi\mu f^2}{\cdots} = \frac{fB}{\cdots}$	Specific dielectric energy: kG^2 GD	Specific power:

APPENDIX

P. 83.

This law of Stienmetz does not hold good for high values of B, the inaccuracy beginning to be important when B is about 10,000 lines per sq. cm. The most recent results point to an index of 1.7 for armature iron and lower values for cast-steel and cast-iron.

P. 110.

A revolving armature alternating current can be arranged from any direct current generator by fitting suitable collector rings mounted on proper insulators.

P. 114.

Disadvantages: --(2) It is liable to become reversed in polarity owing to the failure of a Prime-mover, and be driven as a motor. It may then be short circuited by a Reverse Current Circuit Breaker.

Ideal Prime-movers give a constant torque such as a steam engine with fixed cut off and constant steam pressure.

In series dynamo the speed and therefore the pressure at terminals would adjust itself to suit the motor load. Regulating devices are provided to maintain constant current. Regulators controlled by the main current or part of it can be made to act on mechanism designed to vary the speed of the Prime-movers. Where the speed ofthe prime mover cannot be varied, automatic

devices are made to alter the magnetic field cut by the armature conductors (1) by shunting a portion of the main current, (2) by shifting the position of the current on the commutator.

The first method leads to sparking troubles and greater armature reaction due to the weakening of the field, so it is inadvisable to shunt more than 1/3 of the total current.

P. 116.

In the case of series or compound dynamo the polarity of the field would be probably reversed and serious results occur.

P. 118.

Art. 62 :—After 'within a certain limit.' In shunt dynamo it will have probably no ill effect as the shunt dynamo would run as a motor in the same direction as before. Generally a reverse current cutout is installed in the charging circuit,

P. 121.

Art. 66 :—After 'within its range.' If all the power is used so near the busbar that there is no considerable line drop the generator is flat compounded.

P. 122.

After "from the generator" so that the generator voltage at full-load is higher than at no-load, thereby compensating the line drop and maintaining constant voltage at the load.

P. 134.

1. Approximate:—**Rating from dimension and speed** : –

$P = Kld^2n$

where l = gross length of armature core in inches.

d = external diameter of the armature core in inches.

n = speed in revolutions per minute.

P = power rating of the motor in Power House.

k = a factor which ranges from ·00002 for a 5 H. P. motor to .00004 for a 50 H. P. motor.

The rating of an enclosed motor is about 65 per cent of that of an open motor of the same size and speed.

For generators in Kw rating, k ranges from

0.000015 for a 5 kw. generator

to 0.000030 for a 50 kw.

0.000032 for a 200 kw.

50°/。 overload for one hour

25°/。 " " three hours.

P. 142.

Note that generators are assumed to have their highest efficiency at 75 per cent. of their rated output. The highest efficiency of prime movers should be at the same point but the prime mover should be capable of driving the generator at 50 per cent above the most economical output so as to allow for emergent short period overloads.

P. 169.

2. The Chief advantages of D. C. Motors:—

(1) The possibility of automatic speed variation of series and compound motors without excessive power lost in rheostats.

(2) The possibility of speed adjustment in shunt motors by field rheostats, thereby obtaining at high efficiency any number of speeds that remain constant at the required values, at all loads.

(3) The absence of reactance in line drop.

(4) The possibility of connecting motors to the same circuit as storage batteries or other electrolytic apparatus.

P. 171.

Art. 108. A shunt machine will not operate at all if rotated in the wrong direction.

Art. 109. A shunt machine will operate either as a generator or as a motor with the same direction of rotation and this is true of the compound machine. When the machine changes from a generator to a motor the current in both armature and series field reverses. If the machine is cumulative compounded it will work as a differentially compounded motor which last is not generally used. Thus if we want to run a compound generator as a motor it is generally necessary to reverse the connections of its series field. Series motor with a given connection and direction of rotation will never act as a generator. Thus to act as a generator a series

motor, the series machine must rotate in the oppsite direction to its rotation as a motor or else the connection of its field or armature must be reversed.

P. 176.

If when the machine is operating at such a speed that the voltage generated by the machine is the same as that of the line, and (as a consequence the current is zero), the field is strengthend by cutting out some of the field resistance in the field circuit, the voltage of the machine will exceed that of the line. The machine will then force current in the direction of its own e. m. f. or it will become a generator. A further increase in the field strength will furnish still more current.

P. 186.

Art. 118 (a) After:—"requiring a steady speed", driving of line shafts to which a number of machines are belted, the operation of individual machine tools such as lathes, drill presses, planers, milling machines, and where heavy overloads are not common the shunt motor if driven above normal speed will act as a generator and return power to the line. This property may be exploited to return power while the car is descending hills and in some cases the use of shunt motor in cars may thus be justified.

P. 187.

(b) 1st para last lines—It is a constant power machine to a certain point hence it handles torques that would be too great for a shunt wound motor, and light-

load is automatically hoisted rapidly and a heavy one more slowly.

(c) 1st lines—The series coil in a compound motor may be wound to assist the shunt or to oppose it. Where a larger....................maximum, where close speed regulation is not necessary,..............selected. Punch press, compressers for refrigerating plant crushers.

P. 188.

After 'winding.'—It is standard practice with stock motors to supply approximately 15 per cent as many series ampere turns as shunt ampere turns. The motor has a limiting speed as a shunt machine and if driven above this speed it will act as a generator. It, however, slows down more than a shunt motor under load.

P. 188.

Art. 119.—The total Electrical power VI Supplied to the motor is utilised in (a) to Supply the power lost in heating the field magnets and to Supply the armature copper loss $R_a I_a^2$ and (b) to force the Current through the armature in opposition to the induced E. M. F. e The part (b) is equal to eI_a and according to Lenz's law it is all converted into actual mechanical Power which supplies the Stray power loss and the useful Power delivered at the pulley. The ratio of the Mechanical Power eI_a developed in the armature to the total Electrical Power VI delivered to the Motor is called the efficiency of conversion or electrical efficiency of the motor.

$$= eI_a/vI_a$$

The ratio of the useful power delivered by a motor to the total Mechanical Power developed in the motor armature is called the mechanical efficiency of the motor,

$$= (I_a e - w)/I_a \times e$$

The commercial efficiency of a motor is equal to the product of its efficiencies of conversion and its mechanical efficiency.

P. 205.

Art. 130 (1 –)—After 'current.' A series motor with certain alterations to prevent excessive losses and heating may be operated on alternating current.

(2 –)—After 'current.' The direction of rotation is not reversed by reversing the applied voltage. A shunt machine will not generate at all if rotated in the wrong direction.

P. 219.

3. Protections against overspeed of a series motor :—

(1) Gearing or other positive connection to a load that cannot be less than a safe minimum.

(2) An operator who is present, controlling the speed whenever the motor is running.

(3) Sufficient resistance in the motor circuit to keep down the back E. M. F. and therefore the speed of the machine. This makes the machine inefficient and the speed less, specially at heavy loads. It is only suited to small motors.

P. 219.

Art. 140. Add first Art·140 :—It is in danger of running away at light loads.

P. 220.

Art. 141. (1) 2nd para last :—The speed regulation at any speed less than maximum is poor with varying speed.

(2) 2nd para last :—The speed regulation is good because the armature is connected directly across a source of constant e. m. f.

P. 222 and 223.

The speed of a series motor at a given load may be varied by varying the resistance in the motor circuit, if the load on the motor be constant the current drawn from the line will be constant, regardless of the speed.

P. 223 and 224. Shunted Armature Connection :—

If a resistance is placed in parallel with the armature of a series motor the motor will operate at less than the normal speed when all the starting resistances have been cut out. It is useful where a low speed is desired at light loads, and is particularly useful where the load becomes a negative one, i. e., where the load tends to over-haul the motor, as in lowering a heavy load.

P. 228.

4. Specification of D. C motor : (1) What D. C. voltage is available and whether it is a two-wire or threewire system. (2) Where the motor is located (i.e. enclosed or open type). (3) What is the maximum load

on the motor and how the load varies. (4) What automatic variation of speed is desired or allowable. (5) What speed adjustment is required to be made by hand at the will of the operator under various conditions.

P. 240.

Ex. 33. A certain 15-h. p., 220-volt shunt motor with a speed of 1200 r. p. m. has a stray power loss of 800 watts. The resistance of the armature is 0.02 ohm. Find the value of the armature current for which the armature efficiency is a maximum and compute the armature efficiency at this current.

Ex. 34. A 220-volt D.C. shunt motor when running light, i. e., with no load except the losses in the armature takes an armature current of 15 amperes and rotates at a speed of 700 r. p. m. what will be its speed when the output is 45 h. p.? The losses in the field may be neglected and the resistance of the armature taken as 0.03 ohm.

35. A certain motor has an armature resistance of 0.05 ohm. The applied E. M. F. is 230 volts. The full-load speed with 50 amperes in the armature is 1200 r. p. m. At what speed would the motor have to run to take zero current from the line? If the applied E. M. F. is reduced 10 per cent. What will be the speed if the armature current is zero, it being assumed that the field is unsaturated so that the change of flux is proportional to the change of voltage. What will be the full-load speed?

P. 242.

Disadvantages:—After "specially in the case of a breakdown.' Its inherent voltage regulation and its efficiency are not so good as those of a direct current system where electricity is to be distributed for lighting only and not at a greater distance than about a mile from the generating station. D. C. will be most satisfactory and economical.

P. 243.

Art. 149. After 'as direct current.',:—Polyphase constant speed motors are preferable to D. C. motors because they are simpler and cost less to maintain than D. C. motors.

P. 243.

The A. C. fan is somewhat less advantageous to the consumer in the matter of efficiency while it costs more to instal. From the supply company's point of view the low power factor of the single phase fan is a very serious disadvantage.

P. 258.

The maximum value of the current strength attained during the cycle is called THE AMPLITUDE OF THE CURRENT.

P. 260.

After ∴ Crest factor

or Peak factor = ...

P. 268.

After (4) :—

Reactive component is objectionable for :—(1) The greater the reactive component the less is the maximum working load that may be put upon the alternator for the total apparant power (volt amperes) developed with a given E. M. F. is limited by the maximum current the armature conductor can carry.

(2) If the current lags, it tends to demagnetize a field, to compensate for this a stronger field is necessary, and hence more power is needed for excitation.

(3) Owing to the greater current for a useful load, the cross-section of the cable has to be increased to keep the voltage drop and temperature rise within permissible limits.

P. 270.

Power factor due to capacitance applies to capacitive circuit as that due to inductance applies to an inductive circuit. The power factor due to capacity is leading while that due to inductance is lagging. The former is rarely met with in circuits excepting circuits of very high voltages but they can be produced by using an over excited synchronous motor or condenser.

P. 277

If three wires are used and the ammeter is placed in either outer and the voltmeter across either outer and the the common wire the current in the common wire is

that carried by the two outers. Those not being in phase the resulting current is not twice but $\sqrt{2}$ times that carried by outers.

Fig. 6. 17:— $AA = 100$
$AB = 70.7$
$BB = 100$
$BA = 70.7$

P. 295.

Art. 168. After 'of equal capacity:—an ammeter in any one line giving half the current supplied and the product of the ammeter reading and the voltmeter across either pair of lines giving half the apparent watts supplied.

P, 304.

Art 174.:—(1) The two-phase, has the advantage over three phase in requiring only two transformers, and in having two phases that are insulated from each other and independent in their operation.

(2) The two-phase three-wire system, has the advantage over four-wire system, that the wiring is simplified there is a small saving in copper.

(3) The three-phase system has the advantage over the two-phase three-wire that the three voltages between lines are equal, and all conductors are of the same size.

P. 315

5. To start a single alternator.—The following steps should be observed:—

(1) See that there is sufficient oil in the beaings, with the oil rings free to turn; that the switches are open.

(2) Start the exciter and adjust for normal voltage. Then start the alternator slowly. See that the oil rings are turning.

(3) Allow the machine to reach normal speed. Turn the alternator field rheostat so that all of its resistance is in the field circuit. Close the field switch.

(4) Adjust the rheostat of the exciter for the normal exciting voltage. Then slowly increase the alternator voltage to normal by cuttting out the resistance of the field rheostat.

(5) Close the main switch.

P. 320

6. Satisfactory parallel operation is shown by:—(1) correct divsion of load amongst the machines, (2) freedom from hunting.

P. 323.

Art. 189-Last lines:—The use of solid pole leads to excessive eddy current losses, hence solid poles are not considered desirable.

P. 324. Art. 190. (After No. 4.)

Caution.—The field circuit of a generator to be disconnected from the bus-bars must not be opened before the main switch has been opened; for, if the field circuit be opened first, a heavy current will flow between the armatures.

P. 333.

After 'is called SKIN EFFECT.'—It depends upon the shape of the section of the conductor as it is

greater in a circular than in a rectangular cross section of a conductor. It is due to variation of inductance from the surface to the centre of the conductor.

P. 350.

Choke coil of lightning arrester must be very highly insulated.

Iron cores do not add to the damming action of a choke coil in the case of excessively quick rushes of current which accompany lightning discharges for the reason that the iron does not have time to become magnetized. Hence lightning arrester choke coils are always made without iron cores.

P. 420.

7. Effect of Alternating Current on insulation :—(1) The maximum value to which the insulation is subjected in A.C. circuit is $\sqrt{2}$ times the effective value.

(2) In A. C. the stress in the insulating material is reversed and so the material becomes fatigued and so becomes less effective. The dielectric offers a certain opposition to the setting up of alternating charges. This is known as dielectric hysteresis.

P. 421.

The capacity phenomenon is the same whether the capacity is in series or in parallel, only the numerical results differ.

P. 425.

Power is not lost in impelling an A. C. in a circuit containing a condenser only for all the energy expended

in displacing the electricity in one direction and in stressing the dielectric of the condenser as the voltage increases during the first half of the cycle, is returned to the circuit during the last half alternation of a cycle when the voltage is decreasing, as the dielctric exerts its elasticity and pulls back into an unstressed condition. Very small losses of energy occur owing to Dielectric Hysteresis.

P. 448.

The alternating stress in an insulating material is accompanied by a small loss of power. The dielectric offers a certain oppsition to the setting up of alternating charges. This is known as Dielectric Hysteresis.